The flow of emotions in primary school problem solving

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Research into if, and how, students' emotions change during problem-solving, the factors behind the change, and the potential impact of a certain emotional change on mathematical activity, may provide significant indications regarding students' problem-solving difficulties, and regarding the link between affective and cognitive factors "in context". In this paper, we describe the results of a pilot study conducted at primary school level, while also emphasising the methodological choices made in relation to the young ages of the students involved.

Keywords: Emotions, problem solving, affect, primary school, mathematics.

INTRODUCTION AND THEORETICAL FRAMEWORK

The role played by the affective factors in the study of the mathematics teaching and learning process has been little explored for a long time. The success of behaviourism in psychology, the clear division between emotions and rational thinking, and the conviction that this latter sphere is particularly predominant in mathematics marginalised research into affective factors in mathematical education until the 1980s. As emphasized by Zan, Brown, Evans and Hannula (2006, p. 113): "affect has generally been seen as 'other' than mathematical thinking, as just not part of it."

In the 1980s, the gradual affirmation of the interpretative paradigm in the social sciences, related to a greater attention to aspects regarding the complexity of human behavior, has led researchers to abandon the attempt to explain behavior through measurements or general rules based on a cause-effect scheme, and to search instead for interpretative tools. Within this approach, the scientific community in M.E. has recognized the need to go beyond purely cognitive interpretations of failure in mathematics, in particular to interpret the failure of individuals apparently possessing the necessary cognitive resources in mathematical problem solving (Silver, 1985).

In 1989, the book *Affect and mathematical problem solving by McLeod and Adams* opened up a new era in research into emotions in mathematics education. As a matter of fact, the book includes several contributions that, following the theoretical model for the cognitive origin of emotions proposed by the psychologist George Mandler, underline the role of emotion in one of the most important mathematical activities (problem solving). According to Mandler (1989), complex emotions are the result of the cognitive evaluation of a visceral arousal provoked by a discrepancy between the individual's expectations and the demands of ongoing activity.

In the meantime, evidence of the strong interaction between cognitive and affective factors – in particular, of the influence of emotions on decision-making processes – also emerges from studies from other fields of research such as neuroscience (Damasio, 1996) and psychology (Salovey & Mayer, 1990). The study by Salovey and Mayer, introducing the concept of *emotional intelligence,* completely changed the way of looking at the affective component of human personality and its relationship with the cognitive field. Emotions are no longer regarded as a disturbance to correct thinking, but as a potential for effective thinking, particularly when our (and those of other people) affective experiences are recognised and managed appropriately.

On the other hand, according to Mandler (1989, p. 16): "Affectless learning is not a possible goal for a theory or for the praxis of instruction. Common sense tells us that emotions and affective reactions are with us now and forever."

This all leads to the conviction that it is important to teach emotion management in learning, a conviction that is also gaining ground in the mathematics education research field, which confirms how emotions on the one hand affect cognitive processing, biasing attention and memory and activating action tendencies, and on the other have a key role in human coping and adaptation (Evans, 2000; Hannula, 2002). In particular, Op 'T Eynde, De Corte and Verschaffel (2006, p. 194) claim: "from a socio-constructivist perspective, students' emotions and other affective processes are conceived as an integral part of problem solving and learning."

An important objective of mathematics education should therefore be that of teaching how to manage emotional involvement, particularly that which involves negative emotions (Op 'T Eynde, De Corte, & Verschaffel, 2006, p. 204).

Teaching students how to solve mathematical problems then implies that we have to teach them also how to cope effectively with feelings of frustration or sometimes anger. In other words, when teaching and learning mathematical problem solving, the allowing of space for negative emotions might be an educational goal from a cognitive, as well as motivational, point of view. Indeed, only when experiencing negative emotions will students have the opportunity of learning how to deal with them.

The very features of problem solving mean that the process is typified by frequent *interruptions to pre-established plans* and by local failures, which, according to Mandler's model, initiate the emotional experience process. In his overview of the research on emotions in problem solving, Hannula (2012) concludes: "The main lesson to learn from the research on emotions in problem solving is that emotions are an essential part of problem solving."

Mathematical problem solving is therefore often marked by a strong emotional component, which can very rapidly evolve from positive to negative, pleasant to unpleasant, or vice versa, and emotion management profoundly affects students' performances. So, as emphasised by Op'T Eynde and colleagues (ibidem, p. 204) it is very important to conduct further study into these emotions, in order to "become aware of dynamics underlying the succession of several emotions over a short period of time".

The aim of our research was to study these dynamics at primary school level, investigating students' emotions when faced with a mathematical problem, how these emotions change, and the factors that the students recognise as being responsible for the emotions and for the changes.

METHODOLOGY

Population

The study involved five primary school classes in Canton Ticino (Switzerland): one 3rd grade class (18 students), three 4th grade classes (53 students) and one mixed 3rd- 4th grade class (20 students), making a total of 91 students.

Rationale

It is very difficult to study emotions: all instruments are limited in capturing emotional reactions that are not conscious (Schlögmann, 2002) and, as Ortony, Clore and Collins claim (1988, p. 9): "There is as yet no known objective measures that can conclusively establish that a person is experiencing some particular emotion." These same authors emphasise how emotions are nevertheless not linguistic things, but the most readily available non-phenomenal access we have to them is through language and we are willing to treat people's reports of their emotions as valid.

In the context of mathematics education it has been shown that it is important to adopt a multiple approach to data collection in the research on emotions (Evans, 2004), and how: "mathematical activity should be studied in context, and that researchers should take an actor's perspective [emphasis in the original] *that allows the meaning structure underlying students' behaviours and emotions to become explicit*." (Evans, 2006, p. 241)

In order to "take an actor's perspective", we believe that it is important to choose instruments that contemplate open answers, where the respondent is free to express his emotions using his own words; as Cohen and colleagues (2007) underline:

> It is open-ended responses that might contain the 'germs' of information that otherwise might not have been caught in the questionnaire (...) An open-ended question can catch the authenticity, richness, depth of response, honesty and candor which are the hallmarks of qualitative data.

On the other hand, the low number of mathematics education studies on the emotions of "young" students (primary school) is probably due to the difficulties experienced by the students in terms of possessing and managing the "language of emotions". This is precisely why an important item of our research was an emotional literacy course conducted by Antognazza and Sciaroni (2010) in the classes participating in the research project, aimed at the allowing students to acquire an understanding of their emotions, and also to learn a specific vocabulary for clearly expressing which emotions they are experiencing at a specific time.

Procedure

There were three phases to our study:

1) **A pre-test** conducted on some pilot classes in order to assess, on the one hand, if the mathematics problems tested were calibrated, and to what extent (i.e. the average level of difficulty for the students), and, on the other hand, to identify a codification protocol for analysing the open-answer questions.

The questionnaire used in the research, and the codification protocol, were defined at the end of this phase, and six problems were identified as appropriate to the research objectives. The texts of the problems were shown to the class teachers, who were asked to choose one that they considered to have the *correct* level of difficulty for their students: neither too easy nor too difficult. One example was: "Giulio and Andrea play together with their toy cars. They have a total of 48 cars to play with. At the end of the game, they each take back their own cars. Andrea has three time as many cars as Giulio. How many cars does Andrea have at the end of the game? How many cars does Giulio have at the end of the game?"

2) **An activity phase**. The students were asked to read the selected problem on their own, to answer the questionnaire tested in phase 1, and, subsequently, to try to solve the problem. No time limit was given. This phase was conducted individually in school workplaces in which the students felt comfortable (class, creative activity laboratory or support class).

The questionnaire given to the students consisted of three questions investigating three aspects:

a) assessment of the difficulty of the problem. Closed-answer question "*how do you evaluate the problem?*" with the following possible alternative answers: easy, quite easy, medium, quite difficult, difficult;

b) expression of the emotion perceived when having to solve the problem. Open-answer question: "*how do you feel about having to solve this problem?*";

c) identification of the reasons underlying the perception of the emotion expressed: Open-answer question: "*Why do you think you feel like this?*".

3) **Semi-structured individual interview** (transcribed), focussing on the emotional states experienced when solving the problem, any changes from the emotion initially expressed ("*On the paper you wrote that you were* [emotion written down]. *Did you feel any other emotions when you were solving the problem?*"), and the reasons for these changes ("*Why did your emotion change? What happened?*").

The students were observed while they were solving the problem, in order to analyse their behaviour and their facial expressions. This observation took place under conditions of experimenter blindness (the experimenter did not know how the students had answered the questionnaire), and was also useful in terms of allowing specific questions to emerge in the course of the individual interviews.

RESULTS AND DISCUSSION

A priori assessment of the difficulty of the problem

An analysis of the questionnaire showed how, after having read the text, most students effectively classified the set problem as "quite easy" or "medium" (see Table 1 below).

A priori assessment of the difficulty of the problem	Easy	Quite easy	Medium	Quite Hard	Hard
Percentage	12%	43%	31%	12%	2%

Table 1: A priori assessment of the difficulty of the problem

Emotion perceived from reading the text

Analysis of the answers to the second question – open-ended and fluid responses – is more complex. On the one hand the positive/negative emotion dichotomy must be defined, while on the other hand it is necessary to identify the "representatives" in order to categorise various labels that appear to take inspiration from similar emotions.

According to Ortony and colleagues (1988), emotions are considered as "valenced reactions" to consequences of events, action of agents, or aspects of objects, and then it is possible to classify the reactions to events as being pleased or displeased, the reactions to agents as approving or disapproving, and those to objects as liking or disliking. These dichotomies permitted a first classification of emotions into positive and negative.

In terms of identifying the representatives, the codification protocol developed following the pre-test phase was utilised, making it possible to identify 6 categories consisting of 3 dichotomous pairs: *confident*, *worried*, *nervous*, *calm*, *happy*, *sad*, to which, a posteriori, was added the category *bored,* not found in the answers obtained in the pre-test. We emphasise that the answer was open, without any kind of restriction and with the possibility for the students to specify more than one emotion.

Table 2 summarises the quantitative results obtained.

Some interesting observations: the emotions initially stated, and supported by the subsequent interviews, are essentially perfectly divided into "positive" and "negative", but although this equilibrium is also found in the specific nervous/calm dichotomy (22% vs 20%), "worried" appears to predominate over "confident" in terms of the possibility of solving the problem, and "happy" appears to predominate over "sad" in terms of having to tackle the problem.

Another significant aspect consists of the fact that the balance between positive and negative emotions obtained from the sample of the five classes is less strong when analysing the individual classes involved in the study. For example, the 3rd grade class is typified by positive emotions selected by 17 out of 18 students (10 students express happiness, and 7 students express calmness), while in other classes negative emotions predominate. For example, in one of the 4th grade classes, consisting of 17 students, the following emotions were expressed: worry (6 students), nervousness (3 students) and sadness (2 students). This may depend partly on cognitive aspects such as superior competence, but also on affective aspects, such as conviction in one's own understanding regarding the specific problem, and on other aspects (emotions, convictions, attitudes and values) depending on the specific social context of the class: "students' interpretation and appraisals processes that initiate the emotional process are grounded in a specific context" (Op 'T Eynde & et al., 2006, p. 196).

As a qualitative note to the analysis of the second question of the questionnaire, we emphasise that in three cases the students' answers testify contrasting emotions. In one of these cases, Enea states testifies between the fear of not knowing how to solve the problem ("*a little scared*") and his confidence in his own abilities, or, in any case, the desire to muster his courage ("*but I'm sure that I can do it*"). In the other two cases, the contrasting emotions are associated with various aspects: *intrinsic*, in the sense of being related to the specific problem, and, *extrinsic*, in the sense of being extraneous to the specific problem. Martina appears worried about having to solve the problem of the toy cars ("*I am in a little bit of difficulty because I can't find the calculation*"), but at the same time she notes that she feels fine as she always does in class ("*I feel good, like in class*"). On the other hand, Mattia feels nervous because, as he later explains in the interview, he always feels nervous when he has to solve problems, but he feels relatively calm about the toy cars problem.

Correlation between the perception of the difficulty of the problem and the emotion stated

Most of the students who state that they think that the problem is easy/quite easy feel calm or happy; on the other hand, those who think that the problem is quite difficult/difficult feel worried or nervous. Out

Emotion	Confident	Worried	Nervous	Calm	Happy	Sad	Bored	Other
Percentage	7%	18%	22%	20%	22%	2%	8%	1%

Table 2: Emotions associated with the problem

of 91 students, only 13 do not assign positive emotions to presumed ease, or negative emotions to presumed difficulty.

Analysing the answers to the third question (the reasons for the emotions), these 13 students are seen to divide into two "categories". The first category consists of those who, in a certain sense, show that they appreciate the "intellectual challenge" of the mathematics problem (usually good-achievers) and for whom the easy problem is boring (Chiara: "*I feel bored because it's boring when problems are easy*"), and the difficult problem a real challenge (Irene, who thinks that the problem is difficult, writes: "*The emotion that I feel is joy because the problem is nice*"). The second category consists of those who, like Mattia cited above, say that they feel nervous when they have to solve mathematics problems, and although they think that the toy cars problem is easy, are overcome by what we might call a worry consistently associated with having to tackle mathematics. This is a very interesting phenomenon, in terms of both research on affect in mathematics education, and in terms of didactics. In fact, it is precisely when aversion, worry and fear are experienced *regardless* of the intrinsic aspects that a negative attitude of fatalism toward mathematics can develop, an attitude which is very difficult to modify (Zan & Di Martino, 2009).

Causes of the declared emotions

More generally, when analysing the causes of the declared emotions (question 3 of the questionnaire), the aspect we found most interesting was that of distinguishing between reasons referring to the specific problem (*intrinsic*) and reasons not referring to the specific problem (*extrinsic*).

If we consider only the explicit references to intrinsic or extrinsic aspects – not classifying in either sense statements difficult to interpret, such as "*I feel nervous because I am afraid of making a mistake*": the fear of making a mistake might be associated with doing mathematics (extrinsic) and unrelated to the specific problem, or, on the other hand, it might be associated with the assessment of the level of difficulty of the problem to tackle (intrinsic) – what emerges from the positive/negative emotions dichotomy appears to be rather significant. In fact, the positive emotions are motivated mainly by intrinsic aspects (for example "*because it's easy*"): 56% of those who said they felt happy, 62% of those who said they felt calm, and as much as 86% of those who said that they felt confident. The negative emotions are motivated mainly by extrinsic aspects (for example "*because I don't like doing problems*", "*because I don't like mathematics*", "*because I don't like school*"): 58% of those who said that they felt worried, 67% of those who said that they felt nervous, and more than 86% of those who said that they felt bored.

It should also be noted that the 54% of the students who said that they felt nervous, and the 45% of those who said that they felt worried, highlight the fear of making a mistake or the desire to do everything correctly (for example: *"because I worry about making mistakes"; "I don't like them. And if I get them wrong, how will I manage when I'm big"; "because I don't know if I do it all correctly"; "I think that it's because I don't know if the problem is right or wrong")*. This phenomenon is particularly significant because fear of making errors often becomes fear of math, a phenomenon that has serious consequences on mathematics learning (Di Martino & Zan, 2013).

Change in emotions perceived while solving the problem

One particularly interesting feature from what has been observed until now emerges from the interviews: most of the sample (52%) reports emotional changes from the *initial state*. Those who say that they did not experience any emotional changes are mainly those who started with a positive emotion that was confirmed in the course of the problem-solving process, or else those who started with a negative emotion, associated with extrinsic aspects, an emotion that affected their perseverance in trying to solve the problem. This confirms the link between negative emotions associated with extrinsic aspects and poor perseverance (Hannula, 2012).

The following observations emerge from an analysis of the interviews conducted with those who stated that they experienced emotional changes:

i) **changes in 'intensity'** (8%, only one case with positive emotions and 6 starting from negative emotions), like that reported by Chiara, who initially said that she felt worried because after having read the problem she thought she would not manage to finish it, but who in the interview explained that at a certain point she stopped feeling worried and became "*almost desperate, because already I didn't manage to finish it, and then,* *seeing all these calculations …*", or like Martino, who said that he felt happy during the solving process "*because I realised that the calculation wasn't so difficult*";

ii) **changes in 'direction'**: from positive to negative emotion (25%) (like Samuele, who initially said that he felt happy, but who in the interview explained that he felt worried "*because I was trying to do the calculation but I didn't manage*"), and from negative to positive emotion (14%) (like Selene, who said that she felt worried because she felt alone, but who at the end said that she felt happy because she hoped to have solved it correctly and because "*I feel I did it correctly*"). Moreover, 5% of the students expressed a double change in "emotional direction" while solving the problem: for example, Sofia said at first that she felt calm, then nervous *"because I was afraid of making a mistake"*, then calm again because *"it seemed easy to do"*. Sofia's account demonstrates a repeated assessment of the situation and of any progress made, and, in particular, this testifies the fact that her emotions are (also) associated with intrinsic aspects.

In both cases (changes in intensity and in direction) it is seen how, except in very rare cases, when explaining the emotional change reference is virtually always made to intrinsic aspects, to the perception of making, or not making, progress in solving the problem. This confirms the close link between affective and cognitive aspects: in fact, a fundamental role is played by convictions about making (or not making) progress toward solving the problem, about having solved the problem correctly or not, convictions that are profoundly linked to the mathematical knowledge of the students. In relation to this, and as an important comment, it is interesting to observe how many of those who changed from a negative to a positive emotion because they were sure that they had solved the problem correctly, in fact handed in a wrong solution to this problem. It would be interesting to investigate the emotion triggered off by discovering that the solution handed in, and believed to be correct, was in fact wrong.

CONCLUSION

In the analysis of the results we have focussed particularly on what we feel is the most interesting aspect of our study, in terms of both research and didactic practice. That is, the distinction between positive or negative emotions perceived in a specific mathematics problem solving activity and deriving from an assessment of the difficulty of the activity proposed (intrinsic aspects), or from more general aspects (for example, the stance of those who think "I don't like mathematics, you are asking me to do maths and I therefore have a negative emotion").

What clearly emerges is that most of those who express a positive emotion on finding out that they have to solve the problem in fact refer to intrinsic aspects, and the issue is therefore one of problem assessment. Conversely, most of those who express a negative emotion refer to aspects unrelated to the problem, or, in other words, to extrinsic aspects (except for the fact that it is recognised as a mathematics problem).

On the one hand this confirms how, already at primary school level, there is a generalised type of "a-priori" hostility towards all things mathematical, hostility that is very often typical of an unquestionable negative attitude toward mathematics (Di Martino & Zan, 2011). On the other hand it suggests the importance of working with students so that any negative attitudes of this type do not compromise the a-priori cognitive assessment of the mathematical activities to be tackled, and the resulting performance. In fact, the negative effects of this kind of phenomenon are clear: negative emotions associated with any mathematics-related activities, deciding against investing the cognitive resources required in order to tackle the activity.

However, the results of our study tell us something more: emotional changes (intensity and direction) during the problem solving activity occur almost exclusively as a result of considerations related to intrinsic aspects. These emotional changes appear to be fundamental in problem solving activities: both those "in the positive direction" (they play an important role in problem-solving perseverance), and those "in the negative direction" (they have an important function in terms of cognitive control stimulation).

Further investigation into the link between emotional change during problem solving, and aspects as perseverance and the activation of cognitive controls would certainly constitute an interesting research direction. It definitely appears that it may be important to promote emotional literacy and emotional awareness, also in order to develop problem solving skills already at primary school level. Our research also shows how, at the same time, it is fundamental to develop the "proper mathematical understanding": in our experiment we have shown how many of the positive direction emotional changes were related to the erroneous conviction of having solved the problem correctly.

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