Building stories in order to reason and prove in mathematics class in primary school

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As a part of a multidisciplinary research lead by a team from the S2HEP Laboratory, the aim of our work is to explore the potential functions of stories in Scientifics and Mathematics learning. In this paper we focus on the potential connections between the mathematical space and rhetoric space during problem solving activity. We first characterized theoretically, and then tested experimentally, thanks to a didactical engineering, a didactical environment. We characterise a processes-transferring space between the narrative activity and the problem solving activity. Our results show that the narration act supports the student's mathematical reasoning.

Keywords: Problem solving, stories, narration act, processes-transferring space.

INTRODUCTION

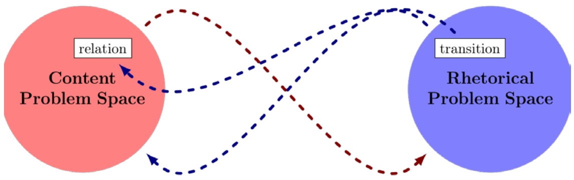
This paper aims to study the contributions of the narrative act in the proving process. There is indeed a natural inclination of the Humans for stories (Bruner, 2003) with a valuable heuristic potential. Our research group has shown, for instance, how the stories' plot and possible worlds brought by fiction can lead children to question their knowledge and build new scientific knowledge (Bruguière & Triquet, 2012). This heuristic prospective leads us to imagine that a reasoning[1] can be built on narrative structures even in mathematics. The reasoning and the narrative acts are both structured process and thanks to Bruner's and Fayol's works, we can assume that the development of a reasoning can rely, from a structural point of view, on narrative structures. Six years old children, who are as able as adults to build complex narratives structures (Fayol, 1985), have to develop mathematical and logical structures. Maybe theses structures and the proof skills related can grow for a part on those already mastered abilities. Following this idea, we developed a model allowing us to anticipate and study connections between the reasoning and the narrative act (Moulin, 2013). In the first part of this paper, we share some theoretical elements about relations between problem solving and story writing activity. We focus on the processes involved and characterize a *processes-transferring space*. Then, we present a didactical environment shaped in order to allow a joint development between narrative and reasoning. We show on chosen examples how the narration-act provides structured guidelines and take part in the proving process.

THEORETICAL ELEMENTS: PROCESSES-TRANSFERING SPACE

The aim of this theoretical part is to define what we call a *processes-transferring space* between the reasoning act in problem solving activity and the narrative act in story building. We rest on Scardamalia & Beireiter's framework (1987) that postulates that during the drafting of a text, the interactions between the rhetoric space (inherent to the construction of the text) and the content's space (concerning disciplinary knowledge) lead to the application of high cognitive functions and to the transformation of the relevant knowledge in both fields (Figure 1).

Outlooks opened by this model lead us to the assumption that during problem solving activity, the commitment to a task related to story building can allow children to initiate, build, structure and/or prove their

1 In this paper, we call "reasoning" the cognitive process that consists in drawing conclusions from facts, evidence, etc. In school, a reasoning is expected form the children when solving a problem.



The rhetorical need to build a transition can, for instance, bring up a relation that was not seen before.

Figure 1: Problem spaces and interactions extract from *Knowledge Transforming Strategy* (Scardamalia & Beireiter, 1987)

reasoning. This assumption brings us to consider a double theoretical framework.

Characteristic elements of mathematical problems2 and solving process

This first point, about problem solving activity, settles down in the field of the didactics of mathematics. Most European curricula consider that the main objective of problem solving is to develop the reasoning and logic skills and to give meaning to mathematical objects. In our work, we study this activity by getting interested in the (cross-disciplinary) skills involvements linked to a heuristic activity (Polya, 1945). To study the processes involved in problem solving, we choose to consider as one object the problem and the solution in a three-block framework (Figure 2).

Determine the whole data's structure with structuring and modelling processes; or build valid conjectures about the nature and the value of the solution involving conception and reflexive processes; or calculate the value of the solution and prove the validity of the calculations using among other explanation and argumentative processes.

The proving process depends on each of theses cognitive processes, which are dependant of the quality of the problem environment. However, school problems are not always enough complex to impose to the pupil the construction of reasoning. Supported by various linguistic tools, we realized an analysis of school problem statement (French textbooks) as if they where stories. As a matter of fact, school problems are often

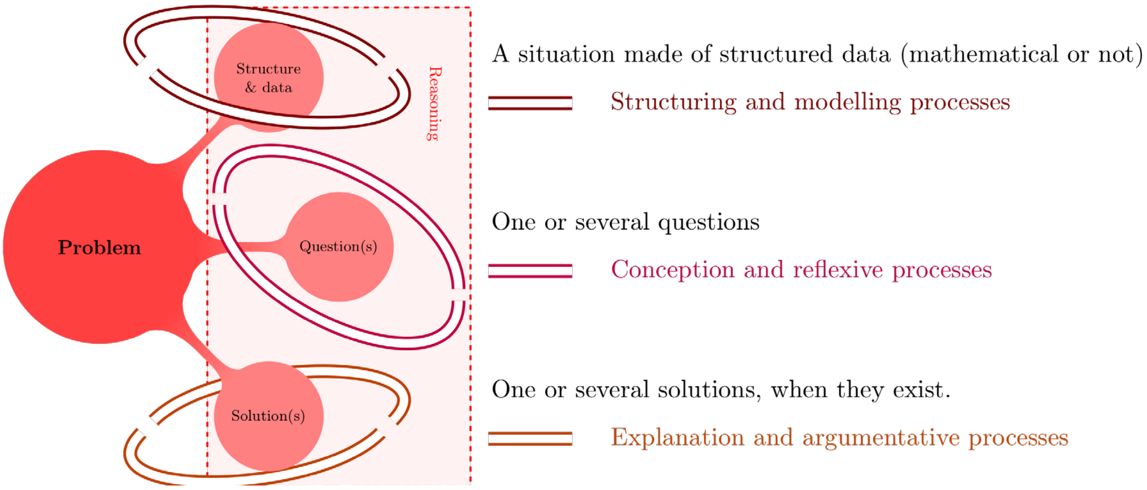


Figure 2: Mathematical problem's components and reasoning process

Problem solving is then a combination of processes developing the necessary actions to determine the solution(s) taking care of the mathematical object(s). Each one of the three problem's components must be handled. The complete processing of each component is equivalent to the problem's resolution. The problem can bee seen as solved when you have either:

(if not always) presented as little stories. But, none of them included neither problematic elements nor triggering factors as intended in the context of stories (Moulin et al., 2012). We could raise here the question of the relevance to present a problem in the form of a story if we remove the element from it that indeed "poses problem". But our main focus here is to grant a larger place to the story dimension already given in the problem's statements. The purpose of this approach is to supplement problem-solving process with *stories functions* that will be presented in the following point.

2 According to us, the name *mathematical problem* group together all the situations involving a mathematical object and asking one (or several) question(s) to which it is possible to answer only after the elaboration of a reasoning.

Characteristic functions of stories and narrative act process

We move our focus off didactics to the heuristic and structuring functions of stories linked to the narrative act. With this opening we want to highlight similarities in the cognitive activities and processes related to both activities (structuring, explanation, problematization, argumentation). When it comes to stories, one first thinks of a linguistic object, telling a story and showing some characteristics of singular shape. However, one can also approaches stories from the angle of their elaboration, as a mindset, with regard to the heuristic, and structuring functions put forward by several researchers such as Bruner and Ricœur. This is the way we chose.

To involve theses functions in the problem solving process, we have to consider in the same kind of way in one hand problems and stories and in the other hand reasoning and narrative act process. Stories have structural characteristics (Reuter, 2009) of their own organized around a plot. The plot may be considered as a question to which it is necessary to try to answer. From then on, the reception of a story, just like its production, involves cognitive processes of recognition and reproduction of this structuring. The solving of the plot therefore corresponds to a (more or less complex) sequence of movements of actions proposed by the story allowing to find a state of balance. Combining Genette's (1972) and Bruner's (2003) work, we came up with a three-block design for stories (Figure 3).

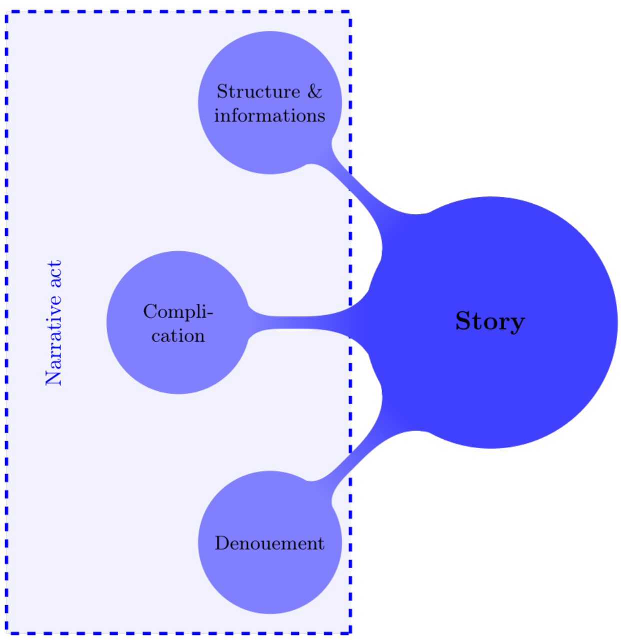


Figure 3: Stories' components and narrative act process

The characteristics of the story object express themselves in the processes of thought allowing to understand it, to build it and conversely. The solver/reader has to imagine from the information different explanatory possibilities. Fiction, as a characteristic of stories, brings a whole space to do it. For Ricœur (1983), the story has moreover this capacity to organize what is disparate into a coherent whole (*holos*). The reception, like the production of a story, therefore imposes to (re)build the temporal, spatial and causal relations of the presented events. These heuristic and structuring functions of stories are strongly connected to problematization and are submitted to an internal logic. So they play a part in similar processes to those that we have described for problem solving. There are three main functions considered in our work:

— The structuring function through situation, structure and information;

— The problematization function induces by the complication and the plot's study;

— The explanation function linked to the solving of the plot and the resolution.

Process transferring space in problem solving and story writing activity

We can now identify a potential *process transferring space* between the content problem space (including the mathematical problem) and the rhetorical problem space within with the story is build (Figure 4). Considering, as Scardamalia & Beireiter (1987) did, that a process is an answer to a local problem we can assume that each local problem (content or rhetoric) and the related process can be either realised in the content problem space or in the rhetoric one. In the context of a mathematic problem, we can make a "transfer hypothesis" assuming that *every processes related to the proving process (explanation, conjecturing, argumentation, etc.) can be handled with the help of tools available in the rhetoric space and vice versa*. In other words, as part of a problem solving activity, the narrative act and the narrative functions going with it can under certain conditions come in the solving and proving process.

In our work, we develop the story scope in integrating it as an essential component of the didactical environment (Brousseau, 1998). We position stories as an antagonistic object of the pupil in problem solving

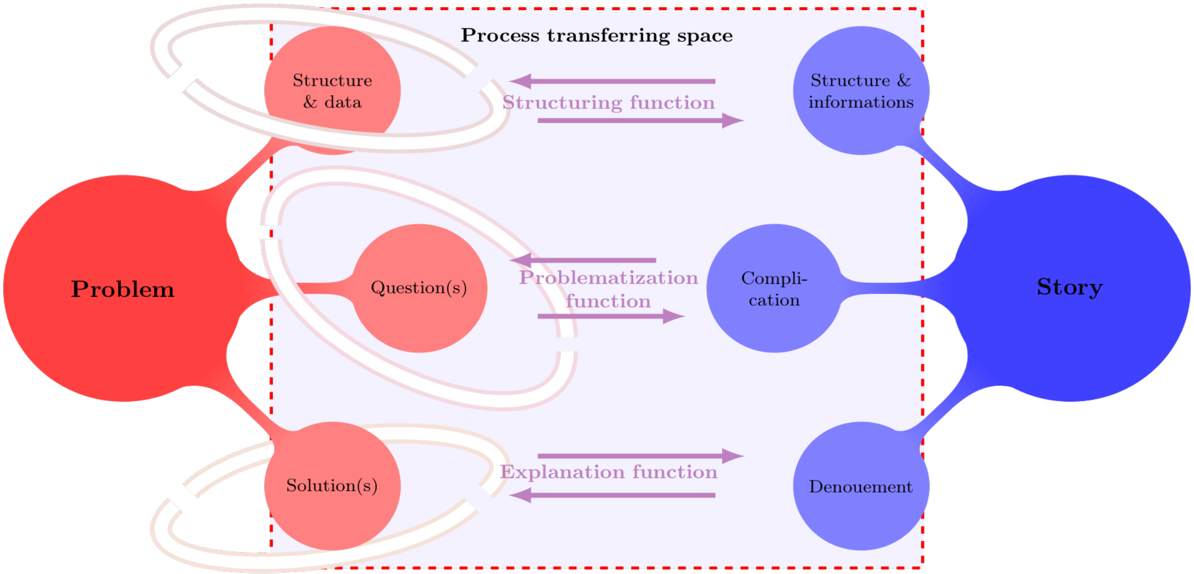


Figure 4: Process transferring space

activity. Indeed, with a structure and logic of its own, story complements the didactical environment. The story structures a space of thought in accordance with the situation at stake in the problem. According to us, this structured space allows the pupil to build and to validate his/her reasoning. We set up an experiment to test various aspects of the interactions between the story and mathematical reasoning. We present the original situation in the following point.

SITUATION AND DIDACTICAL ENVIRONMENT

With the aim of testing our transfer hypothesis, we shaped a didactical environment (Brousseau, 1998) with the objective of ensuring a joint development between narrative and reasoning. The situation of problem solving we offer is built around a game of spinning tops in which two players are in confrontation according to the rules given in Figure 5.

Due to its progress – a sequence of rounds bringing gains and losses of points – this game has a structure that is possible to determine completely through the application of mathematical rules. For instance, the score of the winner of a game is always between 7 and 9 and it is necessary to play a minimum of three rounds to end a game. These properties form, among others, the mathematical and logical structure fixing the possibilities and the impossibilities of the situation. It is on this structure, constituting a local axiom (Tarski, 1969), that we built our mathematical problems.

In a first time, we asked children to produce descriptive narratives based on games actually made. These descriptive stories were meant to be a support for the children to move from a material situation to a more objective one. At this point, the events of the game are the

The game is divided in rounds. To the signal, both players throw their spinning top in a stadium at the same time. It contains a play area and two areas of penalty. The round ends when: One of the spinning tops does not spin any more; One of the spinning tops is in the penalty area; One of the spinning tops is not any more in the stadium; A player touches the stadium. When the round is over, the points are distributed (the rules are applied in order, as soon as a point is given or removed, we move to the following round):
- 1 for the player who throws his spinning top outside the stadium;
- 3 for the player who touches the stadium during the round;
+3 for the player who sends the spinning top of his/her opponent outside the stadium;
+2 for the player who corners the spinning top of his/her opponent in the penalty area;
+ 1 for the player whose spinning top is the last one to stop.
The first player who gets 7 points (or more) wins the game.

Figure 5: Rules

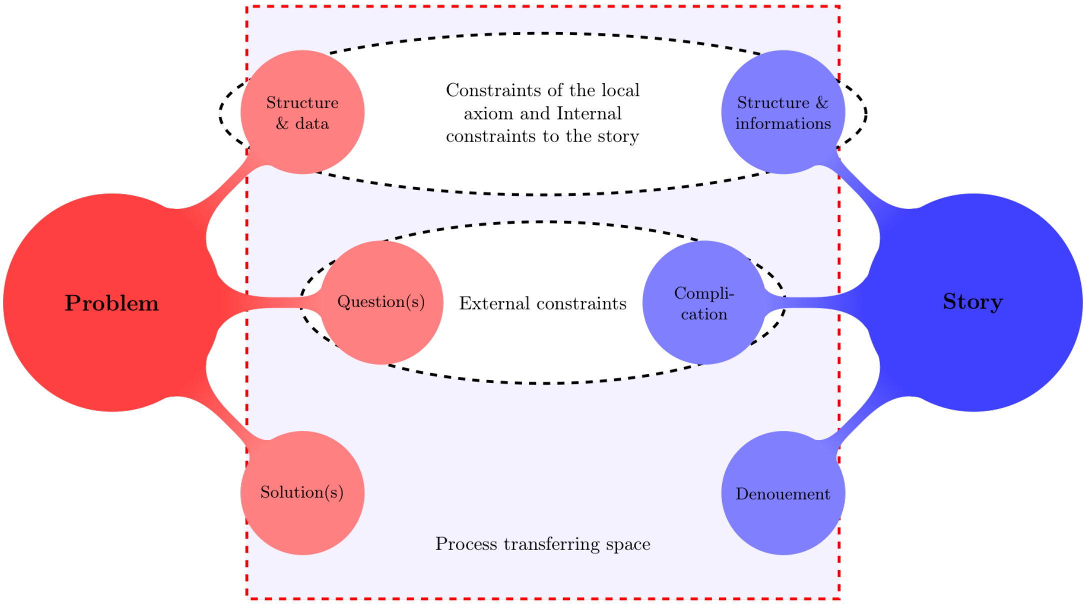


Figure 6: Constraints and didactical environment

events of the story; the scores are included in the story and the game's rules are automatically taking cared of.

Then, in a second time, we confronted them to various mathematical problems about the mathematical structure of the game and the properties presented above (about possible scores, number of rounds, etc.) To address these problematic situations, children could use anticipated-games narratives (based on imaginary games). At this point, the (imaginary) events of the game are still the events of the stories; however, points became mathematical objects and the game's rules are a mathematical structure. The problem's question reflects on the story's complication. There is a room for co-building between the reasoning and the story. In the built situation, the resolution of the problem and the proof process is subject to three types of constraints (Figure 6):

— *Constraints of the local axiom:* These are the constraints carried by the situation (Durand-Guerrier & Diaz, 2005). In our context, they define what can be done or not in the situation and are related to the structure and the data. For instance, the winner always ends the game with a 7, 8 or 9 points.

— *External constraints*: These are the constraints imposed by instructions. They are one or several additional constraint(s), which add up to those of the local axiom. They take place in the question. For example, to impose that the winner wins with 8 points instead of leaving the possibility that he ends up with 7 or 9.

— *Internal constraints to the story:* These are the constraints imposed by the style of the story. As the latter is a structured production, it has to respect an internal logic of time, characters and place.

The interaction between the two structures is the essence of our didactical environment. While solving various problems, children can rely on an environment that includes all the feasible games in accordance with the *local axiom*. We made the local hypothesis that children can produce their results through their stories and validate them thanks to the mathematical and logical rules that govern the situation. The construction through the interaction between the structure of the game and the production of the story constitutes the originality of our experiment. The outcomes presented in the following point are part of a larger research (Moulin, 2014).

RESULTS

We put ourselves in the methodological framework of *didactical engineering* (Artigue, 1988). This one allows us to confront the potentialities of our environment, by an analysis of the choices made determining the "possibilities of action, choice, decision, control and validation that [the pupil] has at his disposal" (p. 258), to the effective productions of the pupils. Therefore, we can validate, in an internal way, our hypothesis thanks to the confrontation between *a priori* and *a* *posteriori* analysis. We conducted our experiment in three primary schools (six class of 10–11 years old children, 138 children). We collected and analysed oral interactions in the class and children written productions. In this part of this article we present some chosen extracts to highlight two significant results regarding argumentation and proof.

Result 1: Children's natural tendency to stories in conjecturing and arguing
The first meaningful result we want to highlight is that, even in a mathematical context, children have a natural tendency to stories.

For instance, after playing and write some descriptive stories children were asked to establish to conjectures about the mathematical structure of the situation:

1. In your opinion, what is/are the score/s that can be obtain by the game's winner?
2. In your opinion, what is the minimum number of rounds needed to end a game.

The only constraint they had in some cases was to justify all of their answer (using the shape of a story or not). They indeed used stories to justify and/or prove. For the most part, 61 over 113, children use an explanatory possible story to justify their answer[3]. Only ten answers are based on mathematics using numbers and calculations. It seems easier for children to work in the narrative space than in the mathematical one (which concerns directly the game's rules). Moreover, the justifications build in the narrative space produce more correct conjectures than the one build on the mathematical space. 85% of the thirty-two complete conjectures came along narrative justification.

When we orally asked children to conjecture about the structure of the situation, they build and used stories in various ways:

— Anticipate potentially feasible games to put forward a conjecture;

— Propose an example (or a counter-example) to validate (or invalidate) a conjecture; the validation (or non-validation) is made by the confrontation between the stories, the mathematical structure of the situation and the constraints of the question.

While producing stories, students embrace the mathematical constraints of the situations and step back from the sensible world. No more restrained by material thinking, the students were more inclined to mathematical approach. Therefore, they develop or enhance their mathematical proving skills (example and counter-example arguments, mathematical conjectures and demonstrations, etc.). Some of them even produced mathematical proof of the impossibility to reach ten points.

Result 2: Children easily travel between the two spaces to solve problems
During our experiment, we asked children to solve various mathematical problems. For the most part, according to Vergnaud's typology for additive structures, they were problem with a composition of transformations (Vergnaud, 1986). The one we present now had this structure and the following wording:

> *Laura plays with spinning tops. During the beginning of the game, she wins 5 points. In total, she gains 3 points. What happened during the end of the game?*

With this formulation, we didn't constraint the children to use a narrative answer. We could have asked them to "tell" like we did in other exercises. However, a lot of them use narrative ways in order to solve this problem and the same type's other problems. We got a good rate of success comparing to the usual rate with this kind of problem4. But, most of all, we want to focus here on the effective practice use by the pupils to solve these problems. The oral correction sessions reveals that children easily travel between two problem spaces: the mathematical one and the rhetorical one. In this specific problem, the difficulty was to identify that you need two transformations (instead of one) to get the lost of two points. In the following example you can notice that the child goes from the narrative point of view, to the mathematical one, then again to the narrative one and finish with the mathematical one.

At the beginning, I though that it needed just one round, just that ... because it's said that she gain [loose]

3 The other answers were based on the games played during the previous session or on the game's rules. For instance, "you can have 8 points because when I played I won with 8 points" or "you can have 7 points because when you have 7 you won".

4 More than 50% against 25% according to Vergnaud (1986) in similar situations.

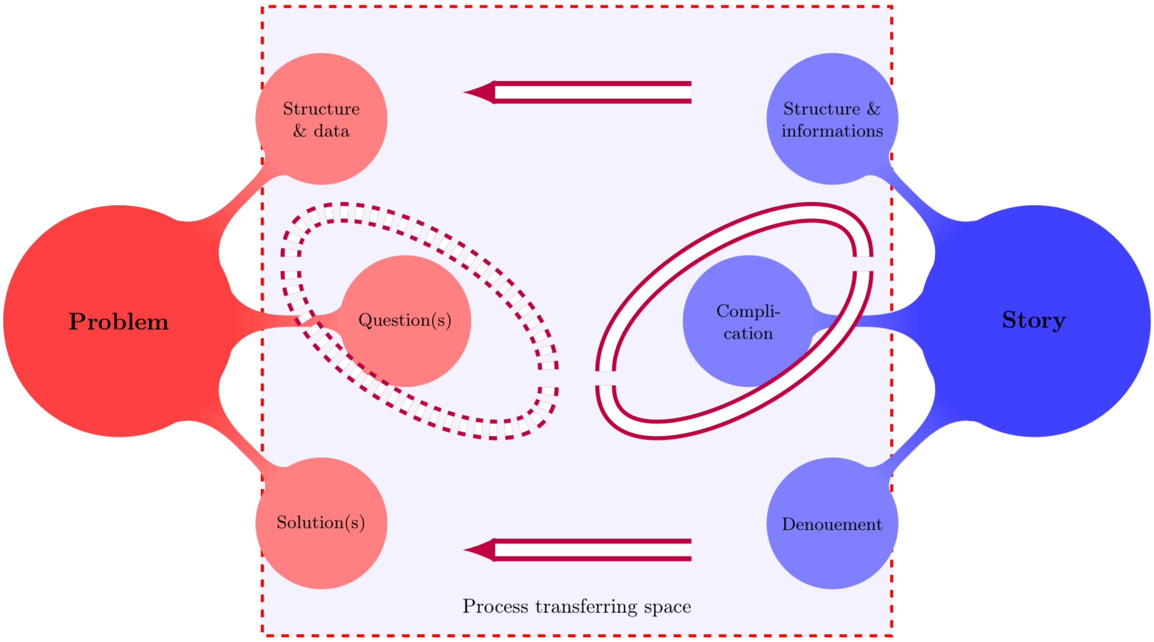


Figure 7: Problematization processes transfer

three points. She had five points in the beginning so she lost two points. And then, because there is no rules bringing the lost of two points, there is only minus one or minus three (...) so I said that I needed two rounds.

It seems that the combination of the narrative and the mathematical space offers a rich environment for children to work in. Instead of being considered only in the mathematical space, the problem can be part-treated in the story space (Figure 7).

The management of the problem in the narrative space brings children to improve their study of the situation. Because of that, their solving and proving are more accurate. Due to their heuristic and structural functions (Bruner, 2003), stories can play a part in problem solving. The act of narration supports the student's mathematical reasoning and justification. The story enriches, with the meaning of Hersant (2010), the didactical environment: there is more possibilities to explore the empirical part of the environment thanks to the fiction brings by stories; there is a need of proof brings by the structural aspects of stories. In accordance with our theoretical framework, story building together with a problem solving activity produces an environment allowing pupils to commit themselves, to structure and to justify the followed reasoning.

CONCLUSION

Our analysis of oral and written children productions reveals that stories are a powerful asset in problem solving activity. By integrating the story into the didactical environment, we offer children a structured space to reason and argue about problematic situations. Taking charge of an explanatory possible or impossible, via story building, allows them to argue and so to conjecture and to get into a proof approach. In a more general way, all the functions of the story can be mobilized as part of problem solving. Structuring, explanation, argumentation are inherent processes for both story writing and problem solving. The possibilities of interaction between these two activities let imagine a joint development, in the pupils, of the capacities needed in the proving process.

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