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Does bodily movement enhance mathematical problem solving? Behavioral and neurophysiological evidence

Diana Henz[1], Reinhard Oldenburg[2] and Wolfgang I. Schöllhorn[1]

1 University of Mainz, Mainz, Germany, henz@uni-mainz.de

2 University of Augsburg, Augsburg, Germany

The present pilot study investigates how bodily movement and mental motion interact during solving of different types of mathematical tasks. In an experimental study, subjects performed algebraic, geometric, and numerical reasoning tasks at three complexity levels under static and dynamic postural control affordances during sitting. Electroencephalographic brain activity was recorded at resting baseline and during all experimental conditions. Results support the hypothesis that bodily movement has a positive effect on cognitive processing of demanding cognitive tasks. Moreover, our results indicate that mental symbolic transformations are processed within a visuo-motor representation that is aligned with the mental representation of physical space.

Keywords: Mental motion, bodily movement, electroencephalography.

INTRODUCTION

Current research in cognitive science demonstrates close interrelations between the cognitive and the motor system. One interesting research question for teaching and learning mathematics is how bodily movement interacts with cognitive processing of mathematical problems, both in learning of concepts and in the application of learned procedures. Recent research shows that spatial abilities predict mathematical performance (Tosto et al., 2014), but it is unclear how this correlation depends on the specific type of the task. Moreover, this influence may result from prior learning that has been influenced by bodily experiences or by bodily movement during cognitive task processing. In the present study, we focus on mathematical thinking that might have been developed using metaphors that ultimately build on experiences of moving in space. We assess students' behavior in terms of mathematical performance, and measure the corresponding electrical brain activity during solving mathematical tasks under different manipulations of movement behavior.

Bodily movement and cognitive performance

Ongoing research has demonstrated effects of bodily movement on cognitive and psychological functioning (for a meta-analysis see Etnier, Nowell, Landers, & Sibley, 2006). Positive effects of moderate aerobic exercise have been shown for attentional, executive, and sensorimotor task performance (van der Niet, Hartmann, Smith, & Visscher, 2014). A large body of research shows close interrelations between the cognitive and the postural control system (e.g., Hwang et al., 2013). To test interactions between the cognitive and motor or postural control system, usually a dual task paradigm is administered in an experimental setting in which participants are asked to perform two tasks, one task taxing motor, and the other task cognitive demands, at the same time. Most studies show that performance is affected in either one or both tasks depending on task difficulty and type of tasks administered compared to conditions where only one task has to be performed. One explanation is that cognitive processing is impaired when two tasks are performed taxing the same informational processing subsystem and therefore occupy its capacity, completely. For instance, the famous working memory model proposed by Baddeley and Hitch (1974) postulates specific subsystems, one for verbal, and the other for visuo-spatial information processing where cognitive resources are allocated depending on their coding format. Several studies have shown that movement information and visuo-spatial information are processed within the same subsystem. This was shown for spatially directed movements (e.g., Logie & Della Sala, 2005), and gestures (Rumiati & Tessari, 2002).

Bodily movement and mathematical problem solving

The famous book "Where mathematics comes from" by Lakoff and Núñez (2000) puts out the strong thesis that "all abstract ideas are built by metaphors that are based on experience made possible by our body interacting with the physical world" (p. 496) and thus has triggered research in mathematics education that investigates the role of bodily movement in learning and explaining mathematics. A similarly strong conception is found in Wittmann , Flood and Black (2012):

> All human concepts, including mathematical concepts, are based in the perceptual motor system experiences we have while interacting with the world around us.

With this background they investigate gestures of mathematics students while solving differential equations and formulate and give support for the hypothesis that algebraic symbols are moved during computation in a way similar to physical objects. When students multiply an equation by a denominator d their behavior is consistent with the view that d moves along some path from one side of the equation to the other.

In judging the strong claim that all mathematical concepts are formed by metaphors that are based on bodily experiences it might be useful to introduce the notion of 'metaphorical distance'. Concepts that are directly linked to bodily experiences have a short metaphorical distance to bodily experiences, while others may have a long distance that spans a chain of metaphors. It seems likely that concepts with smaller metaphorical distance to bodily experience might be stronger related to other concepts with short distance to similar bodily experiences. We suppose, although we have no a-posteriori empirical evidence for this, that the metaphorical distance increases, e.g., in the following sequence: spatial rotation, moving symbols in equations, applying inverse operations, performing algorithms of numeric calculations. This line of argumentation suggests that arithmetical tasks are affected less by bodily movement than spatial geometry tasks or algebra tasks that afford moving symbols mentally.

Besides the metaphorical distance another dimension concerns the question whether bodily experience was essential in forming the concepts but is no longer relevant when these concepts are applied or if actual bodily experience (such as movements or gestures) becomes relevant during applications.

Further, we elaborate the above mentioned point that algebraic sub-expressions may be moved in a way similar to physical objects. It is interesting to note that researchers from different teaching traditions have noted that students tend to use the language of moving objects (e.g., Tall, 2013, p. 12). From the point of view of diagrammatic thinking signs do not refer to mathematical objects, but they are the mathematical objects. If this is combined together with the above theory of moving symbolic objects, the distinction between physical objects and symbolic mathematical objects is completely blurred.

From this discussion, we extract the following *hypothesis of the existence of an algebraic symbol space*: algebraic manipulations are carried out in a visuo-motor representation, either physically or mentally. Mental processes within this visuo-motor representation are supposed to be metaphorically close to experiences of bodily movements.

Moving in physical space may involve similar brain regions as moving in algebraic symbol space. Thus, it may be, that prior exercising one of them may have positive effects on the other, and it may be that simultaneous exercising may decrease (due to capacity limits) or increase (due to psychophysiological activation) performance. Neurophysiological evidence for a pre-motor implementation of metaphorical motion could be demonstrated by Fields (2013). In the present study, we investigate effects of unspecific bodily motion (i.e. motion that is not directly related to the structure of the mathematical task) on mathematical performance by variation of postural control affordances during sitting while participants are working on the mathematical tasks. The corresponding brain activation is assessed as a neural substrate for the postulated common visuo-motor representation that underlies the processing of algebra and geometry tasks. Increases in electroencephalographic (EEG) theta (4.0–7.5 Hz), and alpha (8.0–13.0 Hz) activity in central, and posterior (parietooccipital) brain areas should reflect visuo-motor information processing in algebra and geometry, whereas increases in EEG beta (13.0–30.0 Hz), and gamma (30.0–70.0 Hz) activity should indicate concentrative, attentionally mediated information processing.

In summary, the present paper advances two research questions: The first is whether bodily movement has an effect on mathematical performance measured in terms of behavioral data and corresponding brain activity. The second section tests the hypothesis that transformational algebraic manipulations are processed within a visuo-motor representation by investigating correlations between tasks that differ in the metaphorical distance to bodily movement and measure corresponding brain activation in areas related to processing of visuo-motor information in a frequency range (theta and alpha activity) that indicates working memory processes. We therefore expect increases in EEG theta and alpha activity in central, parietal, and occipital brain areas indicating visuo-motor working memory processes in algebra, similar to brain activation patterns in geometry.

MATERIALS AND METHODS

Participants

In the present study, $n = 15$ university students (mean age = 22.1 years, age range = 19–25 years) were tested. For a sub-sample of students ($n = 6$), electroencephalographic (EEG) activity was recorded for the entire duration of the test. All subjects were right handed, had normal or corrected to normal vision, and no history of neurological impairments. All participants gave informed consent and were naïve as to the purpose of the study. Due to small sample size and its selection, the present pilot study has to be considered as an exploratory study.

Study design and tasks

The laboratory study was carried out in a 3 (mathematical task: algebra, geometry, numerical calculation) x 3 (task difficulty: low, intermediate, high) x 2 (postural control: static, dynamic) within-subject design. We presented three different types of mathematical tasks within each type three levels of difficulty have been distinguished. The last factor was that of control of bodily movement. In the static condition, students were instructed not to move, while in the dynamic sitting condition participants sat on a stool that allows to move in all directions, and therefore promotes a dynamic control of bodily posture. For a sub-sample of students, EEG was recorded for the entire duration of the test.

The three types of mathematical tasks were arithmetics or numerical calculations (Num), algebra (Alg) and spatial geometry (Geo). All tasks were presented in a multiple choice format and processed mentally by the students, i.e. they were not allowed to write down any calculations or notes. The 3 x 3 x 2 item sets were presented in a randomized order. Within each cell students worked on the items for five minutes. To avoid exhaustion the whole test was split up into three sessions.

The arithmetical items were constructed ad hoc but informed by established theories of task difficulty in calculations (e.g. number of digits and carries). Example items are shown in Table 1.

Algebraic items tested the ability to determine the solution of linear equations in one unknown. On the basic level 1 the unknown was located on the left-hand side of the equation and could be determined by arithmetical calculation as read of directly from the equation. At level 2, difficulty is increased by larger number involved and by flipping right and left side of the equation. Both of these levels concern equations that are classified by Filloy (2008) as arithmetical equations, as they do not require to operate on the unknown itself, but only on numbers around it. At level 3, equations are of Filloy's algebraic type, i.e. they require true operating with unknown. All test items require to move the unknown across the equal sign as in $20x + 4x = 50 - x$. Example items for algebra are shown in Table 2.

Performance in spatial geometry tasks was measured by the "Bausteine-Test" (Birkel, Schein, & Schumann, 2002). The test is not designed to have three levels of difficulty but the solution probabilities of all items in a large sample are published in the test manual and we used this to group the items into three level sets.

A constraint of our setup is that all items are presented in a multiple choice form so that taking possible solutions as distractors might lead participants to find the right answer not by transformational algebra but by checking which of the numbers matches the equation. However, it is well known that students usually apply learned transformational methods even when inserting is more effective (e.g., Kouki &

Level	Task	Distractors
1	279 – 69 =	191, 190, 210, 220, 230
2	283 – 144 =	125, 139, 129, 149, 141
3	1980 / 44 =	47, 46, 45, 44, 43

Table 1: Examples for arithmetic items

Level	Task	Distractors
1	$8x + 7 = 47$	1, 2, 3, 4, 5
2	$79 = 11x + 2$	2, 7, 3, 5, 6
3	$x + 15 = x + 10 + x$	5, 10, 15, 20, 25

Table 2: Examples for algebra items

Chellougui, 2013, for a recent confirmation) so that we expect that most students choose transformational strategies. Violations of this assumption would tend to decrease sensitivity of our tests, so that results that we can show would remain valid.

Analysis of behavioral data

As measure of the students' performance in each experimental condition we take the number of correct answers achieved in the fixed time frame of five minutes. These numbers are denoted by a type signifier (Num, Alg, Geo), and the level, e.g., Geo1, Alg3, Num2. Lower levels are easier so that there are more correct answers, e.g., Alg1 > Alg2 > Alg3. When forming sum scores for the types of tasks, we calculated weighted sums to achieve approximately equal weight of all levels, e.g., Alg = Alg1 + 2 * Alg2 + 3 * Alg3. Classical test theory was applied to determine effects of the factors on achievement.

EEG recordings and data analysis

Electrical brain activity was recorded at resting baseline with eyes open before and after experimental tasks, and during each experimental condition. EEG was recorded (Micromed Brainquick, Micromed Systems Evolution) from 19 electrodes positioned according to the international 10–20 system. Vertical and horizontal electrooculogram was recorded from two electrodes. Impedances were kept below 4.0 kΩ. The EEG signal was digitized at 256 samples/s. After removal of oculomotor and electromyographic artifacts EEG data were subjected to Fast-Fourier-Transformations. Power density spectra were calculated for the theta (4.0–7.5 Hz), alpha (8.0–13.0 Hz), beta (13.0–30.0 Hz), and gamma band (30.0–70.0 Hz) for each subject. Data of power density spectra were averaged over all participants and were subjected to a 2 (postural control: static, dynamic) x 3 (mathematical task: algebra, geometry, numerical calculation) x 3 (level: low, intermediate, high) analysis of variance for repeated measurements with Bonferroni-corrected post-hoc *t*-tests.

RESULTS

Behavioral data

For all conditions we found that – according to what one would expect – the higher the level, the smaller was the number of correct answers. Cronbach's alpha for the scales formed by the three levels for each type were 0.79, 0.95, 0.74 for Num, Alg, Geo respectively. However, the Shapiro test for normality of this scales had p-values of 4.9e-05, 0.028 and 0.21 due to the skewness introduced by one exceptional well performing student. In the smaller sample without this student, normal distribution can be assumed. We checked that the results presented below vary only very little when run with the full or the smaller sample. We decided to report results including this student because n is already rather small.

The unspecific effect of bodily motion was positive in all cases, i.e. working in the dynamic condition yield-

Task type	Num	Alg	Geo
Cohen's d	0.41	0.39	0.12

Table 3: Cohen's *d* effect sizes

ed higher scores. The Cohen's d effect sizes for paired samples are shown in Table 3.

However, all of these differences fail slightly to be significant (significance level $p = 0.05$) as measured by the Wilcoxon test due to the rather small sample size.

Considering the first research question we performed a linear regression (R 2013, method lm). Results are shown in Table 4. In this case, it is important to note that with the smaller sample that satisfies normality assumption almost the same results appear. The most interesting β-weight of Geo in Alg3 is 0.064 in this case and significant as well. To complete data analysis, we calculated correlations of the relevant variables (see Table 5). The high correlation between Alg1 and Num is to be expected as reading these equations backwards yields a calculation task. Basically, to deal with Alg1 items one needs to determine by calculation a numerical unknown. This last aspect explains the rather high correlation between Alg1 and Alg3.

In all 3 x 3 combinations of levels and task types students performed better under the dynamic sitting condition. However, due to the small number of participants the effects could not be shown to be significant although effect sizes (Cohen's d for paired samples) indicated at least medium effects going up to $d = 0.49$ for Alg3.

EEG data

Results for EEG brain activity are depicted in Figure 1 (the nose of the head models is directed towards the top of the page). Significant main effects were obtained for posture control, $F(1, 5) = 7.02$, $p < .05$, task, $F(2, 10) = 5.46$, $p < .05$, and level, $F(2, 10) = 5.72$, $p < .05$, with a significant posture control x task x level interaction, $F(4, 20) = 4.13$, $p < .05$. EEG data show increased theta and alpha power in central and posterior regions during algebraic and geometric tasks at high complexity level in the dynamic postural control condition ($p < .05$) indicating an increase of activity in brain regions related to processing of visuo-motor information. Gamma power was increased over all brain regions during numerical reasoning at high complexity level under dynamic postural control ($p < .05$) which is a correlate for an internalized attentional processing mode that is not dependent on sensory modality of information.

DISCUSSION AND CONCLUSION

To our knowledge, this is the first study examining effects of postural control manipulation on mathematical performance. Behavioral and neurophysiological data show positive effects of dynamic postural control on mathematical reasoning performance. Different patterns of brain activation could be observed depending on postural control affordances, mathematical task, and task difficulty. Task-dependent EEG activation patterns indicate that mathematical reasoning is affected differently by manipulation of postural control affordances. We suppose that stimulation of the postural control system activates a visuo-motor representational mode during solving of algebraic and geometric tasks which is indicated by an increase in EEG theta and alpha activity in central and posterior brain areas, whereas attentional information processing is enhanced in numerical reasoning tasks indicated by increases in gamma activity in all brain regions. Therefore, our results confirm the hypothesis that algebraic and geometric tasks are processed in a different mode than arithmetic tasks.

	Alg1 ~ Geo + Num		Alg3 ~ Geo + Num	
	β-weight	p-value	β-weight	p-value
Geo	0.004	0.89	0.067	0.039 *
Num	0.246	0.3×10^{-9} ***	0.135	0.45×10^{-6} ***

Table 4: Linear models for Alg1 and Alg3

	Num	Alg1	Alg3	Geo
Num	1	0.90	0.76	0.47
Alg1		1	0.87	0.43
Alg3			1	0.58
Geo				1

Table 5: Correlations of task types

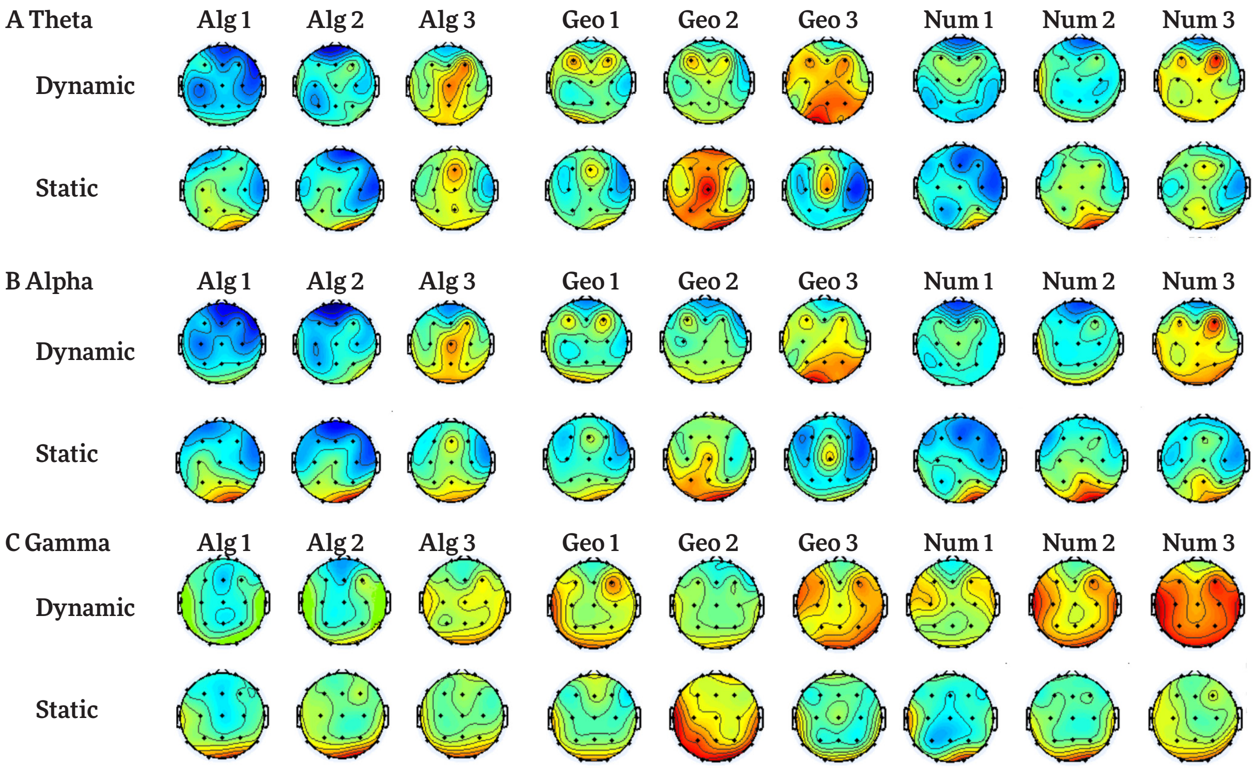

Figure 1: EEG activity during mathematical problem solving in static and dynamic postural conditions. The nose of the head models is directed towards the top of the page. EEG frequency bands: A Theta (4.0–7.5 Hz). B Alpha (8.0–13.0 Hz). C Gamma (30.0–70.0 Hz)

Further, our results support the hypothesis of an algebraic symbol space. The significant non-zero correlation found in behavioral data supports the hypothesis of the existence of an algebraic symbol space. Increased EEG theta and alpha in algebraic and geometric reasoning tasks indicate that both tasks are processed within a common working memory representation when the cognitive system is stimulated by bodily movement. However, further research is needed to clarify this hypothesis.

Finally, we present two theoretical explanations for the found patterns of results: (1) Increased mathematical performance is found in conditions of dynamic postural control affordances due to an increase of level of psychophysiological arousal, and therefore enhanced wakefulness. In a previous study (Maus, Henz, & Schöllhorn, 2013) increased attentional performance during dynamic sitting was demonstrated. EEG brain activation mirrored the found patterns of results as shown by an increase in beta activity in brain areas related to visual processing. EEG brain activation at high task difficulty level under postural control supports the hypothesis of increased psychophysiological arousal. Our results are in line with a study conducted by Vourkas and colleagues (2014) who observed differences in EEG brain activity in arithmetic tasks depending on task difficulty in children. (2) The occurrence of different EEG brain activation patterns in algebraic and geometric tasks in contrast to arithmetic tasks under dynamic postural control confirms the hypothesis that the presented tasks are processed within different cognitive subsystems. Increased central and posterior EEG alpha and theta activity in algebraic and geometric tasks at high task difficulty levels under dynamic postural control indicates that visuo-spatial working memory processes are stimulated by additional bodily movement, and therefore are responsible for the observed enhanced mathematical performance.

With the design of the current pilot study, we present a new methodological approach to investigate the underlying cognitive and neurophysiological processes in mathematical problem solving and their interaction with bodily movement. The found results contribute to a better understanding of cognitive processes that occur during solving of different types of mathematical problems, and encourage to design movement interventions which alleviate mathematical processing in learners. Our results have important implications for designing environments that promote bodily movements in learners of mathematics to increase their academic performance as could be shown in current research (e.g., van der Niet et al., 2014). Further, our results encourage to apply visuo-motor learning and teaching strategies in algebra, such as gestures (for a discussion see Janßen & Radford, 2015), due to the shown physiological preference of the brain for visuo-motor processing of algebra.

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