Computing by counting in first grade: It ain't necessarily so

Michael Gaidoschik, Anne Fellmann, Silvia Guggenbichler

Computing by counting in first grade: It ain't necessarily so

Michael Gaidoschik[1,2], Anne Fellmann[1] and Silvia Guggenbichler[2]

1 Alpen-Adria-University Klagenfurt, Institute for Mathematics Education, Austria, michael.gaidoschik@aau.at

2 University College of Teacher Education Carinthia, Klagenfurt am Wörthersee, Austria

Regarding first-grade strategy use, counting-on is widely promoted as an alternative to counting-all. However, there are concepts of initial arithmetic education that aim at developing derivation strategies systematically from the outset, skipping counting-on. This paper refers to an ongoing study that provides some empirical evidence in support of the latter approach. Students of four Austrian classes whose teachers participated in a professional development programme designed to promote derivation strategies were interviewed to ascertain their computation strategies at the end of first grade. We present first results comparing them to those of a previous study that, using the same tasks, examined a random sample of children whose arithmetic education followed a distinctly different pathway.

Key words: Counting-on, derivation strategies, difficulties in learning mathematics, in-service teacher education.

INTRODUCTION

In the German-language literature on mathematics education it is widely held that the use of counting-based strategies constitutes "a *necessary* stage in each child's learning process" that "*none* can skip" (cf. Lorenz, 2003, p. 105; our emphasis). Less frequent, but still to be found, are explicit recommendations such as by Schipper, Wartha, and von Schroeders (2011, p. 14) that children first be encouraged to use counting-on rather than counting-all. In the respective English-language literature – as far as we oversee it [1] – similar recommendations are the rule. According to van de Walle (2004, p. 164), counting-on is "the most widely promoted strategy" in the USA. In the UK, Australia and New Zealand the working out and consolidation of counting-on constitutes a central element of national programmes to promote numeracy (cf. e.g. New Zealand Ministry of Education [NZME], 2010). Nota bene: Counting-on is promoted merely as a *transitional strategy*; also, there is broad international consensus that children should *eventually* overcome the use of whatever kind of counting-based strategy (cf. Schipper et al., 2011; NZME, 2010). Reliance on counting to add or subtract in higher grades is considered to be a main characteristic of so-called mathematics disabilities (cf. Häsel-Weide & Nührenbörger, 2013).

It is mainly in view of mathematics disabilities which prompts us to call for a revision of notions concerning counting-on like the ones cited above – knowing that we are not the first to do so. Referring to the state of research we shall set forth that promoting counting-on is by no means *necessary* nor, as we argue, is it *conducive*. We shall then discuss first interim results of an ongoing study that promises to provide valuable empirical material for the further exploration of potential alternatives.

EMPIRICAL AND THEORETICAL FRAMEWORK

The recommendation that early mathematics education should *first* systematically foster counting-on is, as a rule, embedded in developmental models suggesting that numeracy evolves through stages of counting-all and counting-on to finally arrive at fact retrieval (cf. NZME, 2010). Usually such models draw upon, although not always explicitly, Siegler's (cf. 2001) "Model of Children's Strategic Choices".

As for the notion of a quasi-natural sequence of stages it has already been pointed out that at least the last stage, fact retrieval, is obviously *not* achieved by *all* children. What is empirically validated is the first step: Most children are able to solve one-digit additions and subtractions already at preschool age, mainly by counting-all (cf. Verschaffel, Greer, & De Corte, 2007). However, also at an early stage they use other strategies as well, e.g., counting-on, which they often, and prior to any classroom instruction, discover as a short-cut (cf. Baroody & Tiilikainen, 2003). In parallel, or even earlier, many children come to devise strategies using their fingers. Some of these strategies do *not* include counting (see below). Many children do soon retrieve from memory at least single tasks. Already at preschool age, some draw on tasks committed to memory to derive other tasks (cf. Gaidoschik, 2010, p. 332–340). Thus, not only is the *usual development* too varied to be captured by a stage model. It also differs from child to child and, most important, does not remain unaffected by exterior influences (cf. Verschaffel et al., 2007, p. 565). In this light, there is no reason to claim that each step is *necessary*: With influences changing, a different development is at least conceivable. That the strategies pursued by children *usually* also include counting-on does not provide sufficient justification to direct the influence of classroom-instruction towards the elaboration or consolidation thereof.

For such an approach to be justified, counting-on would have to prove *conducive* to overcoming counting strategies. This is what Siegler's model of strategic development suggests. According to Siegler (2001, pp. 377–383), each time a child provides a correct solution by counting, the "bonds of association" between task and solution in long-term memory will get strengthened. As soon as the "associative strength" between a task and the resulting number exceeds a certain value, the child would abandon counting and retrieve the result from memory instead. Counting-on is considered to be instrumental for this process to succeed in that it increases both the probability and speed with which correct solutions are obtained (cf. Schipper et al., 2011, p. 16).

What speaks against this model are empirical findings like those by Gray (1991). Gray's theory of the "proceptual divide" distinguishes between children who come to abandon counting following their recognition of relationships between numbers and numerical operations, and those for which the use of counting-based strategies is rather an obstacle in developing viable concepts. The "schema-based view" proposed by Baroody and Tiilikainen (2003) argues in a similar way, maintaining that the decisive factor in abandoning counting strategies is conceptual knowledge of numbers and operations, i.e., the ability to recognise relationships as a basis upon which to derive tasks from other tasks. Intercultural studies also add evidence in support of the targeted promotion of derivation rather than of counting-on. Geary, Bow-Thomas, Fan, and Siegler (1996), e.g., comparing children in the US with their Chinese counterparts found that on problems up to 20 the latter had by and large abandoned counting strategies already by the end of first grade, whereas the former still used counting on 40 % of tasks by the end of third grade. Those differences certainly ensue, at least in parts, from different teaching traditions – unlike in the US, in China it is common to promote derivation as an alternative to counting from early on. As intervention studies show the learning of derivation strategies based on insight facilitates the abandonment of counting strategies (cf., e.g., Steinberg, 1985). To the same direction points a recent study by Rechtsteiner-Merz (2013) which evaluated a similar teaching conception.

This is further corroborated by a longitudinal interview-based study conducted by Gaidoschik (2010; 2012) which investigated into the calculation strategies of 139 randomly selected children at the beginning, in the middle, and at the end of first grade. A significantly higher share of children who by the middle of the school year had solved a task by derivation did retrieve the same task from memory by the end of the year compared to those that formerly had relied on counting-on. Teacher interviews indicated that in all of the children's 22 classes arithmetic lessons followed a rather uniform pattern in two central respects: derivation strategies were widely neglected, while counting-based strategies were fostered at least during the first term. Against this backdrop, by the end of the year some 27 % of children would work out solutions to problems up to 10 mainly by counting whereas some 33 % would use non-counting strategies (cf. Table 2). These children, except for two, had repeatedly used derivations also in the course of the interviews. Children who predominantly resorted to counting were not observed making use of derivation strategies (cf. Gaidoschik, 2010, p. 438).

Interim conclusion: Studies conducted to date do not allow inferring a *necessity* to pursue counting-on for a considerable period of time. Nor do findings corroborate the thesis that fostering the consolidation of counting-on is *conducive* to overcoming the use of counting strategies. Studies on the relationships between the development of strategies and conceptual knowledge as well as theories derived thereof, in fact, suggest promoting derivation as an alternative to counting-on early in first grade.

PRESENT STUDY

The recent study on which we elaborate in the following may be characterised as an ad-hoc field study designed to investigate a number of questions arising from the findings by Gaidoschik (2010, p. 519–521) as set out above. "Ad hoc" means that the four classes covered by the study had actually been selected to serve another end, notably the conduct of a design study on second-grade teaching of multiplication. Decisive for the selection of classes was their teachers' participation in a professional development programme which is offered (so far without any participation of the authors of this study) in the Austrian province of Carinthia under the name "EVEU – Ein Veränderter Elementar-Unterricht" [2]. As one of its main elements, EVEU recommends the systematic working out of derivation strategies on additive basic tasks in first, and on multiplications in second grade. With a view to optimise the cooperation with the teachers, visits to the four classes were made already at the end of first grade. These included sitting in individual arithmetic lessons and interviewing teachers about their teaching derivation strategies over the school year. Inquiries were made also into children's ways of working out solutions to addition and subtraction tasks up to 20. Both the lists of tasks and the procedure were the same as those administered in the interviews conducted at the end of first grade in the framework of the previous study (Gaidoschik, 2010, p. 237–245). This allows us to compare strategies as applied by two different groups of children at the end of first grade drawing on identical tasks. On the basis of guideline-based interviews with their teachers tentative conclusions may be drawn concerning these children's arithmetic lessons. The overall analysis will particularly be devoted to answering the following questions:

> A) During first grade, did EVEU teachers – unlike the teachers of the 2010 study who had not participated in any specific professional development programme – actually work out and consolidate derivation strategies? If so, how, with what intensity and consistency?
>
> B) By the end of first grade, are there any important differences between EVEU children and the sample surveyed by Gaidoschik (2010) regarding the use of calculation strategies? If so, may these differences (also) be attributed to differing teaching concepts?

Sample and design

The sample surveyed for the present study was comprised of teachers and pupils (six and seven-year-old) of four first-grade classes (A-D) from four public elementary schools in Carinthia. The teachers had completed the first year of the EVEU programme (8 half-days) outlined above. The interviews covered *all* children of each class provided firstly that 2013/2014 was actually their *first* school year and secondly they had a *command of the teaching language* – qualifications that had applied also to the 2010 study. In addition in eight cases parents refused to give their consent. Due to these restrictions, the sample covered 11 children (out of 23) from school A, 16 (out of 20) from school B, 19 (out of 22) from school C, and 25 (out of 25) from school D.

The interviews were conducted by the authors themselves in June 2014, towards the end of the school year, in some extra rooms near the classrooms. The children were presented with 22 tasks up to 10 (like 3+4, 3+7, 4+6 or10-9, 7-4, 10-7), and 14 tasks up to 20 (like 6+6, 5+8 or 12-6, 14-9; for more details cf. Gaidoschik, 2010, p. 239–241). Each task was read aloud by the interviewer, at the same time the child was shown the task written on a DIN A7 card. The children were asked to solve the task mentally in the same way as they would usually do and state the result verbally. Immediately thereafter, in case the solution was not provided spontaneously or by using a strategy that could be perceived by observation without any doubt (see below), the child was asked to explain or show how it had obtained the solution.

The video-based evaluation was carried out analogously to Gaidoschik (2010, p. 243–245), i.e. the children's strategies were coded on the basis of the children's utterances, their gestures and facial expressions, and the time needed to produce a solution, as well. The videotapes, without using transcriptions, were analysed repeatedly by one, respectively (in randomly selected 10 % of cases), two members of the interviewing team; disagreements on single codings were resolved through discussion. The main categories that were applied are as follows: *Fact retrieval*, if a correct solution was produced spontaneously (within two seconds); *erroneous retrieval,* if a spontaneously uttered solution was incorrect; *derivation*, if the solution followed an at least short, recognizable reflection and the child described a fitting derivation strategy as his or her solution path thereafter; *counting*, differentiated into counting-all, counting-on and partial finger counting, if fingers were obviously used as a counting-device, or if there were signs of intrinsic counting (nodding of the head etc.) or attempts at using fingers surreptitiously. In the latter case the children were asked to explicitly demonstrate their problem-solving pathway, what they usually did without any further concealing.

We are aware of the flaws inherent in the chosen method (cf. Verschaffel et al., 2007); however, in the absence of a more valid alternative we did our best to classify the children's strategies as appropriately as possible.

First results with regard to participating teachers (cf. question A)

The four teachers stated unanimously that enabling children to compute without counting had been one of their priorities and that they had strived to work out derivation strategies based upon a solid conception of numbers as composed of other numbers. As the main element of classroom practices they referred to classroom discussions, which set the stage for children to put forward proposals as to the most appropriate solution to a task. The teachers also noted that efforts had ever been directed at enabling possibly all children to use non-counting strategies, whereas counting-strategies were not encouraged at any stage of arithmetic lessons.

A content-analytical evaluation of the semi-structured interviews yielded clear evidence of differing intensities and consistencies of the instruction in mainly two aspects. Firstly, with regard to "helping facts" such as doubles (e.g., 4+4=8), ten facts (e.g., 3+7=10), or partitions by the power of five (e.g., 8=5+3), teachers A, B and C stated that, once elaborated, these facts were continuously exercised as an important derivation basis with a view to automatisation. Teacher D, on the other hand, conceded that she might have failed to make sure that these facts were thoroughly known by all children. Secondly, teachers A, B and C made a point of emphasising how essential it was to push children tenaciously, virtually in every arithmetic unit, to explain their solving strategies and again and again put single derivation strategies centre stage. Teacher D reported self-critically, that the effortlessness with which many children went about derivations during classroom discussions, misled her into believing that other children would solve problems in the same way, i.e., by non-counting strategies. Not least, she confessed, did she often feel overtaxed given the rather large size of her classroom (25 children).

First results with regard to the participating children (cf. question B)

Table 1 shows the frequency at which counting strategies were used on problems by the children of the four EVEU classrooms and those surveyed in 2010. In the

	Number of instances in which tasks were solved by counting out of 14 nontrivial tasks with sums/minuends up to 10				Number of instances in which tasks were solved by counting out of 8 problems with one digit-numbers and totals greater than 10			
	mean/ median	standard deviation	min	max	mean/ median	standard deviation	min	max
2010 overall	5.5 / 5	4.6	0	14	3.7 / 4.0	2.6	0	7
2014 class A	0.1 / 0.0	0.3	0	1	0.0 / 0.0	0.0	0	0
2014 class B	0.0 / 0.0	0.0	0	0	0.0 / 0.0	0.0	0	0
2014 class C	1.0 / 0.5	1.6	0	5	0.9 / 0.5	1.5	0	5
2014 class D	1.9 / 1.0	2.6	0	10	1.7 / 1.0	2.0	0	6

Table 1: Problems solved through counting in different groups of students

framework of the 2010 study, a total of 14 problems up to 10 had proved nontrivial, i.e., were known by rote by less than two thirds of children. Table 1 compares means, medians, standard deviations, as well as the minimums and maximums of solutions to these 14 problems as well as to 8 additions and subtractions with one-digit numbers and totals greater than 10 that were achieved by counting-strategies.

As can be seen from Table 1, in the course of interviews conducted in classrooms A and B there was only one single instance of counting-on. In classrooms C and, more significantly, D, besides the vast majority who mostly or entirely used non-counting strategies, there were single children who were still making relatively abundant use of counting. The Kruskal-Wallis test reveals significant differences ($p < .001$) between medians across the five groups of students with regard to both sorts of tasks. Post-hoc pairwise comparisons based upon the Mann-Whitney U test show that in each of the four EVEU classrooms problems from both sorts of tasks were solved by counting to a significantly less extent ($p < .001$) than in the previous sample. Taking into consideration teachers' statements cited above it appears legitimate to draw a distinction in terms of the quality of mathematics education between classrooms A, B and C as a subgroup on the one hand and classroom D on the other. Differences are significant ($p < .001$) also between these subgroups within EVEU classrooms.

With regard to the children's strategy preferences during their first year of school, within the random sample surveyed by Gaidoschik (2010, p. 425–461) six types of strategy development could be distinguished (empirically grounded construction of types, cf. Kelle & Kluge 1999). Given the fact that EVEU children were interviewed only once, assignment to a certain type must be done with caution. Still, it is instructive. Table 2 shows frequencies only of the two types representing the poles in strategy preference at the end of the school year on problems up to 10.

The type "Counting, no derived facts", while not occurring at all in EVEU classrooms A and B, is rare also in classrooms C and D comprising only 5, and 8 %, respectively. In the previous sample, the percentage of children who could be assigned to this type, with solving more than two thirds of problems up to 10 by counting and not a single one by derivation, was about 27 %. In EVEU classrooms A and B, conversely, *all* children belonged to the type "Retrieval and derived facts", i.e., they displayed a high level of retrieval complemented by a flexible use of several derivation strategies. In the 2010 sample, only one third corresponded to this type. In EVEU classrooms, two subtypes could be distinguished within this type – the first being comprised (mostly) of children with a clear prevalence of direct fact retrieval even on tasks involving going-through-ten. Derivation, therefore, was not needed any more in most cases; if, however, it was done quickly and could be clearly explained. On the other hand, there were a few children who frequently resorted to derivation even on tasks up to 10. For a few, this obviously was an arduous process with single derivations taking 30 seconds and longer. Still they would not regress to a counting strategy. These children too were able to provide plausible explanations of their derivation pathways.

In classrooms C (3 children) and D (4 children) a type could be identified that the random sample surveyed in the previous study did not display so distinctly: children who, although not resorting to counting, used fact retrieval or derivation on less than two thirds of tasks up to 10. On the rest they would use non-counting finger strategies – e.g., to figure out the solution to 9-8 they would, without counting, put up nine fingers *in one move* and subsequently, again in one move and with obvious routine, put down eight fingers (four of each hand) to "read off" the result from their finger pattern.

DISCUSSION AND OUTLOOK

The study provides some further empirical support for van de Walle's (2004, p. 164) dictum according to which counting-on "is not necessary if other strategies are used". The EVEU children, according to teachers' statements, had not been encouraged in the classroom

	2010 overall	2014 class A	2014 class B	2014 class C	2014 class D
Counting, no derived facts	27 %	0 %	0 %	5 %	8 %
Retrieval and derived facts	33 %	100 %	100 %	63 %	44 %

Table 2: Distribution of strategy preferences on tasks up to 10 at the end of first grade

to use counting-on as a strategy at any point of time. Rather, and deliberately, computation was addressed only after it had been attempted to consolidate children's conceiving of numbers as being composed of other numbers. From the very beginning, tasks were dealt with in relation to other tasks; relationships were used to derive other tasks. In two out of four classrooms students had almost entirely abandoned counting even on problems with totals greater than 10; in the other two classrooms only one and two children, respectively, relied predominantly on counting strategies. Such level of achievement by the end of first grade is not at all a matter of course – it is at least *this* which can be derived from the comparison with the random sample surveyed by Gaidoschik (2010).

Given the ad-hoc character of the present study, conclusions on the EVEU children's levels of achievement at the *beginning* of first grade must be drawn mainly based on teachers' statements. Teacher A noted that her students were "high performers" in comparison to previous classrooms. Teachers B, C and D described their classes as "average". All the four schools are characterised by a mixed catchment area; socioeconomic backgrounds, however, could not be established for all children. Actually many spheres of influence on children's learning remain largely obscure. While we are far from attributing the children's differing uses of counting/non-counting strategies *exclusively* to the respective classroom practices (which were, moreover, established with limited methods), still we find it plausible that these may *also* have been important.

Evidence of didactically important differences regarding classroom concepts can be found also between the four EVEU classes. Thus, in classroom D both the preparation and reinforcement of derivation was obviously done less consistently than in the other classrooms. This may at least partially account for the significantly higher share of counting strategies use in classroom D. The frequent occurrence of non-counting finger strategies in classrooms C and D, on the other hand, corresponds with teachers' C and D statements that this kind of strategy was explicitly encouraged any time children were observed using their fingers for counting. Whether children who, by the end of first grade, still rely heavily on non-counting finger strategies will in second grade move on to fact retrieval and derived facts strategies; whether this requires targeted support, and what kind of support, are just a few of the many questions we intend to address in a follow-up study.

Implementing the classroom concept set out above was perceived as a great challenge by each of the four teachers surveyed. It is indeed plausible that the larger the classroom and the greater its heterogeneity, the more demanding the teacher's role (cf. teacher D quoted above). This is why we consider it all the more important that teachers in implementing innovative concepts regarding *central* contents of elementary school mathematics be given *long-term* support within the framework of design studies. Professional development programmes, while providing didactic stimuli, cannot, as a matter of principle, translate into a technology. An analysis of teacher interviews against the backdrop of the theory of recontextualisation (cf. Fend, 2006) reveals a mismatch, particularly in case of teacher D, between the knowledge explicated in the EVEU programme and the teacher's implicit knowledge [3]. Such kind of difficulties should not come as a surprise, so this is why expert teachers' visits on a weekly basis form an integral part of the EVEU approach. As to our discipline of mathematics education, we would essentially have to provide scientific expertise to such kinds of measures seeking to work out, together with teachers, solutions to concrete questions arising in the classroom day-by-day and to evaluate the impact of relevant decisions in order to create the basis for the design to be developed further.

Particular attention should be paid to children with learning difficulties. In our study, there were three students in classroom C, and five in classroom D, who had considerable difficulties especially with sums greater than 10. Unable to apply any of the non-counting strategies taught in the classroom, they – as some explicitly admitted – regarded counting strategies as something they were not supposed to use. As a result, they seemingly did not know what to do at all. All teachers convincingly stated that solving tasks by counting had not been *forbidden* at any point of time. Yet, with classroom practices persistently pursuing alternatives to counting, it may be difficult for some children not to think of counting as something they are simply not allowed to do. This might be disregarded if at the same time non-counting strategies were available for all students, which in classrooms C and D was not the case. That for these students counting is a necessity which must not be withheld from them is a conclusion we think is premature – especially in view of the encouraging findings of this study. We do see, however, the need for further development of designs that promote alternatives to counting as a computation strategy from the very beginning.

REFERENCES

Baroody, A. J., & Tiilikainen, S. H. (2003). Two Perspectives on Addition Development. In A. J. Baroody & A. Dowker (Eds.), *The Development of Arithmetic Concepts and Skills: Constructing Adaptive Expertise* (pp. 75–125). Mahwah, NJ: Lawrence Erlbaum Associates.

Benke, G., Kittner, A., & Krainer, K. (2014). Facilitating stakeholders: Experiences from the Austrian IMST project. *The Mathematics Enthusiast*, 11(1), 123–134.

Fellmann, A. (2013). *Handlungsleitende Orientierungen und professionelle Entwicklung in der Lehrerbildung – Untersucht in einer Studie zur Umsetzung eines innovativen Lehr-Lernformats im Mathematikunterricht der Klassenstufen 1–6.* Münster, Germany: Waxmann.

Fend, H. (2006). *Neue Theorie der Schule. Einführung in das Verstehen von Bildungssystemen.* Wiesbaden, Germany: Verlag für Sozialwissenschaften.

Gaidoschik, M. (2012a). First-Graders' Development of Calculation Strategies: How Deriving Facts Helps Automatize Facts. *Journal für Mathematik-Didaktik*, 33(2), 287–315.

Gaidoschik, M. (2012b). *Wie Kinder rechnen lernen – oder auch nicht*. Frankfurt, Germany: Peter Lang.

Geary, D. C., Bow-Thomas, C. C., Fan, L., & Siegler, R. (1996). Development of Arithmetical Competences in Chinese and American Children: Influence of Age, Language, and Schooling. *Child Development*, 67, 2022–2044.

Gray, E. M. (1991). An Analysis of Diverging Approaches to Simple Arithmetic: Preference and its Consequences. *Educational Studies in Mathematics*, 22, 551–574.

Häsel-Weide, U., & Nührenbörger, M. (2013). Replacing Counting Strategies: Children´s Constructs on Number Sequences. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the Eight Congress of the European Society for Research in Mathematics Education CERME 8* (pp. 303–312). Ankara, Turkey: Middle East Technical University.

Kelle, U., & Kluge, S. (1999). *Vom Einzelfall zum Typus*. Opladen, Germany: Leske+Budrich.

Lorenz, J. H. (2003). *Lernschwache Rechner fördern*. Berlin, Germany: Cornelsen.

New Zealand Ministry of Education (2010). *New Zealand Curriculum and Standards*. Retrieved from http://www.nzmaths.co.nz/nzc-and-standards.

Rechtsteiner-Merz, Ch. (2013). *Flexibles Rechnen und Zahlenblickschulung. Entwicklung und Förderung von Rechenkompetenzen bei Erstklässlern, die Schwierigkeiten beim Rechnenlernen zeigen*. Münster, Germany: Waxmann.

Schipper, W., Wartha, S., & von Schroeders, N. (2011). *BIRTE 2. Bielefelder Rechentest für das zweite Schuljahr. Handbuch zur Diagnostik und Förderung*. Braunschweig, Germany: Schroedel.

Siegler, R. (2001). *Das Denken von Kindern*. München, Germany: Oldenbourg.

Steinberg, R. (1985). Instruction on Derived Facts Strategies in Addition and Subtraction. *Journal for Research in Mathematics Education*, 16(5), 337–355.

Van de Walle, J. A. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Pearson.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning 1* (pp. 557–628). Charlotte, NC: NCTM.

ENDNOTES

1. We admit that our knowledge is incomplete with regard to English-language literature and completely lacking as regards literature in languages other than German or English.

2. "A changed way of instruction in elementary school"; for details see (Benke, Kittner, & Krainer, 2014).

3. Going into greater detail as to the possible implications for the implementation of innovative forms of teaching and learning is beyond the scope of this paper. For a theoretical embedding see Fellmann, 2013.