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This paper aims to highlight the different aspects of a discussion on creativity promotion from the point of view of an educator involved in professional development of pre-service teachers. Challenging the prospective teachers with open mathematical problems provides data on their beliefs and behaviour concerning creativity and creativity encouragement in the classroom. We emphasize a certain manner of revealing elements of relative creativity during students' activities. The final remarks suggest relevant agenda for further discussion and research.

Keywords: Mathematics creativity, teachers' education, problem solving, problem posing.

INTRODUCTION AND BACKGROUND

The promotion of creativity seems to be the central issue of mathematical education at all levels. As mathematics teacher educator, I believe in a crucial role of the teacher in promoting students' creativity, therefore, the development of creativity-inviting environment (Sinitsky, 2008) and analysis of students' activity in this environment are the issues of my primary attention.

During my long-term pedagogical practice, I have collected a vast amount of scattered empirical data on different approaches utilized by prospective and in-service teachers for treating open mathematical situations. The course called "Development of Mathematical Thinking" is a subject of my explicit interest, as its attendance consists of prospective elementary school teachers. This paper refers to various aspects of behaviour and the activities of future teachers themselves as the students of the course. With almost two hundred prospective elementary school mathematics teachers involved in different stages of a study, it may be regarded as fairly wide albeit not a very systematic one. Obviously, some aspects of the study have been related to creativity promotion, and provide an empirical basis for the discussion below.

Despite widespread declarations on cultivation of creativity as the core of mathematical education, there is no single accepted definition of creativity (Mann, 2006; Sriraman, Yaftian, & Lee, 2011). Since creativity is related to the process of problem solving, some research papers focus on in-depth investigations of several, mainly intermediate, stages of cognitive and mental processes - in the spirit of a four-component model of preparation, incubation, illumination and verification proposed by Wallas (Dodds, Ward, & Smith, 2003). Following this paradigm, researchers pay major attention to the structure of an 'Aha!' moment (Liljedahl, 2009; Prabhu & Czarnocha, 2014). At the same time, Leikin (2009) has enriched the model of creativity as a specific combination of fluency, flexibility and originality of Torrance, making it possible to measure various components of creativity. In the frame of this paper, we refer to this description of creativity that enables us to analyse the creative elements in mathematical behaviour of prospective mathematics teachers for elementary and secondary schools.

Recent researches on teachers' component of creativity have been carried out both on macro- (Leikin, Subotnik, Pitta-Pantazi, Singer, & Pelczer, 2013) and micro-levels (Pitta-Pantazi, Sophocleous, & Christou, 2013). Up to now, however, teachers' conceptions and practice in relation to creativity has not been studied systematically (Lev-Zamir & Leikin, 2013).

The aim of the paper is to propose additional issues for further research agenda in the field from the point of view of teachers' educator. The discussion refers to the empirical data concerning the following questions:

— What do prospective and in-service teachers think about the possibilities of promoting creativity through everyday learning of mathematics in elementary school?

— How would they deal with various creativity stimulating activities?

— Which features of creativity could be associated with the process of solving multi-step mathematical tasks by prospective mathematics teachers?

BELIEFS VS. DECLARATIONS: POSSIBLE REASONS OF THE GAP

Studying teachers' beliefs regarding the encouragement of students' creativity was not our primary goal. Yet, the issue has arisen when pre-service teachers were asked to list the reasons for the importance of the course "development of mathematical thinking". "We need to encourage creativity in framework of mathematics lesson in school; and the competence of mathematics teacher in aspect of creativity promotion is a key factor in this process," – they stated. This composite proposition has almost become an axiom in the last decade, and pre-service teachers are broadly familiar with both components of it. In the open-form questionnaire, almost 80% of them declared that teachers are required to develop mathematical thinking through learning mathematics rather than focusing solely on standard algorithms and procedures. Significant part of prospective teachers also delivered interesting thoughts on the necessity of involving all the students in the learning process by using suitable pedagogical tools. Nevertheless, it is known (Lev-Zamir & Leikin, 2013), that the likeness of declarative conceptions concerning creativity does not provide the similarity of pedagogical practice related to promoting creativity.

Let us examine the 'real value' of these claims. The same group of students discussed the well-known problem: divisibility of sum of consequent addends:

> Assignment 1. Construct different sums of three consequent addends. What is the common property of these sums concerning divisibility? Try to prove your assumption.
>
> Check the situation with another quantity of consequent addends. Try to generalize.

Following a multi-stage process of problem solving, discussing the results and summarizing the conclusions, students have been asked about possible ways of introducing this activity (partially or as a whole) in elementary school mathematics lessons. Only 20% of respondents have described more or less suitable arithmetic situations or problems.

The reasoning of the remaining 80% of students as to why they could not use such a task in a regular mathematics lesson in an elementary school classroom can be summarized as follows:

— This activity is a difficult one; it is suitable for advanced students only.

— I have no idea how to fit this activity to the needs and the abilities of elementary school students.

— The activity does not belong to any school curriculum.

We observed similar replies when a group of in-service teachers had been discussing analogous tasks during a professional training course. Moreover, many of them have added that "it seems to be a waste of time".

Let me put here two notes and to propose some related questions.

The first remark concerns the current elementary school curriculum. Is it possible that the extensive familiarity of teachers with textbooks and other teaching resources rules out the option of creativity-stimulating activities? In other words, *does the actual content of (elementary) school mathematics invite those activities and learning styles or at least provide a suitable environment for introducing them?* A survey by Sheffield (2013) contains some significant remarks on this topic, but the problem, indeed, requires a separate discussion concerning both evaluation standards and curricula issues.

Another comment is on the structure of pedagogical content knowledge (PCK). In terms of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008), we can easily conclude that pre-service teachers do not have enough PCK in the field of creativity promotion. However, *which specific component of pedagogical knowledge can be associated with tools that a prospective (or current) teacher might use in a classroom to encourage creativity?* It certainly does not fit into the 'knowledge of content and teaching' and cannot be included into the 'knowledge of content and students' category that deals with strategies of learning about specific issues. Since the ability of problem solving is both a principal component of mathematical reasoning and its main measure (Borasi, 1994; Silver, 1997), non-routine activities that stimulate creativity have to appear throughout all the content of subject matter. It seems that awareness on components of creative thinking lies beyond the topics of school mathematics and needs to be built as a core and cross-subject pedagogical mathematical knowledge. By analogy with horizon knowledge (HK) of subject matter content knowledge, it can be determined as HK component of PCK.

Let us turn now to the practice. How do prospective teachers deal with mathematically challenging situations? It is accepted that pre-service teachers need to construct their pedagogical knowledge about encouraging creativity by using their own experience of 'mathematical challenge and discovery' accompanied by further practice in the elaboration of creativity-stimulating activities for students (Shriki, 2010).

"WHAT DO I NEED TO DO?"

For years, students have started the course "Development of mathematical thinking" by filling in a closed-type questionnaire on the nature of problem solving in elementary mathematics. I have observed throughout the years that about 70% of prospective teachers have accepted the following description of mathematics problems and the ways of solving them:

— Each problem belongs to a specific mathematical topic, and there are explicit tools that are suitable for solving the problem within given mathematic areas only;
— To solve the problem, one needs to execute a series of operations (in the right order), similar to a sample, typically known to a student.

These data correlate with the findings of Zazkis and Liljedahl (2002) on perceiving school mathematics as a collection of isolated propositions and the tendency of forthcoming teachers to focus on formulas and algorithmic procedures.

All students had enough knowledge in elementary mathematics to progress with proposed assignments through the research. Nevertheless, the title of a section is, not surprisingly, a citation of central theme and a leitmotif observed in students' replies once they had faced a mathematical situation without an immediate solution algorithm. The pre-service teachers were real newcomers to the field: only 2% of respondents have acknowledged having practice with non-standard mathematical situations and/or open problems.

During a six-month course students were typically challenged with 7–9 assignments of open type (Silver, 1997). Certain assignments were multiple-solution tasks (Leikin, 2009) while others included the search of multiple solutions as an essential stage of the overall inquiry. Each assignment was presented as a multi-stage problem with auxiliary questions acting as a natural way to generalize results derived from the earlier stages. The set of assignments was designed according to a criteria elaborated by author (Sinitsky, 2008). I present here two of these assignments alongside the analysis of students' solving strategies. The first activity and a subsequent extensive discussion on the nature of open-problems has in fact served as a 'pedagogical preparation' for the following assignments.

> Assignment 2. Student has solved the following task: "For a given chain of natural numbers 1, 2, 3, insert signs of arithmetic operations between numbers in order to obtain arithmetic expression which equals zero." He has produced the solution as follows: $1 + 2 - 3 = 0$.
>
> Try to solve the similar problem for longer chains of natural numbers that start with 1, i.e. find suitable arithmetic operation signs to obtain the equality for

> 1 2 3 4 = 0; 1 2 3 4 5 = 0; 1 2 3 4 5 6 = 0; … etc. For each chain, find as many solutions as you can.
>
> Try to find solutions with addition and subtraction only. What can you say about the number of those solutions?
>
> Come back to the same chains with a number 1 as a target result. Explore each of the previously solved chains (for instance, 1 2 3 4 = 1).

When dealing with the first part of Assignment 2, the students have used two principal strategies, i.e. 'balancing' of added and subtracted operands (as $1 - 2 - 3 + 4 = 0$), and multiplying by an arbitrary factor of an algebraic sum that equals to 0 (as $(1 + 2 - 3) \times (4 + 5) = 0$). Each strategy provides a significant amount of relevant solutions, thus we suppose that the fluency in this problem is a function of the number of proposed solutions of a given type and of time the student spent on finding them. After a couple of preliminary stages, many students have worked more or less in an algorithmic manner in compliance to the scheme mentioned by Ervynck (1991). Yet, some students have 'rediscovered' the leading principle for each solution without using any routine procedures.

In contrast to fluency, the flexibility of solutions in this assignment is associated with a shift to another type of solution (for example, switch from expression $1 - (2 + 3) + 4 = 0$ to $(1 + 2 - 3) \times 4 = 0$). We also regard the ability to adjust previously constructed solution to another situation as a feature of flexibility. Thus, we interpret the transition from 'balanced' equality $[(1 + 2) \times 3] - (4 + 5) = 0$ to the equality with unity as a quotient of two equal numbers, $[(1 + 2) \times 3] : (4 + 5) = 1$ as flexibility as well.

Typically, for different groups of students, about 80% of future teachers have shown several degrees of fluency and almost 40% revealed some flexibility in their solutions for Assignment 2. Additionally, small portions of students (about 8–10%) have constructed a handful of surprising and non-trivial solutions that will no doubt belong to unconventional solution space (Leikin, 2007). I would like to present two notable expressions as an example: $((1 + 2) : 3 + 4) : 5$ and $(1 - 2) \times (3 - 4)$. Remarkably, these solutions have served as a starting point for further fluency, as in $(1 + 2) : 3 = ((1 + 2) : 3 + 4) : 5 = (((1 + 2) : 3 + 4) : 5 + 6) : 7 = \ldots$, and flexibility as in $(1 - 2) \times (3 - 4) = 1 \times (2 - 3) \times (4 - 5)$, with a relevant search for limits of possible generalization. In terms of Koestler (1964), progress in understanding provides the basis for the exercise of understanding, and can even lead on to the "next level" of understanding.

According to the accumulated data, Assignment 2 had served as a reasonable tool to evaluate the components of creativity through constructed solutions. This assignment also invites a discussion in classroom about introducing some creativity-related concepts to the reference group. After reaching such a promising conclusion let us turn to the next assignment and analyse the student's activities within the framework of the following example:

> Assignment 3. We assigned the label 'exceptional' to number 11 because it can be expressed as a difference of two square numbers: $11 = 6^2 - 5^2$. Is this the only 'exceptional' number? How can we find other 'exceptional' numbers? For a given natural number, is there a way to write it as a difference of two perfect squares? Can we state that each natural number is an 'exceptional' one?

At the first stage of the solution, pre-service teachers have demonstrated a very limited repertoire of tools and ideas. Almost half of them claimed they 'can do nothing' with a problem and the following dialogue was a typical stimulating tool to start some progress towards a solution:

> Tutor: Do you really believe that 11 is the only "exceptional" number?
> Student: No, I think it is not the only example.
> Tutor: How can you *calculate* an additional "exceptional" number?
> Student: Aha! I can take any pair of perfect squares and subtract one from another. For example, $10^2 - 9^2$, therefore 19 is also an "exceptional" number.

Despite the fact that various pairs of squares give numerous "exceptional" numbers, the ability to record a series of technical results hardly seems to be associated with a fluency of thinking. Alternatively or additionally, some students have shown a fluency in their search for solutions, constructing chains of differences with some regularity. For example, students have constructed a series of differences of squares of consequent numbers or a series of differences of squares with a constant difference between bases, but in all their suggestions students did not *use algebraic expressions*. As a result, those students have derived a distinctive series of desirable representations of natural numbers as "exceptional" ones.

Since representation of any odd number as a difference of two (adjacent) square numbers was the ultimate outcome of the above scheme, a number of students have changed the pattern to produce solutions of diverse types. Following this route, they have discovered "multi-exceptional" numbers – those that have more than one representation as a difference of square numbers. Flexibility may certainly be attributed to this step of the solution.

Can we find any elements of originality among the routines explored by the students? Which mathematical tool did they use in order to complete the solution? Notably, the factorization of squares' differences alone almost immediately leads both to a list of possible wanted decompositions and a discrimination of criterion as an opportunity of such a representation for each natural number. Since a suitable formula has been applied, one may continue with a simple routine. In this context, the breakthrough is connected to a switch towards simple algebra, and not a single prospective elementary school mathematics teacher succeeded in making this switch.

Discussing this disappointing result of the last assignment is especially interesting given the fact that the situation was fairly similar to other assignments, but it is not the scope of this paper. Instead, let us come back to the main goal of our case: assignments as a room to explore creativity. Did we really construct the set of assignments in order to identify gifted students (those with extraordinary creativity) in a population of pre-service teachers? Note that I did not present any numerical data on measuring fluency and flexibility of students' solutions, and I did that for a reason. We want prospective teachers to deal with creativity-promoting assignments in order to make them familiar with the field and to equip them with principal notions and components of 'everyday' creativity (Kaufman & Beghetto, 2009). It is a matter of fact, that through such an experience prospective teachers have meet "global" mathematical ideas and concepts (Safuanov, 2015).

On the other hand, pre-service teachers' own experience may serve, to some extent, as a reasonable model of mathematical behaviour of students faced with challenging activities. Analysing the collected data, we can suggest some specific features for the characteristics of creativity-connected activity of non-experienced students in non-standard mathematical situations.

Most of these students *need* certain *guiding hint or solution as a starting point* to go further and explore the problem. This hint may simply be a minor reformulation of a problem in the terms of possible answers (as one can see from the presented Assignments). In the absence of this 'push', students typically come back to the above-mentioned question "What do I need to do?" in full compliance with the statement of Sternberg (2009) on the role of a supportive environment as a condition for demonstrating creativity.

Furthermore, we can attribute a number of particular features to the dimensions of creativity that have been developed by pre-service elementary school mathematics teachers when exploring multi-step problems. As we have seen, *fluency* is associated with a series of solutions produced on the basis of a discovered sample or pattern. Contrary to this, *flexibility* reflects the ability to switch to a completely different pattern or to take the search to another direction. It seems that the discussion on originality is not relevant in this case, since some straightforward algorithmic solutions have to be accepted as "original" ones: they do not belong to the set of solutions produced by similar groups.

INTEGRATION OF PROBLEM POSING AND PROBLEM SOLVING

The multi-stage and open character of proposed assignments invited and stimulated the students to pose specific and general questions on mathematical situations and also on the results of the preceding steps of their inquiry.

Since problem posing is a central component of creative processes (Silver, 1997), we have constructed and researched the situation of 'pure' problem posing in the frame of above-mentioned course for prospective teachers. A group of 21 pre-service teachers learned the course "Development of mathematical thinking" in self-regulated learning (SRL) format. Following exemplification with several assignments, students were requested to construct *their own open-type mathematical problem* and to explore it. With accordance to 'pure' SRL approach (Goodwin, 2010), students have been asked to design their study, including the choice of a relevant mathematical situation itself as well as the tools and rate to explore it. In this approach of 'absolute free problem posing', the mission proved to be impossible to complete: only 30% of students were able to even formulate a task, and all the tasks submitted had a form of enrichment questions concerning curriculum items of different levels. This result is not a surprise: prospective teachers needed to 'work like real mathematicians' (Shriki, 2010) through generating and solving new problems. Nonetheless, after the tutor's intervention and intensive group discussions, 75% of respondents were able to construct open mathematical situations of different grades of complexity and relevance.

Another item concerning mathematical creativity awareness of teachers is their ability to construct challenging activities for students. As a pilot test, we asked four prospective teachers to adjust their successfully constructed assignments for students of an elementary school. One of the results is presented below.

> Assignment 4 (original version). I have a set of 40 items that look identical but have different weights. The weight is a whole number in kilograms, from 1 to 40. In order to find the weight of each item, I can use a scale in one of the two manners: to balance an item on one side with one or more weights on another side; to balance an item and weights (if necessary) with a set of weights on another side. For each manner, what is the number and the value of required weights?
>
> Assignment 4 (adapted version). Represent arbitrary natural number from 1 to 40 with numbers 1, 3, 9, 27 and using operations of addition and subtraction (each number may be not be used more than once).

Unfortunately, the result is somewhat of a profanation of the initial assignment, and a total loss of the creativity component can be observed. This means that even when prospective teachers try to adapt their own problems to a "real-world" classroom, they are expected to meet some crucial difficulties. Recent research that was focused on the ways teachers posed problems for their students (Pitta-Pantazi, Christou, Kattou, Sophocleous, & Pittalis, 2015) proves that we still need to consider a wide range of cognitive, psychological and social factors.

FINAL REMARKS

Let me emphasize below the major questions dealing with encouraging creativity that, in my opinion, require further clarification.

- Part of pre-service teachers believe that the content of elementary school mathematics is not suitable for activities that promote creativity. Does (and to what extent) the current curricula allow, invite and encourage mathematically challenging activities?
- Pedagogical content knowledge emphasizes the importance of issue-dependant ways of teaching and learning. What is the place of awareness on creative thinking and which are the ways to promote it in the structure of pedagogical mathematical knowledge?
- According to our findings, external support at the initial stage has a crucial role for launching the process of creative thinking. Additionally, the elements of *fluency*, *flexibility* and *originality* appear: utilizing patterns and samples are two examples of that. Do those peculiarities depend on reference groups and/or on the nature of the proposed assignments?
- Posing of mathematically challenging problems must be a part of teacher's repertoire. How does this ability relate to the experience of problem posing through prospective teachers' own handling open mathematical situations?

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