The level of understanding geometric measurement

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The perimeter and area are two important geometric concepts, which are taught through many years in schools. Although the curriculum and the textbooks in Hungary consist of both qualitative and quantitative approaches by teaching area and perimeter, the students' performance is low. The main goal of this research is to investigate students' ideas of the concepts area and perimeter from 5th to 8th grade. We identify typical solving strategies in order to understand students' imagination connected to these mathematical objects.

Keywords: Perimeter, area, geometric measurement, concept formation.

INTRODUCTION

The perimeter and area are two important geometric concepts, which are taught through many years in schools. We know from the works of many researchers and from our experience too that the teaching-learning process on this topic poses many problems. Although the curriculum and the textbooks in Hungary consist of both qualitative and quantitative approaches by teaching area and perimeter, the students' performance is low. In Hungary quite the same misconceptions and troubles are observed as in other countries.

The main goal of this research is to investigate students' ideas of the concepts area and perimeter from 5th to 8th Grade. We find the students' long term memory interesting, because it forms a correct view about the level of understanding concepts examined in the study. It happens in schools usually that teachers examine students' new knowledge in a few days after teaching a certain topic and they find that the students acquired the new concepts and procedures. The good result of such kind of tests can be misleading: some month later students don't even remember the basic ideas.

THEORETICAL BACKGROUND

One of the goals of education is to help students store information in long-term memory and to use it in order to solve problems. There are three different parts of long-term memory. Episodic memory refers to our ability to recall personal experiences from our past. Semantic memory stores concepts, rules, principles, and problem-solving skills. Information is more easily stored in semantic memory when it is easily related to existing, well-established schemata. Procedural memory refers to the ability to remember the steps of performing a task or employing a strategy (Baddeley, Eysenck, & Anderson, 2010; Skemp, 1975).

The topic of measurement is very useful to develop students' skills in problem solving, spatial sense, estimation and concept of numbers (Korenova, 2014).

In primary school by teaching measurement, general measurement principles are used: quantity conservation; direct comparison of quantities without measuring; the need for repeated (standard or non-standard) units; estimation before measuring; exploration of the inverse relationship between the size of the unit and the number required to measure; choosing an appropriate standard unit for a concrete quantity (Curry, Mitchelmore, & Outhred, 2006; Herendiné-Kónya, 2013). In Hungary in the teaching of length, mass or capacity these principles are accepted, but they are not followed in the teaching of area and perimeter. The steps listed are mainly left out; the emphasis is usually placed on the measurement of length and mode of calculation.

The perimeter and area are at the same time geometric concepts and measurable quantities too. This is the reason we use two approaches in the teaching of this topic, a qualitative and a quantitative one.

> In the teaching of area two approaches are generally used. One that can be considered as formal which refers to the calculation of areas with formulae and another, informal, that emphasizes the conservation of area in figures of a different shape. (Acuna & Santos, 2012, p. 1)

We agree with Acuna and Santos, that there is a gap between these aspects of the area, furthermore there is also a gap between the qualitative and quantitative aspects of the perimeter. We know very well, that students used to have deficiency of area and perimeter measurement. Researchers describe that these problems arise among others from the early teaching of formulas (Baturo & Nason, 1996; Vighi & Marchini, 2011; Zacharos, 2006).

Usiskin (2012) speaks about a multidimensional approach of understanding, which helps to clarify the meaning of a certain concept and elaborate teaching materials to develop it. The dimensions of understanding of area and perimeter according to Usiskin are the following:

1) Skill-algorithm understanding: Knowing how to get an answer, i.e. choosing an appropriate algorithm to calculate the area and the perimeter of a given figure.

2) Property-proof understanding: Knowing why the way of obtaining the answer worked, i.e. knowing the derivation of the basic formulas and the relation between area and perimeter of the same figure.

3) Use-application understanding (modelling): Knowing when doing something, i.e. recognizing area and perimeter measurement in everyday life problems.

4) Representation-metaphor understanding: Knowing represent the concept in some way, i.e. area with congruent tiles, perimeter with the length of a fence.

According to Usiskin, the dimensions are relatively independent, and there is no precedence among them in terms of difficulty. In the present study the focus is on the 1st and 4th dimension. We compare the level of the skill-algorithm and the representation-metaphorical understanding in different grades.

RESEARCH QUESTION

The present research is looking for the answer to the following questions:

What do the concepts of area and perimeter mean for students at different ages? What are the students' typical strategies and misconceptions by solving area and perimeter tasks?

Our hypothesis is that the older students perform better with respect to the two investigated dimension of understanding (skill-algorithm and representation-metaphor), due to the expanding knowledge. We assume furthermore that the most frequent mistakes arise from identifying the formulae with the concept itself.

METHODOLGY

The research sample comprises 84 students from the same school in Hungary, from the following classes: 26 students from the class 5/A, 29 from the class 6/A, 19 from the class 7/A, 21 students from the class 8/A. The age of the students was from 11 to 14. The four classes are special language classes, they have more language lessons per week as usual and only the minimum mathematics lessons; 3 lessons per week. Students involved in this study didn't show particular talent and interest in mathematics.

We made interviews with the mathematics class-teachers in order to know the exact teaching-learning situation connected to our topic. By studying the curriculum and relevant textbook we can outline the teaching-learning process in the previous school years.

4th class, 8–10 lessons per year: Recognizing area and perimeter as an attribute of plane figures. Measuring perimeter of polygons by adding the side lengths. Measuring area by counting congruent tiles. Finding the perimeter of a rectangle, applying the formulae $(a+b)\cdot 2$, or $a\cdot 2+b\cdot 2$. Measuring areas by counting unit squares, finding the area of a rectangle with whole-number side lengths by multiplying the side lengths and applying the formulae $a\cdot b$.

5th class, 10–12 lessons per year: Finding areas of rectilinear figures by decomposing them into rectangles and adding the areas of the parts. Finding the area and perimeter of rectangles/rectilinear figures in the context of solving real world and mathematical problems. Recognising rectangles with the same perimeter and different areas or with the same area and different perimeters.

6th class, 4–6 lessons per year: Finding the area of right triangles, other triangles, and parallelograms by cutting and rearranging into rectangles.

7th class, 10–12 lessons per year: Knowing the formulas for the area and perimeter of triangles, special quadrilaterals, circles and use them to solve problems.

To investigate and compare the actual level of the understanding of area and perimeter, we designed a 30-minute written test. In one part of the tasks, the area and the perimeter of concrete shapes had to be calculated with the use of known formulae, where we either gave the required lengths or they had to be measured. In case of solving other part of tasks the development or representation of qualitative images were required. The first three tasks were the same in every class. As these tasks are considered very easy routine tasks for those in the 7–8 grades, we assumed they would need shorter time to solve them, so we assigned two further tasks for them. Students were familiar with the type of the tasks, because they solved such kind of problems earlier. It was considered to be also important to have a real-size picture to every task.

While we are interested in students' long term memory, we did the test on the first week of the new school year, on the 3rd of September 2014. This date provides that the students haven't dealt with mathematics especially with area and perimeter for at least 3 months, so we can consider our test as a delayed test.

TASK ANALYSIS AND RESEARCH FINDINGS

The structure of the test

Task 1: Students had to calculate the perimeter and area of three rectangles with the sides 3 cm x 7 cm, 5 cm x 5 cm, and 2 cm x 8 cm. The rectangle's position on the worksheet was usual; the sides were parallel or perpendicular to the paper edges. We wanted to know if students use the adequate calculating method correctly. The perimeter of the rectangles was the same, the area was not. We asked the following question: "What do you observe?" We wondered whether students notice that the same perimeter not necessary results the same area.

Task 2: *Calculate the perimeter and area of the plane figure.* (Figure 1)

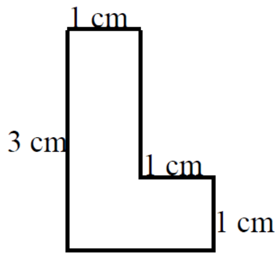

Figure 1

In this case there are no concrete formulae for perimeter and area. Students have to know that the perimeter of a polygon means the sum of the lengths of every side.

They also learned about the additivity of the area and finding area of rectilinear figures by decomposing it. Following Vighi and Marchini (2011), only the necessary numbers are given. We were looking for the level of understanding of these two concepts and typical solving strategies.

Task 3: "*Find the area of the triangle if the distance between two adjacent grid points is 1 cm. Draw two other plane figures with the same area as the triangle has.*"(-Figure 2)

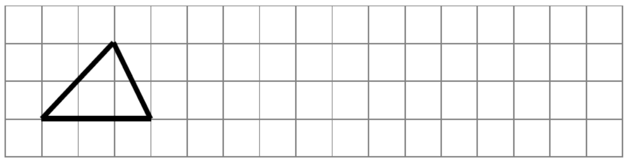

Figure 2

The area of a plane figure means the number of unit squares which cover the figure without gaps and overlapping. Determining the area often requires appropriate cutting and rearranging. There is also a possibility to apply the area formulae for triangles: $b \cdot h/2$. Drawing figure with a given area means more than formal understanding of the procedure for calculating the area.

Task 4 (for Grade 7–8): "*Calculate the area of the triangle.*" (Figure 3)

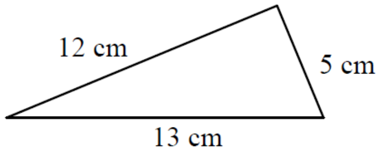

Figure 3

We wonder whether students know the concept of altitude, and apply the formulae learned last school year. The question was the following: is there any student who recognizes that the triangle has a right angle?

Task 5 (for Grade 7–8): "*Calculate the area of the parallelograms. Measure the necessary data by ruler.*" (Figure 4)

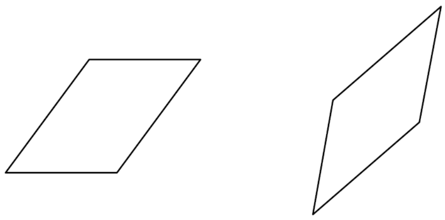

Figure 4

The parallelograms have the same sides but different angles. The first was in "usual" position and the second was rotated. We assume that the unusual position causes problems by determining the altitude of the parallelogram. While the parallelograms have the same perimeter, it's easy to recognize that the areas are different.

The understanding of the idea of perimeter

Every student from the Classes 5 and 8 determined the perimeter of the rectangles in Task 1; except 3 students from Class 6 and 2 from Class 7 gave correct answers too, which means that they are familiar with the calculating method of rectangle perimeter.

The solution of the Task 2 indicates whether the student understands the perimeter concept well or not. In this case there isn't any formula; students have to add all the sides of the rectilinear figure. The diagram below (Figure 5) shows the result of students in different classes. We found that the percentage of the correct answer (10 cm) is not more than 40% in the classes, and what is more, the 5th Graders performed the best. We detected four mistakes as typical. (1) Students often left out one or maximum two sides from the sum. This side is obviously the horizontal side of 2 cm. (2) They added only the given numbers, which indicates the lack of the perimeter meaning. (3) Relatively lots of students applied wrong formulae blindly which is analogue to the rectangular formula. For example: (3 cm + 1 cm + 1 cm +1 cm) · 2. (4) The "additivity of perimeter" also appears in a following way: some students divide the figure in two parts, calculate the perimeters of these parts than add them. It means that the procedure used when calculating the area doesn't work when calculating the perimeter. Vighi and Marchini (2011) call this symptom as "area-perimeter conflict", i.e.: "the use of a procedure for area to compute perimeter". The only distinction is that we experienced this problem not in the Grade 4, but Grade 8.

Reviewing the results of the classes we can establish that the level of understanding perimeter doesn't increase and from the detected solving strategies the wrong rectangle analogy disappeared in Class 8, but we came up with another: using perimeter as an additive quantity.

We noticed that there are 4; 1; 2 and 1 student in Classes 5; 6; 7 and 8, who mixed the words "perimeter" and "area" consequently through all the tasks. The present test was not able to say whether they interchanged only the words, i.e. the label of the concepts or the

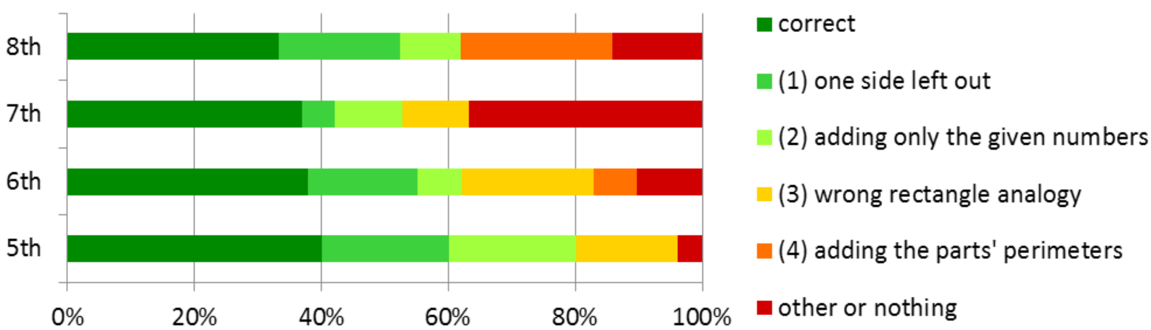

Figure 5: Calculating the perimeter of a rectilinear figure

concepts itself. This finding requires further research on real life word problems.

The understanding of the idea of area

The solution of the Task 3 gives information about the representation-metaphorical understanding (Usiskin, 2012) of the concept of area. The grid suggests the unit squares (*1 cm²*) for determining the area of the triangle. If a student is able to draw another figure with the same area it means that he/she has a correct mental image of the concept even if he/or she can't make the connection between this image and the calculation (Figure 6). Studying the result on the Figure 7 we can see, that the percentage of the correct answer is significantly lower than in case of perimeter. Furthermore students from the Class 8 achieved the best result (close to 50%), it's more than they achieved in the perimeter task.

The most frequent misconception was measuring the sides of the triangle. Most of the students who measured the length of the sides multiplied the three lengths in order to achieve the area. Some of them added the sides or completed the triangle to a minimal rectangle which includes it. The percentage of incorrect answers in Class 6 and 7 are remarkable. 6th Graders forgot about the square grid which is a tool for area measurement, so their solutions were unsuccessful. 7th Graders learnt about decomposing figures and about the formulae of the triangle last year, but the grid-context was "new" for them, so they tried to apply something similar to the well-known rectangular formulae.

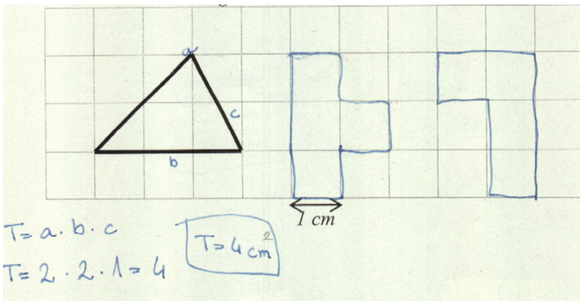

Figure 6: The solution of ***Dorina*** from Class 7

Task 2 is based on the knowledge that the area is an additive quantity (area conservation). The diagram on Figure 8 shows that the rate of correct answers in Classes 5; 6 and 7 are very low. Two typically wrong solving strategies can be detected: multiplying sides (some or all) and applying wrong rectangle analogy. In this last case we recognised not only using a procedure for rectangle to compute the area of the rec-

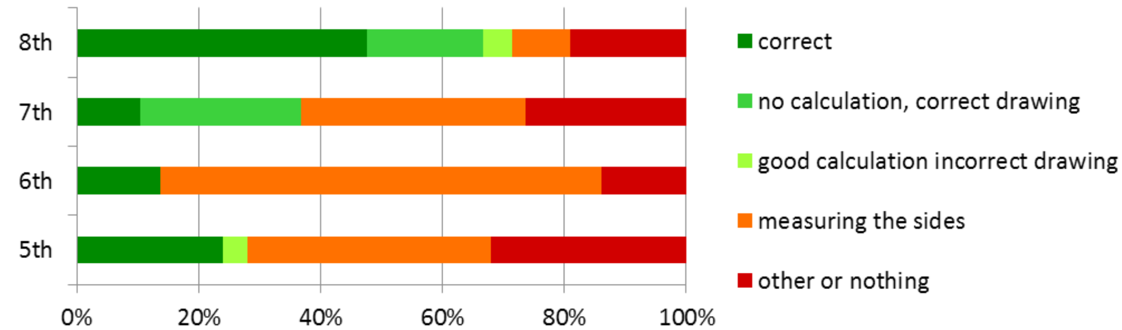

Figure 7: Calculating the area of a triangle drawn on a grid

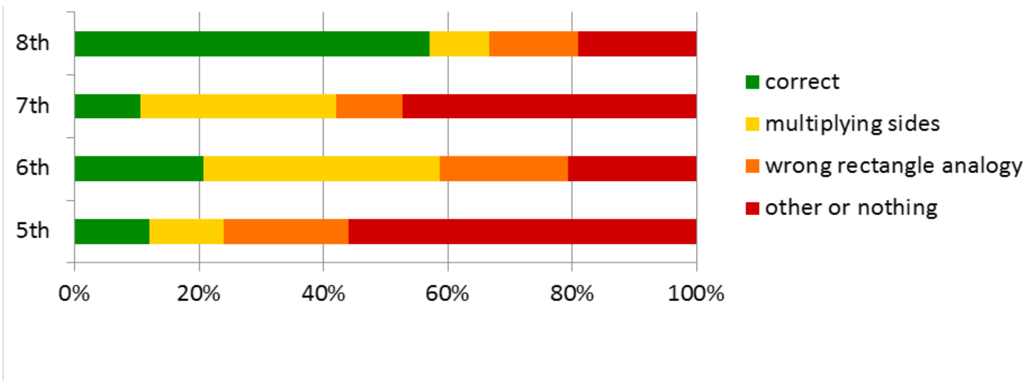

Figure 8: Calculating the area of a rectilinear figure

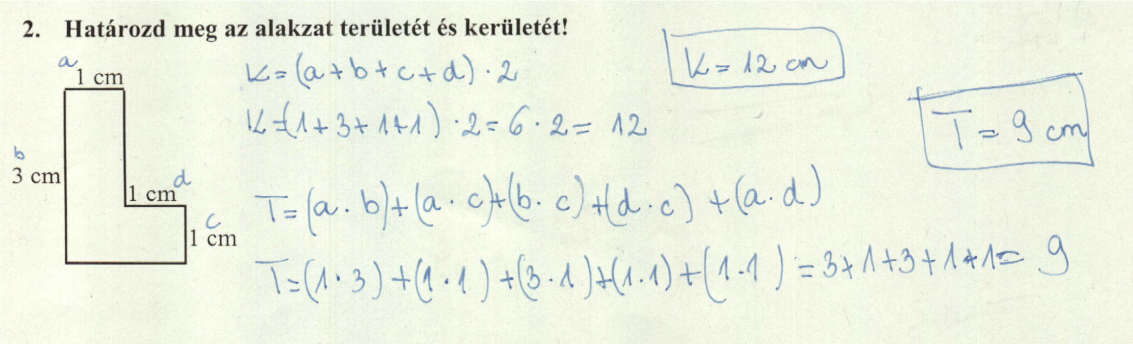

Figure 9: Dorina's (Class 7) solution

tilinear figure, but using a procedure similar to the surface area of a rectangular solid.

For example in the work of *Dorina* (Class 7) (Figure 9), we can notice the use of a procedure for computing the perimeter (K) of the rectilinear figure, analogue to the perimeter of a rectangle. She determines the area (T) with special formulae: she produces each different product of two factors from given sides then ads these factors.

Students who gave the correct solution for Task 2 or 3 were able to solve Task 1 too.

Relation between the area and the perimeter of rectangles

In Task 1, after determining the perimeters and areas of the three rectangles we asked "What do you observe?" There were only a few students (3; 3; 2 and 4 in Classes 5; 6; 7 and 8) who described explicitly that the same perimeter doesn't imply the same area in case of rectangles: "The perimeter of all of the rectangles are the same while the areas are not." One student in Class 8 gave some more explanation:

Lili: The perimeters of the three rectangles are the same, but the areas are different. It depends on the ratio between the sides of the figures.

Determining the area of triangles and parallelograms

While students learnt about the area of triangles and parallelograms in Grade 6, we wonder whether students in Class 7–8 are able to solve such kind of simple tasks.

We constructed the Task 4 and 5 to observe the use of appropriate formulae, the understanding of the concept of altitude and the influence of the position of the figures. In Class 7 there was only one student who calculated the area of the triangle correctly ($12 \cdot 5/2$) and nobody was able to calculate the area of the parallelograms. In Class 8 the result was better: 11 students had success with the triangle and 2 students with the parallelograms. 10 students (Class 7) and 2 students (Class 8) calculated the area of the triangle as the product of the sides. Nearly the same was the situation in Task 5: 10 (Class 7) and 5 (Class 8) students thought that the area of the parallelogram is the product of the two different sides. So the area of the two parallelograms became the same and the students didn't use visual control: it was easy to see that the areas are different. A new example confirmed our earlier opinion that the formulae without any meaning substitute the understanding of the concept itself. 5 students from Class 8 who tried to use the diagonals of the parallelograms ($e \cdot f/2$), did not take into consideration that this parallelogram isn't a rhombus, and the diagonals aren't perpendicular. Of course the position of the figures leaded to misconception too.

CONCLUSION

The findings of our study show the lack of understanding the two geometric concepts we investigated. We thought that the older students perform better due to the expanding knowledge. In contrast we can see for example from the analysis of Task 3, that the mental image of the area concept didn't necessarily develop, students even forgot the ideas established earlier. The formulas cover the meaning of the concepts and cause many misconceptions. One of the main findings of our research is that students think of the concept of area as the product of the sides. The new knowledge also brings mistakes if this knowledge hasn't a strong basis. The additive feature of the area implies the additive property of the perimeter. The introduction of a new formula (e.g. $e \cdot f/2$ for the area of the rhombus) causes trouble in finding the adequate calculating method. We observed and confirmed many misconceptions which were mentioned in the literature before.

Our experiences related to this research highlight the fact that students can easily forget concepts and procedures if they do not have the possibility to establish and practice it. The efficiency requires meaningful and continuous practice. The present study is the part of a wide research which aims at developing a complex teaching experiment in classes 3–8, on the topic of geometrical measurement.

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