Students' language repertoires for prediction

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For communication about prediction (both relating to probability and to conjecture), language is by nature recursive – language is an indicator of meaning as well as a force that shapes meaning. We describe how this recursive nature of language impacted the choices we made in a cross-sectional longitudinal study aimed at gaining insight into children's language repertoires relating to conjecture. We then use some of the data from the project to identify issues relating to interpreting data in such a context. Finally, we raise questions about implications for educators.

Keywords: Language, mathematics education, prediction, conjecture.

INTRODUCTION

The understanding of possibility, risk, and certainty, like the understanding of any mathematics, is mediated by language. Certain language repertoires are necessary to convey the ideas. At the same time, the language used to describe risk shapes the way people conceptualize it. This recursive nature of language compelled us to develop a research project to investigate children's language repertoires in relation to conjecture. Having noted similarities in the language of conjecture and of prediction, we structured classroom and interview prompts to encourage students to make predictions and we talked with them about the meaning of the things they said. In this paper, we focus on research choices in relation to this endeavour. First, we describe choices we made to gain insight into children's language repertoires. Second, we use some data from the project to identify issues relating to interpreting data in the characteristically mathematical contexts of conjecture and prediction.

Moving beyond our academic interest in mathematics education, we will argue that the issues we identify may be significant for understanding everyday experience. In particular, we will raise questions about the impact of mathematics class experiences with uncertainty. We will also raise questions about the impact of intertextuality between uniquely mathematical ways of communicating about conjecture and everyday ways of interacting about authority.

COMMUNICATING ABOUT UNCERTAINTY

The investigation of conjectures (hypotheses) is one of the most important mathematical processes. Much mathematics teaching focuses on enabling students to perform particular mathematical procedures, such as adding fractions, factoring polynomials, and calculating probability. These skills appear as standards in curriculum documents and frameworks (e.g., CCSSO, 2010) that are used by curriculum planners and teachers. Research and professional literature, including curricula (e.g., New Brunswick Department of Education, 2010) and curriculum frameworks, point to the necessity of students learning these intended outcomes through the exploration of mathematical problems.

When people explore a mathematical problem together, as with mathematical investigations in classrooms, it is necessary to have a way of suggesting an idea before knowing it is true. Rowland (2000) noted the centrality of such conjecture to mathematics, and coined this "space between what we believe and what we are willing to assert" (p. 142) as the Zone of Conjectural Neutrality (ZCN). Because of the recursive relationship between language and experience, the language resources available affect the possibilities for making conjectures.

Our theoretical perspective for this research draws on the work of Vygotsky and Wertsch related to the connections between thought and language, and, in particular, the central role that language as social interaction plays in the process of learning. Nevertheless, we have found it a challenge to avoid deficit framing because of the shaping force of one's language repertoire. Deficit framing suggests that one's own way of speaking or thinking is superior by evaluating whether or not others have acquired the same skills. In the study of linguistic variation for numbers, the one area of mathematics register variation that has been documented significantly, Swetz (2009) pointed out how cultures have been rated on the extent of their number systems. In our research we are more interested in the potential for linguistic variation to open up opportunities to understand mathematics differently – for example, how does thinking of numbers as verbs (e.g., Lunney Borden, 2010) change one's conception of counting and arithmetic operations? In our case, how does linguistic variation express itself in relation to understanding probability? Because linguistic variation in mathematics (besides the area of number) has not been researched significantly, the discussion requires careful research to move forward.

Our focus in the article is on participants' repertoires for expressing modality. Modality refers to linguistic tools for expressing degrees of certainty, for example the use of modal verbs like *must* and *could*. "It must be six" is stronger, and thus has higher modality than "It could be six." Some modal verbs—e.g., 'can'—are ambiguous. "You can be excused from the table" indicates a degree of obligation; "You can finish the race" indicates ability; "I can help you" indicates inclination; and "It can be a six (because one of the remaining cards in the deck is a six)" indicates probability. When students hear the word *can*, what does it mean to them?

In an example of research that may appear deficit framed, Shaffer (2006) explained how deaf children with hearing parents did not develop what she called "The Theory of Mind" because of the absence of modality in their vocabulary. Theory of Mind relates to the understanding that different points of view are possible. Linguists Martin and Rose (2005) described the effects of modality (sometimes called 'modulation') this way: "it opens up a space for negotiation, in which different points of view can circulate around an issue" (p. 50) – a description that bears close resemblance to Rowland's ZCN. Shaffer reported that once the children developed vocabulary for modality (in American Sign Language) it became clear that these tools facilitated their quick development of this Theory of Mind.

We are especially interested in the way children use language to express modality in mathematics contexts (and beyond) because modality is important in conjecture (Rowland, 2000), to describe uncertainty, and to understand other points of view (Shaffer, 2006). In our research, we did not aim to look for holes in children's language repertoires. Rather, we focused on attending to the ways they talked about their understanding, to help us see a range of ways to talk about and understand conjecture and uncertainty.

METHODOLOGICAL CHOICES

The data for our cross-sectional longitudinal study comprise audio- and video-recordings from English-medium and French Immersion instructional contexts in an Anglophone region in Canada. Students worked in groups in class and were subsequently interviewed, extending the group work. At the end of the interviews we asked students about the meaning of words they used to describe degrees of certainty.

For each mathematical context we tried to avoid using specialized mathematical language ourselves. We know from second language acquisition literature that learners are generally good at noticing and, subsequently, using the language used in interactions with more able speakers (Long, 1996). We wanted to hear what language skills the children in our research used to communicate their ideas without setting them up with the specialist language to build on. As we struggled to construct problems without use of specialist language, we found that larger narrative contexts made this possible. Other strategies we considered became grammatically awkward. These narratives also made the problems accessible to very young children, perhaps partially because of the lack of specialty language, but mostly, we think, because they connect to children's experience. In addition to embedding our questions in a narrative context, we attempted to avoid specialized uncertainty language when we interviewed participants about their predictions in contexts based on uncertainty. We agreed it would be acceptable to use a language strategy after the participant did, but not before. This proved very difficult; indeed, in the interviews we often used words we intended to avoid, and sometimes used incorrect or awkward structures in attempts to avoid this. After completing most of the first year's classroom and interview interactions, we agreed amongst our team that we should be less paranoid about avoiding specialty language, but knew that this issue would plague interpretation of the data (from before and after our decision to loosen up.)

The first year's participants were in Grades 3, 6, and 9. We had them play a modified version of skunk, which is a game often used in the teaching of probability (e.g., Brutlag, 1994; Neller & Presser, 2004). We had them play in pairs so that they would be more likely to talk with each other about their ideas and strategies. We introduced the game with a narrative like this, varying slightly between contexts because we did not script the narrative: "I was picking strawberries in the forest. After a while, when my basket was quite full, a skunk wandered into the berry patch. I ran away so the skunk would not spray me. I lost the berries in my basket when I ran off." (This narrative also gave a reason for calling the game *skunk*.) Participants had a pile of beans (representing the berries), a cup (the basket), and a bowl (home). When the researcher rolled the die and called out the number, participants put that number of berries in their basket. A six represented the skunk. When it was rolled, everyone would lose the berries in their baskets. On the other hand, if they "went home" (dumping their beans into their bowl) before the appearance of the skunk, their berries were safe. We played seven rounds – one berry-picking expedition for each day of the week. We played the game with participants in their classrooms first. The following day we interviewed groups of students and played again but with six cards bearing the numbers one to six instead of the die. The interviewer would not replace the cards into the deck until the deck was completely played out, at which time it would be reshuffled. Thus the participants experienced the difference between independent and mutually exclusive events in probabilistic situations. During the game, the interviewer would ask the participants to say why they made their choices about when to "go home." After the game, the interviewer would ask participants about specific things they said, asking for clarification on meaning. The camera operator was helpful in this regard, acting as a second interviewer. She or he could make notes on what participants said, which was relatively difficult for the primary interviewer who was busy with the cards and interaction.

In the second year, participants from Grades 4, 7, and 10 (catching some of the same students as the previous year, one grade earlier) predicted the 50th car on different trains based on the first seven cars. The narrative context of this situation had the researcher tell a story about waiting with a friend for a train at a level crossing, and deciding to predict what kind of car the fiftieth car would be. Trains were then shown using presentation software, with an engine and the first six or seven cars, each labelled with their number. After students made their predictions about the 50th car, we had the train accelerate and then decelerate to settle on the 50th car. As with the game of skunk, we had students work in groups to draw out communication.

The sequences presented to students varied considerably to defy expectations of certain kinds of patterns. The cars were distinguishable by colour and shape – Yellow (Y) cars were rectangular boxcars, green (G) cars were tankers, and blue (B) cars were flatbeds carrying big triangles. Train 1 showed Y,G,B,Y,G,B,Y and continued with a pattern of threes (YGB). Train 2 showed Y,G,Y,Y,G,Y and continued with a pattern that increased the number of Ys before each G – i.e., Y,G,Y,Y,G,Y,Y,Y,G, etc. Of course, the initial seven cars could have suggested a pattern of threes (YGY) similar to the previous train – i.e., Y,G,Y,Y,G,Y,Y,G,Y, etc. For this train, we stopped the train at around the 25th car to let students reconsider their predictions. Of course, we invited students to tell us their reasoning whenever possible. Train 3 showed B,G,B,B,G,G, etc. and continued with B, B, B, G, G, G, etc. with increasing groups of B and G. The interviews on the following day started with Train 4 showing Y,B,G,Y,Y,B,G. It continued with groups of four (YBGY) – i.e., Y,B,G,Y,Y,B,G,Y etc. Train 5 started with Y,B,G,B,P,B,Y and continued with a random collection of cars, in which the colours started to misalign with the shapes and new kinds of cars appeared. As with Train 2, we stopped train 5 at around the 25th car so we could hear the students reconsider their predictions. In addition to the confounding randomness of the fifth train, there was no 50th car – it had only had 42 cars. As with skunk, we ended these interviews with questions about distinctions among various language choices we heard the students use.

For this paper, we focus on one interview with four Grade 6 students playing the game of skunk. However, we make some references to other data within the project to illuminate certain findings through comparison. This group of students was not identified by their teacher as exceptional in any way. The school is in an area that has relatively low socio-economic indicators. As noted above, these four students played skunk in class the day before, and subsequently one of our research team interviewed them – first playing skunk with cards instead of a die, and then asking them about some language meanings. We asked them to play skunk in pairs, and they somehow came to an implied agreement that the pairs were competing against each other.

LANGUAGE USED TO EXPRESS UNCERTAINTY

These four 11- and 12-year olds show considerable language repertoires, which we found to be the case for even the most mathematically and linguistically novice students in this project. We were the most careful about and attentive to modal verbs because of our earlier research and teaching work.

The modal verb *have* expresses high modality because it refers to events that must occur. The interviewer used it first (though trying to avoid doing so) in turn 111 and it was used again in turn 229 when Chris talked about the difference between playing skunk with cards and with the die: "It's easier this way because when the skunk first came you just don't have to worry." It wasn't used again until the interviewer asked questions about its meaning. Here is a short version of that discussion.

319 Interviewer: [Yesterday] I heard Terry say when you're working in your groups, "Do we both have to write this down?" So what's the difference between "it has to be the skunk" and "she has to write this down"? Is the "has to" the same? "This has to be the skunk." "She has to write it down." Do you notice a difference between them? ...
346 Terry: Do we both need to, like, do we both need to write it down?
347 Interviewer: No, but it's a proper use of the word. But is it the same as "this has to be the skunk"?
348 Terry: No.
349 Interviewer: No? Why not?
350 Terry: Because you know it has to be.
351 Dale: It absolutely has to be.
352 Interviewer: It absolutely has to be.
353 Terry: Yeah
354 Interviewer: But when asking "do you have to" it's not absolutely
355 Terry: No, yeah
356 Interviewer: Okay
357 Dale: Because the fire bell or something could ring or something and you all go outside and you don't have to write it down.
358 Interviewer: Don't have to write it down but if the fire bell rung this would still be the skunk.
359 Dale: It would still be the skunk.

We note that, to clarify meaning, the students introduce new vocabulary that was not part of the interview up to this point. Terry used the modal verb structure "need to" to emphasize the necessity of "have to." Dale introduced the adverb *absolutely* to further emphasize this sense. The students distinguished between instances of 'have to' depending on context.

We had a similar conversation about the modal verb *can* which had been used in its various forms, including *can't*, by the students in the interview. We started this part of the conversation by referencing Dale's writing in class earlier. When asked what is the most number of berries they could get in a day, Dale wrote, "You can get any number because it could just keep going." (This was with playing skunk with a die) The researcher referred to Dale saying in the interview that it is different with the cards because "we can't keep going." What follows is again a short version.

391 Interviewer: That *can't* – If you're wanting to go visit your friend, and your mother or father says that you can't go over to your friend's house, is it the same kind of *can't*?
392 Terry: No, that means you're not allowed. ...
395 Interviewer: You're not allowed to
396 Terry: Yeah.
397 Interviewer: Or how do you know it's not the kind of can't that Dale said? Where it just can't possibly happen? How can you tell the difference?
398 Terry: By the way she says it.
399 Chris: Yeah. ...
418 Interviewer: When you said earlier "you can't win", which one is that closest to? Remember, when you looked at your basket and you said, "Oh, we can't win." Is that like the "you're not allowed" or is it
419 Terry: It would be you can't
420 Leslie: You don't
421 Terry: Like you, it's impossible, like
422 Leslie: Yeah, it's impossible.
423 Terry: Well, it was because if you added it all up, the skunk
424 Dale: You'd only get, like, fifteen.

425 Terry: The skunk would have come.
426 Chris: Yeah, you'd only get fifteen so if the skunk is that…
430 Terry: Because the skunk was gone.
431 Interviewer: It would have been impossible.
432 Terry: Yeah, yeah
433 Interviewer: So if someone says *can't*, … if I told you that you can't divide by zero in a lesson on dividing would you think that that means that you're not allowed to or that it is impossible to do?
434 Chris: That it is impossible.
435 Interviewer: Why would you think that?
436 Terry: Because you can't divide by zero.
437 Interviewer: Why can't you?
438 Terry: Because it is impossible.
439 Interviewer: How do you know?
440 Chris: Because you can't
441 Terry: Because you can't
442 Chris: If it is zero, you can't put it in any groups.

In this case, Terry introduced the adjective *impossible*, to clarify the meaning of *can't*. No one had used the word before this in the interview. As with "have to", the students distinguished among instances of *can* and *can't* based on context. During and after this interview, we wondered how students could make this distinction for instances in which they do not know a convincing logical argument for the assertion. With the example of division by zero, the students now knew that it is impossible, but how might they have thought about it the first time they heard their teacher say "you can't divide by zero"?

The students introduced three adverbs/adjectives that indicate degrees of probability into the interview. The word *probably* was first used by Leslie and not used again by others. When Leslie and Terry were considering whether or not to make the same choice about going home as the other group, Terry remarked, "One of [our groups] won't lose everything and the other would" (turn 204), and Leslie replied, "It is probably going to be us" (turn 205). The adverbs *absolutely* and *impossible* came up in the conversations about language choices when the students were trying to explain what the modal expressions meant, as noted above.

Other modal verbs used included *would*, which was first used (accidently) by the interviewer and used liberally later by the students, and *may* as in "you may be able to win" (Dale, turn 263). Another specialized linguistic form used by a student was the if-then statement, first used by Chris: "If it was two numbers then it would make a difference" (turn 39). This was in the discussion about the playing skunk with a die.

In addition to the relatively specialist terminology for modality (the modal verbs and adverbs), students expressed degrees of certainty in other ways. Terry introduced the expression "I think" in a conversation about playing skunk with the die. The researcher had asked if the number of berries they got would be different if the skunk came on a one instead of a six, to which Terry replied, "I think it would because we roll the one a lot" (turn 45). Terry introduced another expression to describe the differences between playing skunk with cards and with the die. In turn 236 Terry said, "You never know what is going to happen" (with the die). Terry also said "the odds are harder" (line 273) when the probability of success became lower. Dale was inventive too, and used the expression "I had a feeling" (turn 126) after "going home" to stay safe. This was in reply to the researcher asking, "Did you know that this was the skunk?"

Finally, the absence of any modal expressions is significant in the consideration of modality as well. The use of bald assertions can replace strong modal verbs or adverbs. Dale said, "the skunk is right there" (line 74) while pointing at the skunk card, as yet unrevealed but evidently the skunk by deduction. We might expect "the next one has to be the skunk" or "I am certain that the next one is the skunk" but the bald assertion serves the same function. Chris did the same on line 82 saying "it's there." In this interview (and others), there were many instances of this method for expressing certainty.

DISCUSSION

The four students in the interview described above demonstrated a wide repertoire of language for expressing degrees of certainty. Each of them used a range of expressions, and each of them introduced expressions that no one else had used before. Terry was the most talkative in the discussions about language meaning, but we caution that it would be unwarranted to make conclusions in comparison to the others on this basis. Many of the expressions introduced by the students came late in the interview, which tells us that if the interview had been shorter, we would not have known whether or not the students had these expressions in their repertoires. This serves as an exemplary caution against deficit-based assessments. Similarly, when one student said something, there was no need for the others to say it again or even speak about it unless they disagreed. Also, if students use an expression that has just been used by the teacher or tester, it may not be fully "acquired." We cannot assume someone does not possess certain language simply because they do not use it. However, we can claim that a student has an expression in their language repertoire if they introduce it. This is why we went to the lengths that we did for structuring our prompts carefully.

In addition to using (and introducing) specialist language, the students in the interview at times demonstrated ability to convey their meaning using very limited technical language. In particular, they could make their ideas clear when talking about the extremes of certainty – when events were impossible or certain. The more specialised language seemed to be relied upon either for describing events that were somewhere between impossible and certain, and for clarifying meaning on the extremes when pressed to do so. As noted above, Rowland (2000) introduced the idea of the zone of conjectural neutrality to describe language that specifies degrees of certainty, which is "in defiance of the cultural norm that the pupil is judged to be 'right' or 'wrong'" (p. 211). He claimed it to be helpful for a conjecturing atmosphere. We note that the same terminology is used to describe probability, and thus specialised modality language can defy situations in which predicted results may be between impossibility and certainty. We have only begun to consider the implications for pedagogy considering that the language is shared for both conjecture and probability spaces.

This brings us to discussion of the second research context, which was set up to be similar to but distinct from the game of skunk – a twist on the context. In both contexts, students were making predictions. What is the difference between a train and a pile of cards, both of which are sequences of physical objects? One difference is that the cards are shuffled and train cars are sequenced with some sort of intention. Nevertheless, our experience of real trains is that the sequence of cars seems to be quite random, or in groups (e.g., the boxcars first, followed by a bunch of tankers, followed by a few flatbeds, and finally the rest of the tankers). We have never seen trains with patterns similar to the ones introduced in our research – patterns like yellow boxcar, green tanker, blue flatbed, yellow boxcar, green tanker, blue flatbed, etc. A Grade 4 student in the second year of research involving the trains became increasingly frustrated with the rest of the class identifying what the 50th car would be. This student kept saying that it is impossible to know, while the class continued to ignore him. This student refused to make predictions.

This tension points to the presence of some sort of pedagogical contract in which students generally expect intention from their teachers. Even in the game of skunk, when the interviewers showed all the cards to the students and shuffled the cards directly in front of them, the students sometimes expected some kind of lesson – the appearance of a second skunk card, for instance. With the trains the phenomenon was more obvious; the students (with some exceptions, most notably the Grade 4 student noted above) assumed that the patterns would continue even though the researcher and teacher never said that these were patterns and the described context was one of a real-life train. The anger displayed by participants when they saw the fifth train (the random train) made clear to us the students' expectations for pattern. There is something about the transposition of a narrative into a mathematics classroom that changes it to a scenario in which everything should be predictable (and known by the teacher, or researcher).

In our research project, student predictions were based on both the probabilities inherent in the given scenarios and the students' second-guessing of teacher/research choices in constructing scenarios for pedagogic or other reasons. This raises questions about how students experience probability learning. Uncertainty in the mathematics classroom is experienced differently than outside the classroom. Furthermore we note that the language of conjecture shares language with probability, and so we wonder whether this ought to confound similarly our understanding of the way students experience proof and reasoning.

Finally, we turn our attention to implications beyond the classroom. Increasingly significant social phenomena, such as climate change, involve both calculations of risk, which are based on assumptions, and conjectures (hypotheses). The fact that risk calculation and conjecture share terminology may complicate communication about such social phenomena. Furthermore, both risk calculation and conjecture language about certainty is also used to express authority, as demonstrated in the above conversation about authority. When people in the public sphere who appear scientific make claims that sound authoritative, how are listeners to know whether these claims are warranted? It is incumbent upon mathematics teachers to be aware of these shades of meaning and the risk of ambiguity on such important social issues.

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