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The significant role of language in mathematics teaching and learning is not a new phenomenon. Investigating bilingual mathematics learners is complex and research has demonstrated that language switching practices are also complex and involve not only social and cultural aspects, but also cognitive aspects. However, little investigation has been undertaken into the specific role of languages and their influences on conceptual activity at undergraduate level. The framework, and future research directions, presented in this paper aim to investigate further the cognitive aspects of bilingual learners and their use of their languages, when engaged in conceptual mathematical activity.

Keywords: Language, conceptual mathematical activity, bilingualism, framework.

INTRODUCTION

In this paper we will present a theoretical framework to support empirical research investigating bilingual students' use of their two languages as they engage in conceptual mathematical activity at undergraduate level. The need for such a framework has arisen from the authors' previous research findings and development of new research directions within the area of language and mathematics education. Comparisons of the Irish and English languages demonstrates that there are differences between the languages in terms of syntax and semantics and that this may impact on the processing of mathematical text, and advantage those learning through the medium of Irish. However, what is difficult to conclude, without further investigation, is whether differences between the languages, and when/how they are used, have a differential impact upon cognitive processing (Ní Ríordáin, 2013).

Previous studies in the Irish context demonstrate that a significant relationship exists between performance on mathematical word problems and language proficiency, with bilingual students with high proficiency in both languages performing better mathematically (Ní Ríordáin & O'Donoghue 2009). In particular, high proficiency in Irish had a strong correlation with performance in mathematics (through the medium of English) for students in the transition from Irish-medium primary to English-medium second level mathematics education. National testing in mathematics and English at primary level reveals that students in Irish-medium primary education perform the same or better than students in all-English medium education in both mathematics and English (Gilleece, Shiel, Clerkin, & Millar, 2011). Similarly, at third level education, when examining high-ability bilingual students, it was found that some students found it easier to undertake operations and to process ideas in Irish (as opposed to English) and displayed greater comprehension of the mathematics problems, an ability to self-correct, to select appropriate features in the problem and displayed knowledge of their strategies (Ní Ríordáin & McCluskey, 2012).

The significant role of language in mathematics teaching and learning is not a new phenomenon. Given the marked growth of cultural migration, the focus on education for economic development and the emphasis on English as a language for learning, we have become acutely aware of the importance of recognising the significance of language in learning mathematics (Barwell, Barton, & Setati, 2007). However, little investigation has been undertaken in relation to the specific role of learners' different languages when engaged in mathematical learning. There has been a focus more on the social, rather than cognitive functions of code switching/use of languages. In particular, there is a need for the development of a coherent and integrated interpretive framework for investigating whether differences in languages, and their use, by bilingual mathematical learners have a differential impact upon cognitive mathematical processing, while recognising the social aspects of learning. Fundamental to this is the commognitive approach for the study of mathematical learning by Sfard (2008, 2012).

SFARD'S COMMOGNITIVE APPROACH

Sfard's (2008) interpretive framework for examining learning is founded on the premise that thinking is a form of (interpersonal) communication, and that learning mathematics entails extending one's discourse. If assuming the premise that mathematical learning involves initiation into the discourses of mathematics, then learning mathematics involves substantive discursive changes for learners (Sfard, 2008). Accordingly, mathematics teaching involves facilitating such changes. Sfard also emphasises that communication, and being part of a community, is central to facilitating such teaching and learning activities. A discourse is distinctive in terms of a community's practices in relation to *word use*, *visual mediators*, *endorsed narratives* and *routines* (Sfard, 2008, pp. 133–135). Sfard (2012, p. 3) distinguishes between two types of mathematical learning (change in discourse): *object-level learning* (expansion of what is known already, mainly accumulative) and *meta-level learning* (change of meta-discursive rules, more radical and complex kind of change).

Overall, the commognitive framework 'provides a unified set of conceptual tools with which to investigate cognitive, affective and social aspects of mathematics learning.' (Sfard, 2012, p. 1). A key purpose is to help make sense of classroom processes, while being responsive to the intricate nature of complex data generated in a teaching and learning setting. What we aim to do in this paper is to build on this approach within a bilingual university mathematics education context and to potentially assist towards the reification of the framework into tools that can help analyses within such a context.

A FRAMEWORK FOR THE STUDY OF BILINGUAL LEARNERS

The following sections firstly provide an overview of key perspectives and aspects of consideration informing the proposed framework. Key principles of the framework are then presented, taking a commognitive standpoint (Sfard, 2008). The approach, proposed in this paper, is the result of a number of research studies undertaken in the Irish context to comprehend the intricacies of bilingual mathematical learning.

Perspective on discourse

Given the central aspect of discourse to the commognitive approach it is important that we outline our perspective of discourse. We see mathematics as a discourse and a type of communication (Sfard, 2012). Discourse is more than just language. As defined by Gee (1996, p. 131):

> A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts,' of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or 'social network,' or to signal (that one is playing) a socially meaningful role.

By employing this definition, Discourses are more than verbal and written language and the use of technical language; Discourses also involve communities, points of view, beliefs and values, and pieces of work. Moschkovich (2012, p. 95) utilises the phrase 'mathematics Discourse practices' to draw attention to the fact that Discourses are embedded in sociocultural practices as they evolve from and involve participation in communities, while also cognitive as they involve thinking, signs, tools and meanings. This concept of Discourse will inform the examination of conceptual mathematical development of bilingual learners, linking both the cognitive and social aspects of use of languages.

Perspective on bilingualism

Our work is with bilingual (Irish and English) mathematics learners and it is essential to incorporate this concept into the proposed framework. Defining bilingualism is difficult, in particular defining whether a person is bilingual or not. Definitions vary between political, social and cognitive perspectives. Grosjean and Moser-Mercer (1997) developed the notion of a "complementarity principle" in which they emphasise that bilinguals use their languages for different purposes and in different domains of life. Dominance in one language over the other is common among bilinguals depending on the use and function of each language. Also, studies involving bilinguals tend to focus on only one language, but due to the complex nature of the issue of bilingualism, aspects of both languages need to be taken into account. For the purpose of the development of a framework for the investigation of bilingual mathematical learners, we utilise Grosjean's (1999) concept of a continuum of modes with monolingual and bilingual at each endpoint. Therefore, by utilising the concept of a continuum of modes (monolingual to bilingual), it facilitates an understanding of bilinguals using their languages independently and together depending on the context/purpose. This is further supported by Cummins' (1980) Common Underlying Proficiency (CUP) model of bilingualism, which is a more apt description of language construction within the mind. Outwardly both languages are different in conversation. However, internally, both languages are merged so that they do not function independently of one another, with a central processing unit that both languages contribute to, access and use. We support a non-deficit view of bilingual learners and view language(s) as a resource and a support for learning.

Language and mathematics

In our research we are primarily concerned with the role of bilingual students' languages in mathematics teaching and learning. We consider mathematical language as a distinct 'register' within a natural language, e.g., Irish or English or French, which is described as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (Halliday, 1975, p. 65). One aspect of the mathematics register consists of the special vocabulary used in mathematics and it is the language specific to a particular situation type (Gibbs & Orton, 1994). However, the mathematics register is more than just vocabulary and technical terms. It also contains words, phrases and methods of arguing within a given situation, conveyed through the use of natural language (Pimm, 1987). The grammar and vocabulary of the specialist language are not a matter of style but rather methods for expressing very diverse things. Therefore, each language will have its own distinct mathematics register, encompassing ways in which mathematical meaning is expressed in that language. The process of learning mathematics inevitably involves the mastery of the mathematics register (Setati, 2005). Developing a learner's mathematical register provides them with analytical, descriptive and problem solving skills within a language and the communicative competence necessary for successful participation in mathematical discourses.

Conceptual mathematical activity

Given that our research is concerned with undergraduate bilingual students, it is essential to examine the mathematics register and discourse development at this level. The nature of cognitive growth in the development of university-level mathematical thinking has borne considerable scrutiny over the past half-century (Thurston, 1990; Asiala et al., 1997). The associated literature presents a strong case that the maturing over time of the mathematical thinking of professional mathematicians is such that mathematical concepts become distilled and perfectly understood - by them. As a result of such sustained processing, all forms of former struggle or lack of understanding are potentially removed from memory. Thurston (1990) captures eloquently the power and satisfaction of arriving at deep understanding after struggle. In the process, he suggests that "once you really understand it and have the mental perspective to see it as a whole... You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process" (Thurston, 1990, p. 847). At its core, these authors refer to what has been identified and named by Tall and Vinner (1981) as *concept image* for any given mathematical concept such as, for example, function or limit.

It proposes a genetic decomposition for a given mathematical concept. This encompasses knowing what it means to understand such a concept and knowing how such an understanding can be constructed by a student, thus providing a model of cognition for the concept. Such a model plays a key role also in alternative cognitive theories for mathematical thinking and learning such as those due to Tall and Vinner (1981) where it is referred to as *concept image.* A genetic decomposition attempts to identify the layers of meaning, robustness and accuracy that arise as a particular concept is revisited in a variety of contexts. Thus, for example, a student may initially recognize a function only when given a specific (single) formula to compute values. The student restricted to this level will have a narrow range of comprehension. A deeper level will allow the student to appreciate and manipulate the notion that a function can receive inputs, operate on them and return outputs, thus giving greater flexibility. Encapsulation into an object occurs when the student recognizes the process as a whole, namely that a function is a rule between two sets of values with certain properties. Once constructed, objects and processes can be interconnected in various ways. Given that language influences thought and thinking, and that each language will have its way of constructing the concept, insight into the role and effect of bilingualism/languages on conceptual mathematical learning is critical.

Language and learning

Developing a student's mathematics register and participation in discourse is facilitated by language(s). Language is an essential instrument of thought and is necessary for understanding and combining experiences, and is required for organising concepts. The general consensus in cognitive science is to presume that thinking is occurring in some language (Sierpinska, 1994). Vygotsky was one of the earliest theorists to begin researching the area of learning and its association with language. He concluded that language is inextricably linked with thought – "the concept does not attain to individual and independent life until it had found a distinct linguistic embodiment" (Vygotsky, 1962, p. 4). Although a thought comes to life in external speech, in inner speech energy is focused on words to facilitate the generation of a thought. If this is the case, it raises an important question – does the nature of the language used affect the nature of the thought processes themselves? The transition from thought to language is complex as thought has its own structure. Thought is mediated both externally by signs and internally by word meanings (Vygotsky, 1962). It is the use of language as an instrument of thinking that is of importance, as well as its effect on cognitive processing. Therefore, thought is intimately linked with language and ultimately conforms to it. The linguistic relativity hypothesis proposes that the vocabulary and phraseology of a particular language influences the perceptions and thinking of speakers of that language (Whorf, 1956). Accordingly, each language (e.g. Irish or English) will have a different cognitive system that will influence concept formation and development. We support the premise that a language influences our mathematical thinking, but not necessarily to a degree that it determines our entire mathematical thinking (Sternberg, 2003). We propose that there are differences 'between linguistically distinct versions of "the same discourse".' (Kim, Ferrini-Mundy, & Sfard, 2012, p. 2) which correspondingly impact on mathematical learning.

Learning mathematics

Mathematics and learning is arbitrated through mathematical discourse practices, spoken and written language, symbols, gestures, etc. (Forman, 1996). Learning is situated within and involves participation in a community. Within a mathematics classroom, learning involves participation in the discipline of mathematics, in conjunction with the specific type of mathematics associated with the context (e.g. school mathematics, undergraduate mathematics, etc.) (Forman, 1996). When examining bilingual mathematics learners, it is important to address the social use of language within the context, not just its role in cognition. Moschkovich (2012) emphasises the importance of learning being illustrated within the sociocultural practices of a given setting. Importance is placed on describing learners and communities, and seeing culture as a set of practices and actively involving participants (Gutiérrez & Rogoff, 2003). Accordingly, bilingualism is described in terms of participation and use of language(s) by learners for different purposes and particularly in the context of mathematical discourse.

Effective teaching and learning is a complex endeavour. In correlation with a teacher's strategies, the teacher's own philosophical beliefs of instruction are harbored and governed by the student's background knowledge and experience, situation, and environment, as well as the learning goals set by the student and teacher. Moschkovich (2012) emphasises the importance of discerning between the *conditions of* learning and the *processes for* learning, and the importance of describing the curriculum, courses/programmes and teaching and learning approaches utilised that yield successful outcomes for different groups of learners. Therefore, it is important to examine and report on the characteristics of the learning environment such as whether there are opportunities for: speaking, listening, reading and writing; constructing meaning and knowledge; high expectations for all students; rejection of a deficit view of learners (AERA, 2006; García & González, 1995). In particular, we adopt a non-deficit perspective of bilingual learners and focus on the strategies that teachers use in developing conceptual mathematical learning.

Key principles of the proposed framework

Adopting a commognitive approach to research, combined with key concepts discussed in the previous sections, give rise to several key principles and methodological considerations for investigating undergraduate bilingual mathematics learners and their use of their languages. Primarily, the authors' framework is underpinned by a non-deficit view of bilingual learners where languages are viewed as a resource and essential for thinking. Within the framework, thinking can be defined as the activity of communicating with oneself (Sfard, 2012). Accordingly, mathematical thinking can be viewed as a Discourse, which in turn is a form of communication and involves being part of a mathematical community. Taking this view, the language or language(s) in which mathematics is being learned becomes an important issue for consideration. Within the framework, development refers to a change in Discourses (Sfard, 2012). Accordingly, we refer to the development of a student's mathematical Discourse as opposed to the development of the student themselves. Development of Discourses is a product of collective human actions and the context acknowledged. Given that the authors are primarily concerned with conceptual mathematical learning/activity, we are concerned with meta-level developments in Discourses. Since our focus is on bilingual mathematics learners, it is important that an analysis of the language(s) in which the discourse is taking place is conducted. The successive meta-discourses relating to topics of interest, for example functions, need to be documented and compared between languages.

By adopting a commognitive approach (Sfard, 2012), there are a number of key principles that need to be adhered to and which have been adapted to reflect our framework. Firstly, *Operationality*: the purpose of the research is to share useful stories. Therefore, it is important that the researcher's articulation avoids misunderstandings and is unambiguous and clear (p. 9). Second is *Completeness:* the researcher must choose the entire discourse related to the topic as the unit of analysis (p.9). Here, we add to the principle in that when examining bilinguals, we must document this discourse (plausible developmental trajectories) in both languages e.g. the discourse relating to 'limits' or 'functions' in both English and Irish. It should involve an analysis of successive meta-discourses in each language. Third is *Contextuality*: any kind of interaction is an event of learning (p. 9). It is essential that the researcher documents the interactions as fully as possible and analyses utterances within the context of the conversation. We extend this, in the given context, to the need to examine when and how bilingual students/researchers use their language(s). The next principle is that of *Alternating Perspectives*: when analysing data, the researcher alternates between being an insider and an outsider of their own ways of using words (p. 9). This is heightened within a bilingual context in that consideration must be given to both languages, their use in the given context and possibility of significant differences between researcher and participant discourses. Finally, the principle of *Directness*: when describing their study, the researcher presents things said (and done) by the participant first, not their own interpretation of the data (p. 9). It is hoped that these over-arching concepts and principles will foster insights into bilingual mathematics learning and contribute to the development of research.

CONCLUSION AND FUTURE RESEARCH

At the National University of Ireland (NUI) Galway, students have an option to study Mathematics through a bilingual approach (Irish and English) during their first year of undergraduate education. This provides an opportunity to investigate language choices made by undergraduate students and to identify how these choices impact on conceptual mathematical activity. We propose to address the following research questions via an investigation that is supported by the proposed framework.

- In what ways do formal discourses in English and Irish on, for example 'limits', follow different developmental trajectories in undergraduate mathematics education? 'Limits' are chosen as an example given its ubiquitous nature; for example, limits of sequences, of functions at points and at infinity, summation of series, derivatives and integrals. Developmental trajectories refer to identifying all of the discourses related to 'limits' that an Irish-speaking and English-speaking person is likely to encounter (Sfard, 2012). This can be in everyday life or specifically related to a teaching and learning context (e.g. second and third level in this case) – all potential trajectories need to be listed.

— What is the nature of and reasons for meta-level developments of mathematical discourses in bilingual students? Meta-level development refers to a change in the discourse that results in expansion of the discourse relating to a particular topic(s) and is a complex type of change, rather than an 'object-level' change that is more accumulative in nature (Sfard, 2012, p. 3). Therefore, meta-level development is primarily concerned with conceptual mathematical activity.

— How are languages (Irish and English) utilised in, and how do they impact on, meta-level developments in mathematical discourses of, for example 'limits', in undergraduate mathematics education? By detailing the developmental trajectories in each language it is expected to demonstrate how learning can be affected by the characteristics of a language, while also examining when and how bilingual students utilise their languages.

Investigating bilingual mathematics learners is complex and research has demonstrated that language switching practices are also complex and involve not only social and cultural aspects, but also cognitive aspects. The framework, and future research directions presented in this paper, aim to investigate further the cognitive aspects of bilingual learners and their use of their languages, when engaged in conceptual mathematical activity.

REFERENCES

Barwell, R., Barton, B., & Setati, M. (2007). Multilingual issues in mathematics education: introduction. *Educational Studies in Mathematics, 64*, 113–119.

Cummins, J. (1980). *The construct of language proficiency in bilingual education*. Washington, DC: Georgetown University Press.

Foreman, E. (1996). Learning mathematics as participation in classroom practice: implications of sociocultural theory for educational reform. In Steffe, L. P., Nesher, P., Cobb, P., Goldin, G. A., & Greer, B. (Eds.), *Theories of mathematics learning* (pp. 115–130). Mahwah, NJ: Erlbaum.

Garcia, E., & Conzalez, R. (1995). Issues in systemic reform for culturally and linguistically diverse students. *Teachers College Record, 96*, 418–431.

Gee, J. (1996). *Social linguistics and literacies: ideology in discourses (3rd Ed.)*. London: The Falmer Press.

Gibbs, W., & Orton, J. (1994). Language and mathematics. In J. Orton & G. Wain (Eds.), *Issues in teaching mathematics* (pp. 95–116). London: Cassell.

Gilleece, L., Shiel, G., Clerkin, A., & Millar, D. (2011). *The 2010 national assessments of English reading and mathematics in Irish-medium schools*. Dublin: ERC.

Grosjean, F. (1999). Individual bilingualism. In B. Spolsky (Ed.), *Concise Encyclopedia of Educational Linguistics* (pp. 284–290). London: Elsevier.

Grosjean, F., & Moser-Mercer, B. (1997). The bilingual individual. *International Journal of Research and Practice in Interpreting*, *2*(1), 163–187.

Gutiérrez, K., & Rogoff, B. (2003). Cultural ways of learning: individual traits or repertoires of practice? *Educational Researcher, 32*(5), 19–25.

Halliday, M. A. K. (1975). *Learning how to mean: explorations in the development of language.* London: Edward Arnold.

Kim, D. J., Ferrini-Mundy, J., & Sfard, A. (2012). How does language impact the learning of mathematics? Comparison of English and Korean speaking university students' discourses on infinity. *International Journal of Educational Research*, *51/52*, 86–108.

Moschkovich, J. (2012). How equity concerns lead to attention to mathematical discourse. In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics –Theories, practices and policy* (pp. 89–105). New York: Springer.

Ní Ríordáin, M. (2013). A comparison of Irish and English language features and the potential impact on mathematical processing. In Ubuz, B., Haser, C., & Mariotti, M. A. (Eds.), *Proceedings of the 8th Congress of the European Society for Research in Mathematics Education* (pp. 1576–1585). Turkey: METU.

Ní Ríordáin, M., & McCluskey, A. (2012, July). *The influence of bilingualism on mathematical problem solving at third level education in Ireland.* Paper presented at the 12th International Congress on Mathematical Education, Seoul, South Korea.

Ní Ríordáin, M., & O' Donoghue, J. (2009). The relationship between performance on mathematical word problems and language proficiency for students learning through the medium of Irish. *Educational Studies in Mathematics*, *71*, 43–64.

Pimm, D. (1987). *Speaking mathematically: communication in mathematics classrooms.* London: Routledge & Kegan Paul.

Sfard, A. (2008). *Thinking as communicating: human development, the growth of discourses, and mathematizing*. Cambridge, UK: Cambridge University Press.

Sfard, A. (2012). Introduction: developing mathematical discourse – Some insights from communicational research. *International Journal of Educational Research*, *51/52*, 1–9.

Setati, M. (2005). Mathematics education and language: policy, research and practice in multilingual South Africa. In R. Vithal, J. Adler, & C. Keitel (Eds.), *Researching mathematics education in South Africa: perspectives, practices and possibilities* (pp. 73–109). Pretoria: HSRC Press.

Sierpinska, A. (1994). *Understanding in mathematics. Studies in mathematics education series: 2.* London: The Falmer Press.

Sternberg, R. J. (2003). *Cognitive psychology* (3rd Ed.). London: Wadsworth

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*, 151–169.

Thurston, W. P. (1990). Mathematical education. *Notices of the American Mathematical Society*, 37, 844–850.

Vygotsky, L. S. (1962). *Thought and language*. Cambridge, MA: MIT Press.

Whorf, B. L. (1956). *Language, thought and reality*. Cambridge, MA: MIT Press.