Learning geometry through paper-based experiences

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Teaching of geometry in the elementary school in Poland is carried out in a very static way. Students learn various geometrical concepts in concrete, typical situations. They rarely meet these concepts in less common situations or not directly related to the geometry. In this paper, I will present a part of activities that have been carried out among pupils in the fourth grade of primary school. During these courses, students had the opportunity to take a look at familiar geometric concepts (square, cube, perpendicular, parallelism) from a new perspective.

Keywords: Geometry, primary school education, spatial cards, teaching geometry, 2D and 3D geometry.

INTRODUCTION

In recent years, there has been a lot of discussion in Poland on changing the education system. The results achieved by Polish students in various tests and exams showed that this topic is a lot of catching up to do. The new curriculum for mathematics has pointed out that it is necessary to develop students' competencies, which are important not only during a math class, but also in everyday life. A lot of space was devoted to geometry, which so far has not been significantly present in teaching, especially at the lower levels of education. And often geometrical problems are ignored or treated unfavorably by teachers at a lower level of education (kindergarten education and I-III grades of primary school). Teaching geometry is confined to familiarize students with the basic geometric shapes, i.e. square, rectangle, triangle and circle. In addition, students learn to measure the length of a segment, draw straight lines and segments which are perpendicular or parallel. At the next level of education geometry appears as a separate branch of mathematics, where the student is expected to have a formal geometrical knowledge. This leap from the first to the second level of education meant that for many students geometry has become as a very difficult subject. An additional difficulty is also the fact that geometry requires a much different approach than other areas of mathematics, e.g., arithmetic or algebra. It cannot be "algorithm and routine" and put in rigid frames. Thus, few questions appear: How to help students in the transition between the levels? How to help students develop their geometric intuition, which will be the basis for their future formal knowledge? Trying to find the answer to these questions

I decided to organize a series of activities with the students, whose goal would be to develop their interest in geometry. The activities were organized in such a way so that the student would recognize geometric issues somehow by accident, while having fun.

THEORETICAL BACKGROUND

Geometry is an integral part of our daily life even if we do not realize it. Geometry teaches the basic skills of logical thinking and reasoning. We can observe that in the latest trends in early education a large emphasis is placed on developing the skills needed for a child to explore and understand the world and to cope with different situations of everyday life. The skills that are particularly useful in various situations include analyzing, critical thinking and putting and verifying hypotheses. The tasks of the school according the new curriculum include the care that a child could acquire the knowledge and skills needed to understand the world and the tools the child needs in math skills in real-life and school situations and for solving the problems.

The most important skills acquired by the student in the course of general education in elementary school should be, inter alia, mathematical thinking, comprehension as the ability to use the basic tools of mathematics in his or her daily live and carrying out elementary mathematical procedures. On the higher stages of education mathematics is presented as structured and ordered formal knowledge, with specified chapters of mathematics. This enables the advanced work both on exploring and developing mathematical knowledge and develop formal (symbolic) mathematical language. At lower levels of education (especially in the lower grades of primary school) students are not presented a finished, formal knowledge from different areas of mathematics, but are introduced them to the world of arithmetic and geometry (Hejný & Jirotková, 2006). The world of arithmetic is ultimately structured and governed by clear rules. Individual records and symbols used in this world are read by all in the same way. The presents the situation in the world of geometry is different, as Hejný and Jirotková (2006) write:

> The world of geometry is a community of individuals or small families and there is a large diversity in the linkages between them. From the didactic point of view, arithmetic is suitable for developing abilities systematically, and geometry is more suitable for abilities such as experimenting, discovering, concept creation, hypothesizing and creating mini-structures. (p. 394)

By analyzing the historical development of geometry we can notice that it was accompanied by human in his activities since the dawn of time (much like arithmetic). Initially, geometry was not a theoretical science, but appeared from the need and desire of people to arrange the space around them, solve many practical problems - from construction by travelling to the ornamentation (Hejný, 1990). However, this geometry was the first "scientific field within mathematics" which was created by human. Its significance for the study of the ancient world was great. It had an important role in mathematics. This historical trait also points the way for didactical approaches to teaching school geometry: geometrical knowledge arises by action. Thus, it is important to gain experience, and practical problem solving.

The importance of geometry in the education of children and young people was the topic of many researchers' considerations. There is a belief that geometry can support the overall development of the child's competence in mathematics. Swoboda (2009) has written the importance of geometry in teaching children and adolescents. She emphasizes that a geometrical approach is closer to a child than an arithmetic one and can open doors to the world of mathematics. It is important that "Geometrical cognition starts from a reflection upon the perceived phenomena and in this way correlates with the basic ways of learning among children" (p. 29). In addition, geometry gives the opportunity to develop the mathematical ways of thinking such as generalization, abstraction, perceiving relations and understanding rules.

Although geometry has a great potential to develop mathematical thinking of students in the school teaching it is not treated with due care. It consists of a number of factors. As Karwowska-Paszkiewicz, Łyko, Mamczur and Swoboda (2001) write, one of them is the very limited number of lessons concerning geometry. It is the reason why during geometric lessons some "ready knowledge" is given and students "did not have the opportunity to learn the properties of the figures through manipulating and even if they had it, it was only apparent" (p. 86). Hence, there is no place in this teaching style for problem solving teaching, and there arise difficulties in the connection between the problem, the procedure of solving it and the solution.

To take advantage of the full capabilities of the geometry in the education of children and youth, you need to change the approach to its teaching. Geometry was born out of the action and of humans needs for development and structuring of space around them. Therefore, an important element in the teaching of geometry should be acting. As we can see in Swoboda (2001):

> Action play an important role in the formation of geometrical concept because there is always correlation between concept and the activity addressed to the concept. The object from the real world are perceived as the gestalt. The way of gathering information is perception, but after that the action with the object leads to the verbal description in their properties. (p.151)

So an important didactical issue is how to organize activity in mathematics lessons in such a way as to encourage students to actually participate in the lesson and to give them a chance to creative thinking and discovering mathematics.

The experience plays an important role in the learning process. Specifically writes about this Hejný in his theory of the General Model (Hejný, 2001). Children create their own knowledge primarily based on the experience they have. For a description of these experiences they use language - as it is close to them. The closest experience for a child is the language of gestures. It is the first language, how a child learns. This language is very helpful during learning of mathematics. Just as Cook and Goldin-Meadow (2006) say: "children who produce gestures modeled by the teacher during a lesson are more likely to profit from the lesson than children who do not produce the gestures". Only at a later stage of learning a child meets the formal language of mathematics. It is important that the experience and action in the acquisition of mathematical knowledge appears first, followed by language, and only at the end there is formal knowledge. According to Burton (2009), the human activity is the first and after that mathematics arise. And mathematical language (mathematical concepts, objects and relationship) arise through natural language, and within particular socio-cultural environments, in response to human thinking about quantity, relationship and space.

METHODOLOGY

Data for this paper were collected during classes with third grade students from primary school. It was a series of meetings, whose main purpose was to develop the students' interest and talents of mathematics. Classes took place once a week and last one school hour (45 minutes). Twenty students from the third grade of primary school (10–11 years old) took part in these meetings. They were students, who coped pretty well with school mathematics. They had no major problems with the mastery of the material carried in the classroom. They were students willing to undertake new challenges. During these meetings the students through fun developed their mathematical abilities. Much of the class was devoted to geometry. Students through play, by using paper and scissors, learn about various properties of figures in the plane and in space. The purpose of these meetings was to develop spatial imagination and the ability to perceive relationships and analogies between objects on the plane and space. Also worked on the development of students' mathematical language, and, in particular, to bring students the terms such as parallelism and perpendicularity.

During each class, students had access to colored cardboard, scissors, duct tape, crayons, markers. Classes were recorded with a video camera. After each meeting a protocol was prepared. The research materials consist of the works done by students, videos and chat records.

In this paper, I would like to present the classes concerning geometry called "play with geometry". The main purpose of the course was to develop spatial imagination and students' interest in geometry. In addition, I wanted to examine:

- How will students cope with the creation of three-dimensional models?
- How will they move between dimensions?
- Will they be able to notice the parallel and perpendicular elements in three-dimensional models?
- Will they be able to describe the work made by them in the mathematical language?

Activities under the name of "play with geometry" are divided into two parts. During the first of them students prepared spatial cards from pre-made templates. They cut out, filed and attached elements so that after unfolded their cards they received spatial composition. These classes were analyzed together, as making certain cuts or submit reflected in the final work. Students have tried to capture all the relationships that exist between objects.

After working with ready-made templates it was the time for students own creativity. They were given colored cards and scissors. Their task was to create any space card. Both themes of work and how to perform it completely belonged to students.

This task apparently had little to do with geometry. My aim was to show students that in such seemingly non-mathematics activities mathematical – geometrical ideas can be found. This kind of geometry we can find in our daily life.

Students began working on the worksheet in half. Then they began to mutilate the card. At first their work was very spontaneous and not targeted. After some time, students began to notice that the method of cutting and bending the paper affects how their work. So they began to think over each of the next move, already analyzed the resulting image and then decide on the next move. Very often the first work done was treated as a "training card". After its execution students looked at it, analyzed it, in order to say what they managed to achieve. Only then they started working on the "final card". When all the students had finished their work, everyone had the opportunity to present what he did. Also a discussion was conducted, in which all the students participated. During this discussion, the teacher drew attention to the manner of carrying out the cuts by the students. There were such concepts as "parallel", "perpendicular". Attention was also drawn to the properties of figures that appeared in the works.

ANALYSIS OF STUDENTS' WORK

The work of Maks

Maks, a student of average ability, he made a very simple job. In an interview with the teacher noted that his foundation was to cut the "three squares". Initially, however, he did not quite know what it really means.

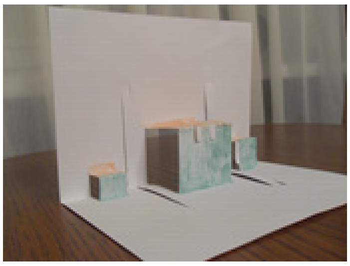

Figure 1: The card made by Maks

Teacher: Maks did the card and he colored it. Please, tell us about your work

Maks: I cut three squares.
Teacher: What does it mean?
Maks: I do not know.
Teacher: [goes to the Maks' bench, takes his work consists in half and shows the class] Maks cut three squares. But how did you do it?
Maks: I fold it and cut here, here, here, here, here and here. [he shows a pair of scissors, how to carry out cutting]
Teacher: Ok, and what is next?
Maks: I opened it and pull out [he shows the way of making the card]. I watched how it looks and coloured.
Teacher: Ok.. And what were these cuts that created a square?
Maks: [silence]

No one in the class could answer the question posed by the teacher. Everyone knew how to make a "square", most of the students had it in their work. They could not, however, describe how to construct this object using mathematical language. To help students teacher presented the experience. The teacher took a piece of paper and scissors and made a few cuts. Only then the students began to pay attention to the geometrical relationships.

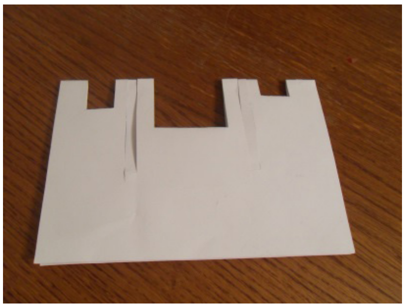

Figure 2: Squares in Maks's work

Teacher: If cutting can be free? Well, look: I take the card, fold it in half and I will cut it. Here and here [holds the card in two places]. Maks said, that for every square he made two cuts. So I have two cuts and I put the cut piece to the center of my cards [he shows]. Look, did I leave the square?
Students: No.
Teacher: Look, if I want to make a square, the same as Maks did, how should I cut.
Student 1: Well... like a square.
Teacher: And what does it mean „like a square"?
Student 1: Well ... squarely.
Teacher: And what is characterized by square?
Student 2: Because it has all equal sides
Teacher: Well, what else can we say about the square?
Student 3: That is edgy.
Teacher: That is edgy?
Student 2: Because it has all sides perpendicular and the right angle.
Student 1: And it has four sides
Teacher: Very well. So now let's get it all together: a square has four sides. You said that it has right angle.
Student 2: All sides are equal.

Teacher: Yes, all sides are the same length. And what else can we say about the square?
Maks: [he is looking at his work and pointing to the opposite sides of the cut square] And these are parallel.
Teacher: Great Maks. So what had to be done?
Maks: You have to make two parallel cuts of the same length.
Student 2: Why had not I thought of it?

Square is the first figure, faced by students. It is well known to them, everyone can indicate a square and describe its properties. However, during their work students forgot about the "mathematical" side of a square. In everyday language the square "is such a squared (edgy) thing, which has equal sides". Such a description is sufficient for students at some stage. And this one students applied during the class. The parallelism of the opposite sides and the perpendicularity of the adjacent sides was a secondary issue for them. Both concepts have been widely discussed during the lessons of mathematics. Students had drawn perpendicular and parallel segments. They even marked on the square (using colored pencils) which the sides are parallel and perpendicular.. However, these situations were "pure mathematical". Students know that now they are talking about perpendicularity and parallelism. During the course of creating the spatial cards nobody pointed them that "this is math". They initially did not see the mathematical context in what they were doing. It was for them just a normal execution of simple art work. Only a common conversation with the teacher allowed students to call in an appropriate way what they have made. Only then they began to use mathematical language in their descriptions. These experiences have allowed students to realize new opportunities for creating future cards.

Filip's work

Filip first created a "test card". He worked quickly, did not pay attention to accuracy. When the card was ready, he looked at it, analyzed the arrangement of individual elements, and then created a new card. This time, however, he proceeded very carefully. The effect of his work is presented in picture 3.

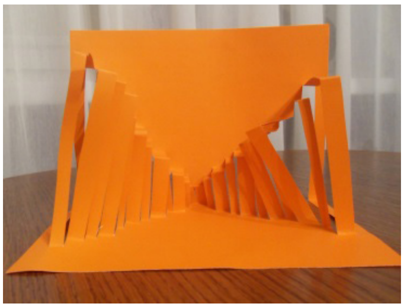

Figure 3: Filip's work

The boy was also aware of how it should carry out further cuts to achieve the desired end result. The following conversation with the teacher in an evidence of this:

Teacher: Great job, Filip. Tell us, how did you make it?
Filip: I cut strips
Teacher: And how did you cut these strips?
Filip: Getting shorter. Once a thinner, once a wider.
Teacher: Ok.. And on what more do you pay attention while preparing the work?
Filip: [silence]
Teacher: Well, how do these cuts take place?
Filip: Well ... straight
Teacher: Straight? What does it mean straight, Filip?
Filip: [long break, he looks at his work] Perpendicular to this line [he puts on the edge of the sheet] (...)
Teacher: And how are these cuts are related to each other?
Filip: Parallel.

The boy knew the concept of perpendicularity and parallelism. He could also use in practice the relationship of the perpendicularity and parallelism of objects. However, initially he could not name these concepts. He used the terms "straight, equally". These are the terms taken from everyday life, from everyday language. During creating the cards, it was enough for him. Just in describing what he did, he had to start using mathematical language. Students initially treated the task posed to them as a typical manual-plastic one: I have to make a spatial card. In their work they were rather to the directed to the visual aspect and did not refer to geometrical knowledge (although classes were held in the framework of mathematics lessons). A common discussion on the work made by students made them realize that what they were doing was hidden geometry.

Students were very surprised about that. Often appeared the statements: "oh, here is also a mathematics" or "Can we use the geometry here?" Therefore, in their further work they tried to use knowledge from geometry to create new spatial cards.

CONCLUSION

For 9–10 years old students the classes were in a new form. During the classes the students turned out to be open to new challenges. They approached the problem creatively. The care taken on the details of the created cards can prove the huge interest in the presented subject.

Although the concept of parallelism and perpendicularity were well known for pupils, they had problems with the indication of perpendicular or parallel objects. It seems that there are two different aspects: knowing concept definition and using it in practice. An additional difficulty was also a movement between two planes: the students during cutting worked on a flat plane, and the effect of their work was viewed in three dimensional space. When students were cutting out a “square” they received as a result a “cube”.

When discussing the concepts of parallelism and perpendicularity during math class the focus was on two-dimensional geometry. On a flat sheet of paper the students were shown perpendicular and parallel segments and by using a ruler they drew perpendicular and parallel lines. Meanwhile, during the course presented here they “cut out” perpendicular and parallel lines which was a new experience for them.

Hejný (2004) in his theory of the development of student’s mathematical knowledge writes that it is very important in the learning process to gain experiences, which are the basis for the creation of formal mathematical knowledge. The more different experiences, the better assimilation of knowledge. To the knowledge of students which is stable, flexible and operational, you need to provide them with as much variety of experiences related to a given concept, recorded in the different planes.

For primary school students the issues connected with geometry are not easy. The classes presented here were an attempt to find a way to present these issues in such a way that students could experience, see. By following the view that it is best to learn by experience, I tried to organize the activities in such a way that students can manipulate the materials given to them and solve the task by themselves. The open question is whether this is the right way to introduce the student to the world of three-dimensional geometry. In my opinion – yes, it is good direction. In my further work I would like to focus on the development of a geometrical environment which will be on the one hand appropriate and conductive for teaching and on the second hand - student-friendly. What should be this geometrical environment? What tools should be used in this environment? What kind of tasks and problems help to teaching and learning geometry?

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