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1 Introduction

Production Planning (PP) aims at making production decisions optimizing the Manufacturing and Supply Chain performances.

Given a limited amount of manufacturing resources, raw materials, and inventory space, PP minimizes the overall manufacturing and supply chain costs while meeting service level requirements and obeying operational constraints. In this research we focus on the sequence-dependent set-up costs which occur when we change a product production during manufacturing. In real-world manufacturing, this cost typically depends on both the predecessor and current production. The set-up costs are also called clean-up costs, changeover costs or start-up costs (in the literature). These costs are one important part of the production costs for many real-world production systems. The set-up costs optimization may dramatically affect the quality of the production plans in terms of machine operational efficiency, cycle time, service level and inventory turns. Finally, different levels of details are typically taken into consideration which largely affect the complexity of the resulting models. In this work, we study a real-world PP problem with sequence-dependent set-up costs and we propose different formulations with different levels of details of the operational constraints.

2 Modelling

One of the most general problem in PP is the *Capacitated Multi Product Lot-sizing Problem* (CMPLP) (we refer the interested reader to [1, 2, 3] for further details). We are given a set $I = \{1, 2, \ldots, n\}$ of products that must be produced during a predefined time window. For each product i and time slot t, a set-up cost sc_t^i, a production cost pc_t^i and a holding cost hc_t^i are prescribed. A maximum capacity p_t^i is given for each period and product. The problem asks for satisfying a foreseen demand d_t^i (per period and product) at the lowest possible cost. The formal mixed integer programming model is shown below :

$$\min \quad \sum_i \sum_t (pc_t^i x_t^i + sc_t^i y_t^i + hc_t^i s_t^i)$$

$$\begin{aligned}
s_{t-1}^i + x_t^i &= d_t^i + s_t^i && \forall t, \forall i \\
x_t^i &\leq p_t^i y_t^i && \forall t, \forall i \\
x_t^i &\geq 0 && \forall t, \forall i \\
y_t^i &\in \{0, 1\} && \forall t, \forall i \\
s_t^i &\geq 0 && \forall t, \forall i,
\end{aligned}$$

where the x_t^i, y_t^i, s_t^i are decision variables representing the producing quantity, the decision of producing and the storage quantity of product i at time slot t respectively. In many literature works, the set-up costs depend on the current job and time slot under consideration. In this work we consider the Sequence-Dependent Set-Up Costs Production Planning Problem which requires planning of lot sizing and sequencing of production lots (see e.g. [4, 5]).

3 Contribution

In this paper, we present different methods to model sequence-dependent set-up costs and analyze the resulting effect. First, different ways to represent set-up costs are explored, which are shown to be memory and computation efficient in comparison to a naive matrix format. The naive matrix format considers storing the changeover costs between every pair of product productions in a matrix. Second, for production planning with time buckets, various ways of modelling and approximating sequence dependent set-up costs are provided. The models are evaluated and compared through the effect on lot sizing, production grouping and production wheel, as well as from the perspective of complexity and accuracy. More important, we propose a framework to help determining the best model to be used depending on the characteristics of the production planning problem and the time horizon. Finally, we are taking into consideration an application which comes from a real world planning project delivered by DecisionBrain. Extensive computational experiments are reported and discussed.

Références

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