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# THE CO-CONSTRUCTION OF A MATHEMATICAL AND A DIDACTICAL INSTRUMENT

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*Spreadsheet is not given as didactical tool to serve mathematics education. It may progressively become such an instrument along a professional genesis of use on teachers' side. Using both the notions of distance and double genesis, the case study described in this paper illustrates the beginning of such a genesis and the complexity that comes along with it.*

Keywords: ICT, instrumentation, double (professional / personal) genesis, instrumental distance.

## INTRODUCTION AND DATA PRESENTATION

For several years now, research communities have paid an increasing attention to technology use in math education (see Drijvers *et al.*, 2010 for a historical overview). Theoretical frames as the instrumental approach (Artigue, 2002; Guin, Ruthven & Trouche 2004) have been developed around the concept of *instrumental genesis* focusing on ICT impact on pupils' work. This paper moves from this focus towards the impact of technology on *teachers'* practices. It focuses on the case of the spreadsheet which is not given as a didactical professional tool but that may progressively become such a didactical instrument through a professional genesis on teachers' side. In order to study these geneses, I introduced two theoretical constructs within the frame of Instrumental Approach. The paper illustrates a case study on how these two concepts are useful in analyzing the teacher's geneses with the spreadsheet and in describing the complexity of these geneses. The data on which we are illustrating the concepts are based on observations (on two consecutive years) of an experienced teacher, named Dan in the following, integrating spreadsheet for the first time in her classroom. The observation of Dan's spreadsheet integration shows some evolutions from a year to the next.

### Few words about Dan

Dan is an ordinary teacher, having more than 10 years of experience, also involved in teacher training and having integrated dynamic geometry software, but spreadsheet is a new tool for her. During the first year, Dan was motivated by her participation in a research project focusing on spreadsheet use for *algebra* learning (Haspekian, 2005a). At the end of the research, an interview collected her thoughts and feelings about this experience. The following year, she used the spreadsheet by her own choice, without any research protocol. On that occasion, we recorded her first spreadsheet session and the following session in a paper-pencil environment. Some phenomena during this observation and the way Dan evolved in her practice with

spreadsheet as a didactical tool provide interesting data. Let us first present the evolutions at stake and then describe the theoretical frames to analyse these data.

### **Dan, year 2, or “the second trial of using spreadsheet in mathematics lessons”**

During the second year, Dan introduced spreadsheet not with algebra but with statistics (headcounts, frequencies and cumulative frequencies after having seen these notions in paper-pencil). In this context, some of the observed elements are surprising: the lesson, showing very little statistics, is mostly centred on the tool use and functionalities, and reveals unexpected mathematics (notions of variable, formula, distinction between “numeric/algebraic” function...). The latter reflect the influence of the year 1 experience, centred on algebra, but this does not explain completely the evolution year 2 (variations and regularities) summarized in Table 1:

Use of spreadsheet	Year 1	Year 2
<b>VARIATIONS</b>		
Class level	7 <sup>th</sup> Grade (12 year old)	8 <sup>th</sup> Grade (13 year old)
Old/new content	New	Old
Mathematical Domain	Algebra	Statistics
Spreadsheet location	Limited to computer lab	Computer lab +ordinary classroom
Synthesis	No	Yes
Interactions Teacher-Students	Mostly individual	Individual and collective
Use of the video and collective presentation	Piloted by teacher, limited role	Teacher and student. Important role
Students Configuration	Work by pairs	Work by pairs + collective work: one student at the board
<b>REGULARITES</b>		
Maths objectives, teacher aims	Algebra	
Additional material	Worksheet for pupils and pre-organised spreadsheet file	
Institutionalisation	In an ulterior lesson, in ordinary classroom	

**Table 1: Comparison Year 1- Year 2**

How can we explain these observations, the variations and the emerging regularities?

### **THEORETICAL FRAMES TO UNDERSTAND THE OBSERVATIONS**

To understand teacher’s practices, the “didactic and ergonomic approach” developed by Robert and Rogalski (2002) describes teacher’s activity through 5 components: personal, mediative and cognitive dimensions, as well as institutional and social constraints. The *cognitive* and *mediative* components relate to the choices made by the teacher in the spatial, temporal and mathematical organisation of the lessons. For Robert and Rogalski (2002) teachers are not totally free in these choices; they are more or less constrained by *personal*, *institutional* and *social* dimensions: the personal component relates to the teacher as a singular subject, with his own history, practices, vision of mathematics learning. The institutional and social dimensions relate to curricula, lessons duration, school social habits, math teachers habits etc.

This first frame is used at a global level. In the case of ICT practices, instrumental aspects seem to interfere with each of these components. This leads to the use of the instrumental approach in order to analyse at a more local level some of the phenomena observed during the year 2.

### **What are Dan's evolutions through these 5 components?**

Table 1 shows an evolution of the mediative and cognitive components (math domain chosen, way of introducing spreadsheet, class level, etc.). This indicates (and it is confirmed by the phenomena observed during the lesson) that Dan's personal component evolved too. How can we explain this evolution? Our analyses lead us to consider two main results: first, a phenomenon of *double instrumental genesis* takes place in the evolution of Dan's personal component, and second, all changes operated by Dan in her mediative and cognitive choices go towards a *reduction of the instrumental distance*. Let us precise more these theoretical issues.

The idea of *distance* has been introduced (Haspekian, 2005a) to take into account, beyond the "computer transposition" (Balacheff, 1994), the set of changes (cultural, epistemological, institutional) introduced by the use of a specific tool in math "praxis". For a given tool, a too big distance to the "current school habits" is a constraint on its integration (Haspekian, 2005b). On the other hand, didactical potentialities of technology rely on the distance it introduces as regards to paper-pencil (providing new representations, new problems...). In (Haspekian, 2005a) we have brought out 4 types of elements that can generate some distance. Some are directly linked to the *computer transposition*, as the representations and the associated symbolism. They can also be of an *institutional nature* [1], *didactical nature* (vocabulary, field of problems they allow to solve...), or *epistemological one* (what gives the tool an epistemological legitimacy). This is linked to teacher's personal component (her representations of math, of math teaching, of the role this tool plays in the development of math).

Then, the way teachers orchestrate and support pupils' instrumental geneses evolves year after year. Considering spreadsheet as an instrument for the teacher, allowing her to achieve some teaching goals, we consider a process of instrumental genesis *on teacher's side*. The same artefact, the spreadsheet, becomes an instrument for pupils' mathematical activity and an (other) instrument for teacher's didactical activity.

In Dan's case, this process is even more complicated since it is split in two. There is a *double instrumental genesis* because Dan is not developing one but two instruments from the artefact spreadsheet. The personal instrumental genesis leads to the construction and appropriation of a tool into an instrument for math work, which differs from the *professional instrumental genesis* that leads to the construction and the appropriation of the previous instrument into a didactical instrument for math teaching. The didactical functionalities of this tool are not pre-defined, the teacher must develop and integrate them in her usual teaching practices and habits. My

hypothesis is that these two processes are not independent for Dan as they happened simultaneously in her case. Neither are they independent of pupils' geneses as we will see. Let us first describe more precisely the *professional genesis*:

Applying the instrumental approach to the spreadsheet seen as a *teaching* instrument that the teacher builds along a professional genesis, we can bring out two processes:

- An instrumentalization process: the tool is instrumentalized by the teacher in order to serve her didactic objectives. It is distorted from its initial functions and its didactical potentialities are progressively created (or "discovered" and appropriated in the case of an educational tool);
- An instrumentation process: a teacher will have to incorporate in her teaching schemes that were relatively stable some new ones integrating the tool use. She will progressively specify spreadsheet use to a particular class of situations (as "take advantage of spreadsheet for algebra learning") and organise her activity in a way that will become progressively invariant for this class of situation (the Dan's case already shows some regularities from year 1 to year 2).

Dan builds up *schemes of instrumented action* [2] aiming at using spreadsheet to teach algebraic concepts (e.g., variables, formulae through the use of the copy, or by taking benefits of the numerical feedback to infer the equivalence of two formulae). This brings into play some usage schemes concerning material aspects, as the tool integration in a larger set of instruments (with the video projector), the organisation of the lessons, that are schemes that will undertake the modes of exploitation and the orchestrations, for instance, using a video projector at the beginning of the session for collective explanations, making pupils communicate and work by pairs, giving a sheet of instructions and a pre-built computer file to gain time, but also regularly "clicking on cell to check whether the pupils have edited a formula or numerical operation, or even directly the numerical result...).

I said this professional genesis was not independent of Dan's personal genesis, but interfered one on the other [3]. I also said that the professional genesis is made more complex by the fact that Dan wants her pupils to manipulate spreadsheet too (one could imagine a spreadsheet usage only under the teacher's control) and learn math through this activity. Thus pupils' instrumental geneses are part of the teacher's instrumental genesis. Here again, the two phenomena are imbricate and interfering. Some of our teacher's activity observations during the year 2 result from these interferences. We will show an example in the next section.

## **TEACHER'S DOUBLE GENESIS INTERFERING WITH PUPILS' INSTRUMENTAL GENESES**

As we mentioned, Dan has inscribed the introduction of spreadsheet in her class within the domain of statistics. Fig.1 is an excerpt of a pupil's exercise that shows the corresponding spreadsheet file with the pre-edited formula built by Dan:

**Step 2 : usage of formulae et the «handle of recopy».**

distance (km)	$0 < d \leq 5$	$5 < d \leq 10$	$10 < d \leq 15$	$15 < d \leq 20$	total
headcounts	16	14	12	8	50
frequencies (%)	32				100

- 1) a) What is the total number of items? \_\_\_\_\_  
 Where is this number located? What is the formula to calculate it? \_\_\_\_\_  
 b) If one changes the headcounts for  $0 < d \leq 5$ , does the frequency change?

	A	B	C	D	E	F
1						
2						
3	<b>Etape 2 :</b>	<b>calcul de fréquences</b>				
4						
5	distance (km)	$0 < d \leq 5$	$5 < d \leq 10$	$10 < d \leq 15$	$15 < d \leq 20$	total
6	effectif	16	14	12	8	<b>=SOMME(B6:E6)</b>
7	fréquence (%)	<b>=B6/\$F\$6*100</b>				100

**Figure 1: Dan's final version of formulae**

It is interesting to notice that this file has been modified three times by Dan. In its first version, the formula calculating the frequency (in B7) was: **=B6\*100/50**. This formula, if copied along line 7, calculates the correct frequencies for the corresponding data of line 6. But it is not adequate regarding the question b) [4].

The day before the lesson, Dan realised the mistake and changed the formula into: **=B6/F6\*100**. She confided she did not feel yet totally comfortable with spreadsheets. If her own instrumental genesis with spreadsheet as a mathematical instrument probably plays a role here, we also see that the key point of the problem comes from the spreadsheet as a *didactic-oriented* instrument. From the spreadsheet as a “calculus-oriented instrument” point of view, the formula was adequate. It is the didactical aim (showing the mathematical dependency between the number and the frequency) that leads Dan to ask the question b), which turns the formula wrong. Dan did not realise this when she built first her formula. At that moment, the personal instrument stands at the front of the scene, and covers up the professional instrument and its didactical aims (the question b).

Interference between the personal and the professional instrument can be seen again in the continuation of the story. The new formula, **=B6/F6\*100**, is now adequate for question b, but it is **still not convenient** if we consider the next question (Fig. 2) for inverted reasons! Dan wants pupils to copy the formula in order to fill line 7 and meet this functionality and the automatic incrementation of cell references (B6 becomes B7...). This time, this is part of her goals for students' instrumental geneses.

- 3) Complete the table using the formula in B7:  
 Recopy the formula on the right. (see instructions below for the “cell recopy”).  
 What is the formula contained in C7? D7? E7?

**Figure 2: The continuation of the task**

But the formula above, if copied on line 7, does not fit anymore, as the cell referring to the total, F6, will be changed into F7, F8... along the copy. A solution to this problem is to fix the cell F6 in the recopy by using the “\$” functionality. But Dan did not want this functionality to appear in the first spreadsheet session. It was over the level of instrumentation she wanted for her pupils at this moment. When she built her new formula for question b, the \$ was not in her mind and she did not put it, forgetting that it will create false results at question 3. The day before the session, after discussing over a phone call she realised the mistake and integrated finally the \$ in a last-minute decision!

This time the formula was wrong regarding an instrumental goal: the \$ symbol was over Dan’s instrumental objectives and she did not keep it in mind. It is neither easy nor trivial to bend to all the constraints, all the more that she had already changed her very first version of formula for a mathematical aim, now she had to change it again for an instrumental aim... This time, it is the professional-oriented instrument that overrode on the personal one, by taking into account pupils’ geneses and the level of instrumentation that she wanted them to reach.

These successive formulae somehow disrupted the session: Dan put the \$ sign in the formula and wanted to avoid speaking about it with pupils, but it has been of course highlighted during the session! Facing pupils’ questions, she was compelled to explain but she just said that it is not important to write it in paper-pencil environment. Then, when a pupil came to the board to write the spreadsheet formula, he forgot the \$, the “division by zero Error” appeared after copy and Dan said “*now you happy?*” but did not explain the message nor the division by zero![5].

### **Interpretation: the complex and “split in two” geneses on teacher’s side**

This example shows how the double genesis on the teacher’s side may interfere with pupils’ geneses: spreadsheet constraints interact with the teacher’s goal and didactical expectations (she wanted to introduce only a basic level of spreadsheet functionalities). She has not yet turned her personal instrument into a math teaching one. This process is made more complex by the different geneses at stake. As we saw in the example, it is constrained by:

- The mathematical learning the teacher aims at (statistics and algebra),
- Pupils’ instrumentation, that is how to make them work math through spreadsheet (as the mathematical headcount-frequency dependence through the change of the frequency cell after changing the value of the headcount cell),
- Pupils’ instrumentalization, that is which functionalities are aimed at, which schemes of use do we want them to build - here: relative references and automated incrementation of cell references with the copy, but not yet the absolute references, the \$ sign and its different behaviour in the copy.

Managing all these constraints at once is not easy: spreadsheet is not given as a

didactical instrument, the case of Dan shows that such an instrument is only progressively built along a complex professional-oriented genesis.

### **Understand the global coherence in the evolution of the teacher's practices**

As a synthesis, we can say that institutional and social components, together with Dan's own reflection on her practices, lead her to evolve in her conception of the spreadsheet use. Here are the observed evolutions:

1. **Higher level of class** : she uses spreadsheet with 8<sup>th</sup> graders instead of 7<sup>th</sup> graders
2. **Lower quantity of « new » concepts**: avoid mixing the introduction of the spreadsheet with the introduction of new math notions
3. **Domain change**: introduce the tool with statistics which seemed to Dan more appropriate than algebra
4. **Contents shifted towards the instrument but conditioned by the level of instrumentation fixed by Dan** (she did not want to stress the \$)
5. **Deeper articulation between social and individual schemes**, the importance of the articulation in instrumental geneses has been mentioned by Trouche (2005). In the interview, Dan says she did not organise moments of mutualisation enough and she explicitly wished to take care of this point the 2<sup>nd</sup> year.

Observing deeper these evolutions, they all appear to converge in the direction of *reducing* the instrumental distance. The next section develops this point.

### **REDUCTION OF THE INSTRUMENTAL DISTANCE ALONG TEACHER'S PROFESSIONAL GENESIS**

At different levels, Dan's modifications year 2 tend to decrease a too big instrumental distance of the spreadsheet.

#### **1. Changing the class level: Higher level of class**

This modification comes with the change of the domain (point 3): in French curricula, spreadsheet is explicitly mentioned with statistics for 8th Grade pupils. In the 7th Grade curriculum, spreadsheet appears in a more general and vague way. It requires from teachers a deeper work and thought to define its potentialities for learning math notions, these latter appear more distant from spreadsheet math than in the 8<sup>th</sup> Grade, where spreadsheet appears clearly in relation with precise notions of the curriculum. Thus, choosing this level allows Dan to reduce the distance and match more easily with the official prescriptions. Besides, year 1, Dan found pupils' instrumentalisation not easy in 7th Grade (difficulty to use the "recopy", select a single cell, edit a formula). Older pupils seem to be more skilful and problems linked to instrumentalisation should be less interfering with the math work. With 7 graders, manipulations of the tool seemed more difficult and the tool appeared less transparent.



## **2. The “old/new” game in the mathematical and in the instrumental contents**

Year 1, Dan introduced a new instrument when she introduced new math contents (algebraic notions). The ratio old/new is different in year 2 and also goes in a direction of reducing the distance by reducing the part of “new”: all the math notions at stake in the spreadsheet session (headcounts, frequency, cumulative frequency) had already been seen previously in paper-pencil. This work (new environment with “already-seen” concepts) will then serve Dan as a basis to work later algebraic notions (new concepts in an “already-seen” instrument).

## **3. Domain changing**

The math domain chosen by Dan in year 2 also reduces the distance as regards to algebra, for at least three reasons. The domain of statistics is usually seen to be more in conformity with the representations of a spreadsheet work than algebra. Furthermore institutional pressure is less important in statistics than algebra, a more classical and traditional domain strongly linked to paper-pencil math. On the contrary, statistics are nowadays seen as more fitted to technologies. At last, in the spreadsheet language, one can find more common terms with statistics whereas the distance to the traditional algebraic vocabulary is important (Haspekian, 2005b).

## **4. A deeper care to instrumental aspects**

Distinguish the instrumental genesis on pupils’ side and the dual genesis on teacher’s side allowed us to understand better the teacher’s activity and the phenomenon observed during the session. Dan’s didactical instrument is improving along the experiences and the examples above (click on the cell to see if pupils edited a formula, use the recopy, introduce the relative references but not the absolute ones, not to introduce the logical functions...) show that spreadsheet sessions require careful consideration of all these instrumental aspects. But this is not evident and the first section illustrates difficulties and interferences between these three geneses. Dan’s evolution shows a deeper care of the instrumental aspects with the pre-determination of a level of instrumentation for pupils, well defined, not too high and not too far from usual (not using the \$, not introducing the “IF” function etc.).

## **5. Moments of mutualisation and articulation with paper-pencil mathematics**

Dan introduced year 2 some moments of mutualisation in spreadsheet sessions. In the interview, she affirmed her will to increase the similarity with the traditional sessions. She said having the feeling that it is necessary to multiply the links with the paper-pencil math (e.g., she started the sequence by a paper-pencil session, then worked the same notions in a spreadsheet session, then she came back on the work done with spreadsheet in a paper-pencil session, etc.). All these actions contribute to reduce the distance with paper-pencil, to mix these two environments in a greater proximity. This is a key point to integrate spreadsheet: in (Haspekian, 2005a), teachers who used to integrate spreadsheet had these characteristics. It is thus

interesting to notice that Dan's professional genesis follows the same line. The "old/new" game mentioned above is another characteristic found in expert practices.

## CONCLUSION AND PERSPECTIVES

Many reports deplore the poor integration of ICT in math teaching and researchers stress a phenomenon of "disappointment" after an enthusiastic period where pioneers claimed ICT benefits to learn math. One of the reasons is the "teacher barrier" (see Ruthven, 2007 or Balanskat, Blamire & Kefala 2006) and particularly the importance of teachers' practices, which is seen as a key issue in ICT integration. This is why it seems crucial to progress in understanding of practices and instrumental geneses. In this understanding, the previous study appears to shed light on two elements.

The notion of distance to the referential environment plays a role in technological integration. As we saw, it explains some of Dan's evolutions in terms of a *reduction* of the distance (either by making this distance more explicit or by multiplying moments that alternate work in the two environments enriching both of them). This constitutes a significant creative task for teachers as the tool is not given with any didactical functionalities, it requires a professional instrumental genesis on teacher's side different from the personal genesis and also different from that on pupils' side.

Thus the dual genesis on teachers' side is another interesting element to consider in order to study both local phenomena that can be observed in a session, and the more global evolution of practices, year after year. Dan's emerging practice with the tool is understood better in this frame of double geneses.

Several questions remain, as delimiting more precisely different criteria that create some distance. To understand practices, it is also necessary to determine which elements may counterbalance the distance and play in favor of the tool integration (such as institutional injunctions, or tool epistemic value, didactical design...). We also have to characterise better geneses on teacher's side compared to pupils' one.

## NOTES

1. Beyond the computer transposition that modifies math objects, the modification, from an institutional point of view, concerns the whole ecology of these objects (tasks, techniques, theories). The idea of "distance" reflects the gap between praxeologies associated to two different environments (paper-pencil being a peculiar environment of the math work).
2. Rabardel (2002) distinguishes two types of schemes: *usage schemes* (related to the *material* dimension of the tool) and the *schemes of instrumented action* (related to the global achievement of the task, with goals and intentions).
3. It may not be the case for all teachers: unlike Dan's case, the first instrument can already be constituted in a more advanced way, long before trying to make it a didactical instrument. .
4. The formula refers to the value 50 for the total. If one changes the value of any headcount, then the total will change and the formula becomes wrong.
5. Increment of references after copying make the formula refer to empty cells, by default, empty cell are treated in formulas as if they contain the value 0, this is an option that can be changed.

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