Analysing teachers’ belief system referring to the teaching and learning of arithmetic

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In this paper, we want to discuss the structure of teachers’ belief systems. Firstly, we discuss teachers’ belief systems from a theoretical perspective including characteristics of beliefs systems like its cluster structure, the centrality of beliefs or the hierarchy of beliefs. Afterwards, we analyse the beliefs of one primary teacher emphasising particularly the structural aspects of this teacher’s system of beliefs concerning the teaching and learning of arithmetic. Finally, we discuss potential benefits of investigating a belief system in detail. We conclude the paper with a brief summary and suggestions for further research.

Keywords: Arithmetic, belief system, central and peripheral, primary and derivative.

INTRODUCTION

Teachers individually define their way of teaching. Thus teachers decide what mathematical content they bring to the classroom and they decide how they teach a specific content. A teacher’s individual reasoning why he/she selects specific content and why he/she prefers a specific teaching style could be understood as a notion of a teacher’s system of beliefs (Eichler & Erens, 2015), in which beliefs represent the intersection of the cognitive and motivational aspects of a teacher’s mathematics related affect (Hannula, 2012). Lerman (2015, p. viii) states that, “exploring teachers’ beliefs and their development are important topics in themselves”. However, the importance of gaining knowledge towards mathematics teachers’ beliefs has also been emphasised by many researchers since teachers’ beliefs about mathematics and the teaching and learning of mathematics potentially have a high impact on the teachers’ classroom practice (Philipp, 2007).

As a prerequisite of analysing both, the development of teachers’ beliefs and the enactment of teachers’ beliefs, there is a demand to investigate the complex structure of teachers’ beliefs in detail. For example, referring to the change of teachers’ beliefs, Liljedahl, Rolka and Rösken (2007, p. 280) state that a “deeper analysis of beliefs in the context of mathematics teachers’ professional growth is needed to penetrate the surface stories of the data and reveal the nuanced and situated belief structures that are often hidden”. The mentioned belief structures are also understood to potentially unfold the relation between teachers’ professed beliefs and the teachers’ enacted beliefs; this relation is not completely explained yet (Furinghetti & Morselli 2011). For example, Wilson & Cooney (2002) suggested that only those professed beliefs would be enacted that are central for a teacher and, thus, they suggested that unfolding the structure of a teacher’s belief system could explain inconsistencies. As an important aspect of an in-depth analysis of mathematics teachers’ beliefs, research results imply to consider the discipline-specifity of teachers’ beliefs (Franke et al., 2007). Consistently, our own research yield considerable differences of teachers’ beliefs about different disciplines like e.g. arithmetic (Eichler & Erens, 2015).

Two aims of this project are to analyse the development of mathematics teachers’ beliefs and the extent in which these beliefs are enacted in the teachers’ classroom practice. For this reason, following the considerations outlined above, we firstly tried to “penetrate the surface stories” (Liljedahl, 2007, p. 280), i.e., to analyse the structure of the teachers’ beliefs in depth. After outlining our theoretical framework, we discuss the method of analysing teachers’ belief structures or rather teachers’ belief systems in depth. This analysis is the main focus of our paper. We further outline the results of our analysis for one teacher. This teacher is part of a sample of 20 mathematics
teachers of primary schools involved in our research project. We conclude this report summarising our findings and suggestions for further research.

**THEORETICAL FRAMEWORK**

Following Pajares (1992) the term beliefs represents an individual's personal conviction concerning a specific subject, which shapes an individual's way of both receiving information about a subject and acting in a specific situation. The specific subject in our research is the teaching and learning of arithmetic. We decided to focus on this specific subject due to two reasons: Firstly, arithmetic is the main subject for the primary teachers of our sample. Secondly, a focus on a specific mathematical discipline is in our research an important aspect for facilitating an in-depth analysis of mathematics teachers' beliefs. As a specific form of beliefs we regard the teachers' beliefs about content, beliefs about ways of teaching or beliefs that represent a teacher's teaching goals (c.f. Eichler & Erens, 2015).

The internal organisation of the mentioned beliefs is called a teacher's belief system (Green, 1971). A belief system is mainly characterised by three aspects:

1) “Beliefs can be either central, which means strongly held, or peripheral, which means less strongly held” (Philipp, 2007, p. 260).

2) A belief system could consist of different clusters that are connected in at least quasi-logical ways. This means that different clusters of beliefs could be isolated, but different clusters of beliefs could also be contradictory (ibid.). For example, central beliefs are not necessarily connected to peripheral beliefs. Further beliefs referring a mathematical discipline like arithmetic could be contradictory to beliefs referring another mathematical discipline (Eichler & Erens, 2015).

3) Beliefs systems could be organised hierarchically including primary beliefs and derivative beliefs (Green, 1971). If teaching goals are regarded (see above) a primary goal could be to prepare students to solve problems in their future life. A derivative goal could be to follow a cognitive guided instruction (Franke et al., 2007). A relation between primary and derivative goals must not be necessarily logical in an objective sense (quasi-logicalness, see above). Further, it is noteworthy that “primary beliefs might not necessarily be more central than the associated derivative beliefs” (Philipp, 2007, p. 260).

Research in mathematics education has reported specific clusters of beliefs that refer to different features of the perception of mathematics in general (Dionne, 1984). Based on these two reports Grigutsch, Raatz and Törner (1998) distinguish four views that could describe teachers’ beliefs about mathematics in general but also teachers' beliefs about the teaching and learning of arithmetic:

- A formalist view stresses that arithmetic is characterised by a logical and formal approach. Accuracy and precision are most important.

- A process-oriented view is represented by statements about arithmetic being experienced as a heuristic and creative activity that allows solving problems using different and individual ways.

- An instrumentalist view places emphasis on the "tool box"- aspect which means that arithmetic is seen as a collection of calculation rules and procedures to be memorized and applied according to the given situation.

- An application oriented view accentuates the utility of arithmetic for the real world and the attempts to include real-world problems into class.

Further we refer to a global distinction of two different ways of teaching mathematics or arithmetic, i.e., a “cognitive constructivist orientation”, and a “direct transmission view” (Staub & Stern, 2002, p. 344). We assume that these two different orientations are two ends of a continuum with three points of orientation: constructivism, co-constructivism and transmission (Strohmer et al., 2012).

**METHOD**

The sample consists of 20 primary teachers. However, in this paper we restrict the discussion to one teacher, i.e. Mrs. A (a young teacher from south of Germany), and her beliefs referring the teaching and learning of arithmetic. This restriction is not based on specific characteristics of Mrs. A, but is based on the main aim.
We collected data in two different ways: Firstly, we used a semi-structured interview including clusters of questions referring to arithmetic content and goals of teaching arithmetic as well as goals of teaching mathematics, students’ learning of arithmetic or materials used, e.g. textbooks. In addition, the interviews incorporated prompts to evaluate given arithmetic tasks or fictitious statements of teachers or students that represent one of the views mentioned above.

Secondly we used a questionnaire referring to teachers’ views (Grigutsch, Raatz, & Törner, 1998). Participants were asked to rate every item (e.g., item 5: Everyone is able to invent mathematics or rather to re-invent mathematics) using a 4-point Likert-scale. We adapted this questionnaire by changing the focus to beliefs referring arithmetic (new item 5: Everyone is able to invent arithmetic or rather to re-invent arithmetic), but also used Likert scale. Finally, we used the existing scale of Strohmer and colleagues (2012) to measure the teachers’ teaching orientation that we also adapted for teaching arithmetic. We conducted the following three steps for analysing the data.

First step of data analysis
We used a qualitative coding method (Kuckartz, 2012) that is close to grounded theory (Glaser & Strauss, 1967) to analyse the data of the verbatim transcribed interviews. We used deductive codes derived from a theoretical perspective like ‘application oriented’ belief and inductive codes for those beliefs we did not deduce from existing research (Kuckartz, 2012). We will describe different inductive codes later in the result section.

Second step of data analysis
We weighted the deductive codes with 1, 2, -1 or -2 due to the following rules:

- If a teacher mentions a goal without a precision we weighted the code with 1.
- If a teacher explains a belief more deeply we weighted the code with 2.
- If a teacher refused a belief with explanation, we weighted the code with -1.
- If a teacher refused a belief without explanation, we weighted the code with -2.

One aim of this step of the data analysis was to develop quantitative evidence for the results that we gained through interpretation of the interview transcripts. For example, if the weighted sum of codes referring to a specific view is much higher than for another view, this could serve as evidence for a different grade of centrality of these two views. Further the sum of the weighted codes facilitates the comparison of the analysis of the interview and the analysis of the questionnaires. The deductive codings as well as the inductive codings were conducted by at least two persons and we found the inter-rater reliability to show an appropriate value.

Third step of data analysis
The four subscales of the adapted questionnaire of Grigutsch, Raatz and Törner (1998) yielded four sums of ratings referring to the application oriented view, the process oriented view, the instrumentalism view and the formalism view. We compared the distribution of these four subscales to the sum of weighted codes referring the same views. To facilitate the comparison we used standardised distribution of the rating sums and the sums of the weighted codes. To compare both standardised distributions we used a correlation coefficient and other measures on association (e.g., Kendalls Tau-b) in an exploratory way and, further, used a U-test for proving differences between the two distributions.

RESULTS
As mentioned before, we restrict the focus to one teacher, Mrs. A, and her belief system to give a comprehensive picture of the structure of one belief system.

Process orientation as a central belief of Mrs. A
Mrs. A expressed coherently a process oriented view. For example, to the question of her favourite style of teaching arithmetic and her preferred methods she answered:

“Truly, it is important that they are able to find the solutions on their own, that they can work individually (...) that they can solve problems, that they can work on open tasks, that they can find their own strategies.”
Later, nearly the same answer ensued when she was asked about pupils and their way of learning arithmetic:

“It is always important for me, that it comes from the pupils themselves, that it includes a problem, I like giving pupils problem statements.”

Again, being asked to the question, which goals she would like to reach with her arithmetic lesson, she answered:

“And then there are strategies, i.e. to be flexible, to adapt oneself to something new. Therefore, you need the right attitude that you have the confidence to try something you don’t know and to put effort into it.”

The three quoted episodes referring to different topics of teaching arithmetic, i.e. the teaching style, students’ learning and teaching goals give evidence that beliefs representing the process oriented view are central in the belief system of Mrs. A.

Further, the prompts given during the interview contain a process orientation. For example, Mrs. A was asked to arrange eight given teaching goals into a hierarchy. Figure 1 shows her arrangement of these goals for arithmetic lessons, where Mrs. A valued problem solving and process orientation as the most important goals.

In Figure 2 we show a further prompt consisting of students’ statements representing the four views towards mathematics. The teachers were asked to arrange the statements from most desired (1) to least desired (4) if the statements represent arithmetic. Mrs. A preferred the second statement representing the process orientation.

Just as the professed beliefs the responds to the prompts referring teaching arithmetic give strong evidence that process orientation is central for Mrs. A.

The results of the second and third step of analysis (sum of weighted codes; questionnaire) are shown in Figure 3 where both distributions are standardised.

Looking at the figure it is obvious that application and process are more accepted than the other two orientations. The interview results as well as the questionnaire results imply this assertion. In conclusion application and process are central in Mrs. A’s belief system. The high degree of coherence in different parts of the interview, the sum of weighted codes and, finally the questionnaire underline that the different instruments all measure the same.
Application as a derivative belief
Although application oriented beliefs are central for Mrs. A, however, her answers concerning the application oriented view gave evidence that application oriented beliefs are derivative beliefs. Thus they seem to be subordinated to process oriented beliefs. That means that application is in some sense a central teaching goal but rather a means to an end for another primary belief:

“The relation to reality is important too, as I said before referring to money and time, but it doesn’t has to be highlighted all the time. Today, for example, I just gave them a mathematical problem...”

Application oriented goals seem not to be derived from process oriented goals in a logical way. However, both (clusters of) beliefs are central and application oriented beliefs are subordinated to process oriented beliefs.

Formalism and instrumentalism as peripheral beliefs
Although formalism is mentioned in some parts of the interview, it just has a peripheral meaning in the belief system of Mrs A. She names the importance of mathematical correctness once. Further, she emphasises that students should understand what they do in mathematics and why they do things. However, formalism seems to be a peripheral belief of Mrs. A and in some sense a desirable but peripheral result of process orientation as the following quotation illustrates:

“Apart from that the process oriented training is more useful, because there is a little bit more reflection and control, so you can see what you are doing and why you are doing it!”

While Mrs. A talks positive about process, application and formalism when she regards her teaching of arithmetic, she often refuses instrumentalism. It is mentioned and it is denoted as important, but instrumentalism often receives a negative connotation, e.g. when Mrs. A talks about “mindless practicing” or when she says “you just have to do it this way”:

“I realize this drill is just mindless practicing, but in some ways it makes sense, for example when you want to establish an algorithm, yes, then you just have to do it this way.”

The critical comments referring the instrumentalism view of Mrs. A show that this aspect is peripheral in her belief system. Referring to instrumentalism, we did not find evidence for a clear relation to another view.

Concluding the analysis of the belief system of Mrs. A the process orientation is central. Also application orientation is central in her belief system but, however, subordinated. Thus, application oriented goals could be understood as a means to an end to reach process orientation. Formalism is a positive connotated peripheral belief that represents a desired but peripheral result of process-orientation. Instrumentalism is also a peripheral but isolated belief with a negative connotation (Figure 4).

The belief cluster of process orientation
Mrs. A’s central belief, i.e. process orientation, is closely connected to a set of further beliefs and, thus, could be understood as a belief cluster consisting of several defining beliefs. These defining beliefs are a result of inductive codes which were formed during the analysis of the interview transcript. We illustrate only two beliefs that constitute the belief cluster of process orientation. The first belief concerns comprehension that Mrs. A explains the benefit of open word problems - so called Fermi tasks - that are based on individual models:

“Comprehension must stand on the top and you can reach it while giving tasks with a problem solving context. This has not to be a big Fermi task but also you can just confront the pupils with something and then see what they do.

The second quotation represents the belief flexibility. Here, the word strategies illustrate the closeness to process orientation:

“...they [the children] should not be afraid of numbers. They should be flexible and fit in their head. And this brings us back to the strategies, they can learn with the help of the “half-written” calculation.”

As well as process orientation also application, formalism and instrumentalism are belief clusters. In Figure 4 we show some of the beliefs that for Mrs. A define the four belief clusters.
In the same way as it was shown above, we investigated beliefs that concern more general the process of teaching and learning. Referring these beliefs, a (co-)constructivist orientation is central for Mrs. A: "Well, it should be oriented on the pupils, the teacher should abstain himself, so that there exists a high amount of work the pupils do, it should not only be training but also the teacher should give them credit so that they can work independently…"

The centrality of this orientation is also shown in the following prompt referring to learning arithmetic. Here Mrs. A should rate three different statements of pupils. The figure illustrates that it is important for her that the pupils work independently.

As discussed to the results referring the arithmetic related beliefs, the qualitative interpretation, the sum of weighted codes and, finally, the results of the questionnaire yield very similar results also referring to the different ways of teaching (Figure 6).

In her statements to her experience in sitting in on classes:

"... and I have seen this in secondary school, [...] but it is so different and boring. The older teacher stand in front of the class, they lead the way and the pupils replicate and practise. And we learned quiet different things, for example at the university – much more pupil activity, much more problem solving and open lessons."

Most of the time Mrs. A emphasises that she wants the pupils to make their own experiences and that she does not want to stand in front of the class and tell the pupils how things work. Still there exist a few episodes where she talks about the relevance of teacher-centred teaching:
“...so teaching should be effectively and therefore it also is important to teach the pupils sometimes in teacher-centred teaching.”

These examples show that a direct transmission view is not rejected but peripheral in the belief system of Mrs. A.

DISCUSSION

Looking at the results we can approve some of the aspects which characterise a belief system referring to Green (1971). Thus the structure of a mathematics teacher’s belief system referring to the mathematical subdomain arithmetic contains beliefs with different centrality. The beliefs can be central (here: application and process orientation) or peripheral (here: instrumentalism and formalism). Further, the beliefs are hierarchically arranged as primary beliefs and derivative beliefs. In the case of Mrs. A the application orientation is a means to an end to facilitate process orientation. Finally, we identified different belief clusters, e.g. the cluster process orientation which contains for example comprehension or flexibility (cf. Figure 4). The theoretical statement that central beliefs are not necessarily connected to peripheral beliefs (Philipp, 2007) was pointed out in the connection between the central and peripheral beliefs of Mrs. A. Formalism could be understood as a peripheral result of process orientation, instrumentalism however has no connection to this belief.

Our results can be used to compare teachers from different type of schools and with different background concerning their professional career. However, the main reason of the in-depth analysis of a teachers’ belief system that we discussed in this paper is to provide a basis that facilitate further research, i.e. investigating both the relation between professed beliefs and enacted beliefs and the development of teachers’ beliefs.

REFERENCES


