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Decentralized Simultaneous Energy and Information Transmission in Multiple Access Channels

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Decentralized Simultaneous Energy and Information Transmission in Multiple Access Channels

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Abstract: In this report, the fundamental limits of *decentralized* simultaneous information and energy transmission in the two-user Gaussian multiple access channel (G-MAC) are fully characterized for the case in which a minimum energy transmission rate b is required for successful decoding. All the achievable and stable information-energy transmission rate triplets (R_1, R_2, B) are identified. R_1 and R_2 are in bits per channel use measured at the receiver and B is in energy units per channel use measured at an energy-harvester (EH). Stability is considered in the sense of an η -Nash equilibrium (NE), with $\eta \geq 0$ arbitrarily small. The main result consists of the full characterization of the η -NE information-energy region, i.e., the set of information-energy rate triplets (R_1, R_2, B) that are achievable and stable in the G-MAC when: (a) both transmitters autonomously and independently tune their own transmit configurations seeking to maximize their own information transmission rates, R_1 and R_2 respectively; (b) both transmitters jointly guarantee an energy transmission rate B at the EH, such that $B \geq b$. Therefore, any rate triplet outside the η -NE region is not stable as there always exists one transmitter able to increase by at least η bits per channel use its own information transmission rate by updating its own transmit configuration.

Key-words: Gaussian multiple access channel, simultaneous information and energy transmission, RF harvesting, information-energy capacity region, Nash equilibrium, stability.

Selma Belhadj Amor is an INRIA Postdoctoral Research Fellow at Laboratoire CITI (Joint Lab between Université de Lyon, INRIA, and INSA de Lyon), Lyon, France (selma.belhadj-amor@inria.fr). Samir M. Perlaza is an INRIA Research Scientist at Laboratoire CITI (Joint Lab between Université de Lyon, INRIA and INSA de Lyon), Lyon, France (samir.perlaza@inria.fr). He is also with the Department of Electrical Engineering at Princeton University, Princeton, NJ 08544, USA. Part of this work was presented at the 50th Annual Conference on Information Sciences and Systems (CISS), Princeton NJ, USA, March 2016. This research is supported in part by the European Commission under Marie Skłodowska-Curie Individual Fellowship No. 659316 (CYBERNETS).

H. Vincent Poor is with the Department of Electrical Engineering at Princeton University, Princeton, NJ 08544, USA (poor@princeton.edu).

**RESEARCH CENTRE
GRENOBLE – RHÔNE-ALPES**

Inovallée
655 avenue de l'Europe Montbonnot
38334 Saint Ismier Cedex

Transmission décentralisée et simultanée d'information et d'énergie dans les canaux à accès multiple

Résumé : Dans le présent-rapport, les limites fondamentales de la transmission décentralisée et simultanée de l'information et de l'énergie dans les canaux Gaussiens à accès multiple à deux utilisateurs (G-MAC) sont déterminées dans le cas où un débit minimal b de transmission d'énergie est requis pour un décodage réussi. Tous les triplets de débits atteignables et stables de transmission d'énergie et d'information (R_1, R_2, B) sont identifiés. Les débits d'information R_1 et R_2 en bits par utilisation canal sont mesurés au niveau du récepteur et le débit d'énergie B en unités d'énergie par utilisation canal est mesuré au niveau d'un collecteur d'énergie. La stabilité est considérée au sens d'un η -équilibre de Nash (η -NE), avec $\eta \geq 0$ arbitrairement petit. Le résultat principal est la caractérisation complète de la région η -NE d'information-énergie, i.e., l'ensemble des triplets d'information-énergie (R_1, R_2, B) qui sont atteignables et stable dans le G-MAC quand: (a) les deux transmetteurs règlent leurs configurations d'émission d'une manière autonome et indépendante dans le but de maximiser leurs débits individuels de transmission d'information R_1 et R_2 , respectivement; (b) les deux transmetteurs garantissent conjointement un débit de transmission d'énergie B au niveau du collecteur d'énergie tel que $B \geq b$. Par conséquent, tout triplet en dehors de la région η -NE n'est pas stable car il doit toujours y avoir un transmetteur qui soit capable d'augmenter son débit d'information par au moins η bits par utilisation canal en ajustant sa propre configuration d'émission.

Mots-clés : Canal Gaussien à accès multiple (G-MAC), transmission simultanée d'information et d'énergie, collecte d'énergie RF, région de capacité d'information-énergie, équilibre de Nash, stabilité.

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1 Introduction

In wireless communication networks, energy and information can be simultaneously transmitted [1]. From the perspective of information transmission in point-to-point channels, the fundamental limit on the information rate (in bits per channel use) is given by the information capacity [2]. Information capacity is defined as the supremum over all information rates for which it is possible to reliably transmit information. Alternatively, from the perspective of simultaneous energy and information transmission (SEIT), a trade-off between the information transmission rate and the energy transmission rate (in energy units per channel use) is usually observed. For instance, consider a binary symmetric channel (BSC) with cross-over probability p , i.e., $P(1|1) = P(0|0) = 1 - p$ and $P(1|0) = P(0|1) = p$. Assume also that the symbol ‘1’ provides 1 energy unit whereas the symbol ‘0’ provides 0 energy units. The capacity of this channel is $1 - H_2(p)$ bits per channel use and is achieved by equiprobable inputs. Here $H_2(\cdot)$ denotes the binary entropy function. Equiprobable inputs induce an energy rate of $\frac{1}{2}$ energy units per channel use. The maximum energy rate is $1 - p$ energy units per channel use, when ‘1’ is always sent. Let b , with $0 \leq b \leq 1 - p$, denote the required minimum energy rate. If $b > \frac{1}{2}$, then equiprobable capacity-achieving inputs are not sufficient to achieve the minimum energy rate and the transmitter is forced to use the symbol ‘1’ more frequently than the symbol ‘0’, which induces an information rate loss. In this case, the maximum information rate which can be achieved is $H_2(b) - H_2(p)$ and is strictly smaller than the capacity and is decreasing in b . The fundamental limit on the information rate for a minimum energy rate in point-to-point channels is given by the information-energy capacity function derived by Varshney [3].

In the context of multi-user channels the information-energy fundamental limits are fully described by the *information-energy capacity region*. That is, the set of all achievable information-energy tuples at which energy and information can be reliably transmitted. The information-energy capacity region in the discrete memoryless multi-access channel (MAC) and multi-hop networks was studied by Fouladgar *et al.* [4]. Recently, Belhadj Amor *et al.* [5, 6] derived the information-energy capacity region of the Gaussian MAC (G-MAC) with and without feedback where there exists an energy harvester (EH), possibly non-located with the main receiver. Analogously to the point-to-point case, these works show that there exist two energy regimes: One in which the energy rate constraint does not have any impact and thus, the set of achievable information rate tuples are those of the classical G-MAC. Conversely, in the other regime, increasing the information rate implies reducing the energy rate and vice-versa. An object of central interest regarding the results in [4], [5], and [6] is that the achievability of these information-energy rate tuples is subject to the existence of a central controller that decides an operating point and indicates to all network components the corresponding transmit-receive configuration that should be used. Unfortunately, this assumption does not hold in networks in which a central controller is not feasible. This is typically the case of *decentralized* or *ad hoc* networks such as sensor networks, body area networks, among others. In this type of multi-user channels, both the transmitters and the receivers are assumed to be autonomous and to be able of unilaterally choosing their own transmit-receive configurations aiming to maximize their individual benefit, e.g., individual information rate, individual energy rate or a combination of both. Hence, from this perspective, the notion of information-energy capacity does not properly model the fundamental limits of simultaneous energy and information transmission in decentralized networks. To tackle this anarchical behavior observed in decentralized networks, the notion of stability is introduced and a new notion is presented: *the energy-information η -Nash region*. This region is the set of all information-energy rate tuples that are achievable and stable. In this case, stability is considered in the sense of an η -Nash equilibrium (NE) [7], with η arbitrarily small. A multi-user channel is stable in the sense of an η -NE if none of the transmitters or receivers

is able to increase its own individual benefit by more than η units by unilaterally changing its transmit-receive configuration.

The remaining of this report focuses exclusively on the analysis of the decentralized G-MAC with an external EH. Yet very simple, this channel model captures the key aspects of the intrinsic information-energy trade-off in the context of multi-user channels. This analysis strongly relies on tools brought from information theory [8] and game theory [9],[10], and [11]. Previous works have studied decentralized MACs using game-theoretic tools when the aim of each transmitter is limited to exclusively transmitting information. For instance, Lai and El Gamal [12] proposed a framework to study the power allocation problem in fading decentralized MACs when the transmitters aim to maximize their own individual transmission rate. Gajic and Rimoldi [13] considered a similar scenario with time-invariant channels in which the transmitters have the choice of adopting any possible transmit configuration and determined the subregion of the information capacity region that is achievable at an NE. Varan and Yener [14] studied two-hop networks in which the source(s) is (are) incentivized to perform energy and signal cooperation to maximize the amount of its (their) own data that is reliably delivered to the destination. A review of the state of the art on fundamental limits of SEIT in point-to-point and multi-user channels is provided in [15].

This report studies the fundamental limits of *decentralized* SEIT in the two-user G-MAC when a minimum energy rate is required for successful decoding. More specifically, each transmitter chooses its own transmit configuration aiming to maximize its individual information rate to the receiver/information decoder while it guarantees an energy transmission rate higher than a given predefined threshold at a given EH. The receiver is assumed to adopt a fixed configuration that can be either single-user decoding (SUD), successive interference cancellation (SIC) or any time-sharing configuration of the previous decoding techniques. This report provides a game formulation of this problem. The main contribution is the full characterization of the η -NE information-energy region of this game, with $\eta \geq 0$ arbitrarily small. Note that when there is no energy rate requirements, the competition between the players (the transmitters) is only through interference because it is the unique source of interdependence among the different players. However, when a minimum energy rate constraint is required for successful decoding, it clearly creates an additional interdependence among the transmitters. In fact, consider a multi-access scenario in which a given transmitter simultaneously transmits energy to an EH and information to a receiver. If this transmitter is required to deliver an energy rate that is less than what it is able to deliver by only transmitting information, it is able to fulfill the task independently of the behavior of the other transmitters since it can use all its power budget to maximize its information transmission rate and it is still able to meet the energy rate constraint. In this case, the minimum energy constraint does not play a fundamental role. Alternatively, when the requested energy rate is higher than what it is able to deliver by only transmitting information, its behavior is totally dependent on the behavior of the other transmitters since its decodable information rate depends on whether or not the other transmitters are sending messages using an average power for which the minimum energy rate constraint at the EH is satisfied. In this case, the minimum energy rate constraint drastically affects the way that the transmitters interact with each other. More critical scenarios are the case in which the requested energy rate is more than what all transmitters are able to deliver by simultaneously transmitting information using all the available individual power budgets. In these cases, none of them can unilaterally ensure reliable energy transmission at the requested rate. Hence, the transmitters must engage in an energy cooperation mechanism through which they reduce their information rate to be able to send an energy rate that is higher than the minimum required energy rate at the EH. In a decentralized setup, such a cooperation is not natural and this shows how the energy rate constraint can clearly change the behavior of the players.

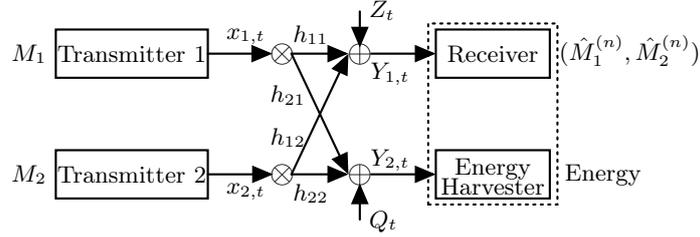


Figure 1: Two-user memoryless Gaussian MAC with energy harvester.

The remainder of the report is structured as follows. Section 2 describes the channel model and provides a game-theoretic formulation of decentralized SEIT in the G-MAC. Section 3 shows the main results of this paper and reports important observations. In Section 4 and Section 5, the proofs are provided. Finally, Section 6 concludes the paper.

2 Gaussian MAC with Minimum Energy Rate b

2.1 Channel Model

Consider the two-user memoryless G-MAC with an EH as shown in Fig. 1. At each channel use $t \in \mathbb{N}$, $X_{1,t}$ and $X_{2,t}$ denote the real symbols sent by transmitters 1 and 2, respectively. Let $n \in \mathbb{N}$ be the blocklength. The symbols $X_{i,1}, \dots, X_{i,n}$ satisfy an expected average *input power constraint*

$$P_i = \frac{1}{n} \sum_{t=1}^n \mathbb{E}[X_{i,t}^2] \leq P_{i,\max}, \quad (1)$$

where the expectation is over the message indices and where P_i and $P_{i,\max}$ denote respectively the average transmit power and the maximum average power of transmitter i in energy units per channel use, for $i \in \{1, 2\}$. The receiver observes the real channel output

$$Y_{1,t} = h_{11}X_{1,t} + h_{12}X_{2,t} + Z_t, \quad (2)$$

and the EH observes

$$Y_{2,t} = h_{21}X_{1,t} + h_{22}X_{2,t} + Q_t, \quad (3)$$

where h_{1i} and h_{2i} are the corresponding constant non-negative channel coefficients from transmitter i to the receiver and EH, respectively. The channel coefficients satisfy the following \mathcal{L}_2 -norm condition:

$$\forall j \in \{1, 2\}, \quad \|\mathbf{h}_j\|^2 \leq 1, \quad (4)$$

with $\mathbf{h}_j \triangleq (h_{j1}, h_{j2})^\top$ in order to meet the energy conservation principle. The noise terms Z_t and Q_t are realizations of two identically distributed zero-mean unit-variance real Gaussian random variables. In the following, there is no particular assumption on the joint distribution of Q_t and Z_t . The signal to noise ratios (SNRs): SNR_{ji} , with $\forall(i, j) \in \{1, 2\}^2$ are defined as follows

$$\text{SNR}_{ji} \triangleq |h_{ji}|^2 P_{i,\max}, \quad (5)$$

given the following normalization over the noise power. Within this context, two main tasks are to be simultaneously accomplished: information transmission and energy transmission.

2.2 Information Transmission

The goal of the communication is to convey the independent messages M_1 and M_2 from transmitter 1 and transmitter 2 to the common receiver. The message indices M_1 and M_2 are independent of the noise terms $Z_1, \dots, Z_n, Q_1, \dots, Q_n$ and uniformly distributed over the sets $\mathcal{M}_1 \triangleq \{1, \dots, \lfloor 2^{nR_1} \rfloor\}$ and $\mathcal{M}_2 \triangleq \{1, \dots, \lfloor 2^{nR_2} \rfloor\}$, where R_1 and R_2 denote the information transmission rates.

At each time t , the t -th symbol of transmitter i , for $i \in \{1, 2\}$, depends solely on its message index M_i and a randomly generated index $\Omega \in \{1, \dots, \lfloor 2^{nR_r} \rfloor\}$, with $R_r \geq 0$, that is independent of both M_1 and M_2 and assumed to be known by all transmitters and the receiver, i.e.,

$$X_{i,t} = f_{i,t}^{(n)}(M_i, \Omega), \quad t \in \{1, \dots, n\}, \quad (6)$$

for some encoding functions $f_{i,t}^{(n)}: \mathcal{M}_i \times \mathbb{N} \rightarrow \mathbb{R}$. The receiver produces an estimate $(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) = \Phi^{(n)}(Y^n)$ of the message-pair (M_1, M_2) via a decoding function $\Phi^{(n)}: \mathbb{R}^n \rightarrow \mathcal{M}_1 \times \mathcal{M}_2$, and the average probability of error is given by

$$P_{\text{error}}^{(n)}(R_1, R_2) \triangleq \Pr \{(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) \neq (M_1, M_2)\}. \quad (7)$$

2.3 Energy Transmission

Let $b \geq 0$ denote the minimum energy rate that must be guaranteed at the input of the EH in the G-MAC. The minimum energy rate b must satisfy

$$0 \leq b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}, \quad (8)$$

for the problem to be feasible. This is mainly because $1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$ is the maximum energy rate that can be achieved at the input of the EH given the input power constraints. This maximum energy rate can be achieved when the transmitters use all their power budgets to send fully correlated channel inputs.

The average energy transmission rate (in energy units per channel use) induced by the sequence $(Y_{2,1}, \dots, Y_{2,n})$ at the input of the EH is

$$B^{(n)} \triangleq \frac{1}{n} \sum_{t=1}^n \mathbb{E}[Y_{2,t}^2], \quad (9)$$

where the expectation is over the message indices M_1 and M_2 .

The goal of the energy transmission is to guarantee that the expected energy rate $B^{(n)}$ is not less than a given target energy transmission rate B that must satisfy

$$b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}. \quad (10)$$

Hence, the probability of energy outage is defined as follows:

$$P_{\text{outage}}^{(n)}(B) = \Pr \{B^{(n)} < B - \epsilon\}, \quad (11)$$

for some $\epsilon > 0$ arbitrarily small.

2.4 Simultaneous Energy and Information Transmission

The G-MAC in Fig. 1 is said to operate at the information-energy rate triplet $(R_1, R_2, B) \in \mathbb{R}_+^3$ when both transmitters and the receiver use a transmit-receive configuration such that: (i)

information transmission occurs at rates R_1 and R_2 with probability of error arbitrarily close to zero; and (ii) energy transmission occurs at a rate not smaller than B with energy-outage probability arbitrarily close zero. Under these conditions, the information-energy rate triplet (R_1, R_2, B) is said to be achievable in the G-MAC with minimum energy rate constraint b .

Definition 1 (Achievable Rates). *The triplet $(R_1, R_2, B) \in \mathbb{R}_+^3$ is achievable if there exists a sequence of encoding and decoding functions $\{\{f_{1,t}^{(n)}\}_{t=1}^n, \{f_{2,t}^{(n)}\}_{t=1}^n, \Phi^{(n)}\}_{n=1}^\infty$ such that both the average error probability and the energy-outage probability tend to zero as the blocklength n tends to infinity. That is,*

$$\limsup_{n \rightarrow \infty} P_{\text{error}}^{(n)}(R_1, R_2) = 0, \text{ and} \quad (12)$$

$$\limsup_{n \rightarrow \infty} P_{\text{outage}}^{(n)}(B) = 0. \quad (13)$$

Note that the minimum energy rate constraint b requires in particular that:

$$\limsup_{n \rightarrow \infty} P_{\text{outage}}^{(n)}(b) = 0. \quad (14)$$

Often, increasing the energy transmission rate implies decreasing the information transmission rates and *vice versa*. An important notion to characterize the fundamental limits on this information-energy trade-off is the *information-energy capacity region* defined as follows:

Definition 2 (Information-Energy Capacity Region). *The information-energy capacity region $\mathcal{E}_b(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ of the G-MAC with minimum energy rate constraint b is the closure of all achievable information-energy rate triplets (R_1, R_2, B) .*

The information-energy capacity region $\mathcal{E}_b(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ is described by the following theorem.

Theorem 1 (Information-Energy Capacity Region with Minimum Energy Constraint b). *The information-energy capacity region $\mathcal{E}_b(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ is the set of all information-energy rate triplets (R_1, R_2, B) that satisfy*

$$0 \leq R_1 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11}), \quad (15a)$$

$$0 \leq R_2 \leq \frac{1}{2} \log_2 (1 + \beta_2 \text{SNR}_{12}), \quad (15b)$$

$$0 \leq R_1 + R_2 \leq \frac{1}{2} \log_2 (1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12}), \quad (15c)$$

$$b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (15d)$$

with $(\beta_1, \beta_2) \in [0, 1]^2$.

Proof: The proof of Theorem 1 follows immediately from [5, Proposition 1] and [5, Theorem 2]. ■

Comments and Observations: In the constraints (15), when the parameters β_1 and β_2 are such that $\beta_1 = \beta_2 = 1$, the corresponding region is characterized by

$$0 \leq R_1 \leq \frac{1}{2} \log_2 (1 + \text{SNR}_{11}), \quad (16a)$$

$$0 \leq R_2 \leq \frac{1}{2} \log_2 (1 + \text{SNR}_{12}), \quad (16b)$$

$$0 \leq R_1 + R_2 \leq \frac{1}{2} \log_2 (1 + \text{SNR}_{11} + \text{SNR}_{12}), \quad (16c)$$

$$b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22}, \quad (16d)$$

where the information rate constraints describe the capacity region of the G-MAC and the upper bound on the energy rate constraint corresponds to the maximum energy rate that can be achieved using independent channel inputs. On the other hand, when the parameters β_1 and β_2 are such that $\beta_1 = \beta_2 = 0$, the information rates are $R_1 = R_2 = 0$ and the energy rate B satisfies

$$b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}. \quad (17)$$

Here the upper bound equals the maximum feasible energy rate. Hence, from this constructive viewpoint, the terms β_1 and β_2 in (15) allow the transmitters to trade-off between information and energy rates. These parameters might be interpreted as the fractions of power that transmitter 1 and transmitter 2, respectively, allocate for information transmission. The remaining fraction of power $(1 - \beta_i)$ is allocated by transmitter i for exclusively transmitting energy to the EH. More specifically, to achieve any rate triplet in this region, at each time t , transmitter i 's channel input can be written as:

$$X_{i,t} = \sqrt{(1 - \beta_i)P_i}W_t + U_{i,t}, \quad i \in \{1, 2\}, \quad (18)$$

for some independent zero-mean Gaussian information-carrying symbols $U_{1,t}$ and $U_{2,t}$ with variances $\beta_1 P_1$ and $\beta_2 P_2$, respectively, and independent thereof W_t are zero-mean unit-variance Gaussian energy-carrying symbols known non-causally to all terminals. The codebook and the encoding-decoding schemes for the information-carrying signals can be those described in [16] or [17].

Note that the information-carrying signals carry both energy and information. These signals are useful to both the EH and the information decoder, whereas the other signals are only energy-carrying and are useful only to the EH. These energy-carrying signals carry only common randomness that allows the creation of correlated signals to increase the energy rate. The common randomness is known to the information decoder and does not produce any interference to the information-carrying signals as its effect can be suppressed using classical successive interference cancellation.

Note that an inherent assumption here is the existence of a central controller that determines an operating point and imposes the transmit or receive configuration to be adopted by each network component. From this global or centralized perspective all points inside the information-energy capacity region are possible operating points. However, in a decentralized network, each network component is an autonomous decision maker that aims to maximize its own individual reward by appropriately choosing a particular transmit or receive configuration. From this perspective, only the information-energy rate tuples that are *stable* can be possible asymptotic operating points. The following subsection describes decentralized simultaneous energy and information transmission over the G-MAC with minimum energy rate constraint b .

2.5 Decentralized Simultaneous Energy and Information Transmission

In a decentralized G-MAC, the aim of transmitter i , for $i \in \{1, 2\}$, is to autonomously choose its transmit configuration s_i in order to maximize its information rate R_i , while guaranteeing a minimum energy rate b at the EH. In particular, the transmit configuration s_i can be described in terms of the information rates R_i , the block-length n , the channel input alphabet \mathcal{X}_i , the encoding functions $f_i^{(1)}, \dots, f_i^{(n)}$, the common randomness, the power dedicated to information and energy transmission, etc. The receiver is assumed to adopt a fixed decoding strategy that is known in advance to all transmitters.

Note that if the aim of each transmitter, say transmitter i , is to maximize its own individual information rate R_i subject to the minimum energy rate b at the EH, it is clear from (15) that

one option should be using a power-split in which the component dedicated to the transmission of information β_i is as high as possible. However, its power-split β_i must also be chosen such that the energy-outage probability (14) can be made arbitrarily close to zero.

This reveals that the choice of the transmit configuration of each transmitter depends on the choice of the other transmitter as they must guarantee the minimum energy constraint required at the EH. At the same time, depending on the decoding scheme at the receiver, the information-carrying signal of one transmitter is interference to the other. This reasoning implies that the rate achieved by transmitter i depends on both configurations s_1 and s_2 as well as the configuration of the receiver, even if it is assumed to be fixed. This justifies the analysis of this scenario using tools from game theory.

2.6 Game Formulation

The competitive interaction of the two transmitters and the receiver in the decentralized G-MAC with minimum energy constraint b can be modeled by the following game in normal form:

$$\mathcal{G}(b) = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}}), \quad (19)$$

where b is a parameter of the game that represents the minimum energy-rate that must be guaranteed at the EH (see (14)). The set $\mathcal{K} = \{1, 2\}$ is the set of players, that is, transmitter 1 and transmitter 2. The sets \mathcal{A}_1 and \mathcal{A}_2 are the sets of actions of players 1 and 2, respectively. An action of a player $i \in \mathcal{K}$, which is denoted by $s_i \in \mathcal{A}_i$, is basically its transmit configuration as described above. The utility function of transmitter i , for $i \in \{1, 2\}$, is $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \rightarrow \mathbb{R}_+$ and it is defined as its own information rate,

$$u_i(s_1, s_2) = \begin{cases} R_i(s_1, s_2), & \text{if } P_{\text{error}}^{(n)}(R_1, R_2) < \epsilon \text{ and } P_{\text{outage}}^{(n)}(b) < \delta \\ -1, & \text{otherwise,} \end{cases} \quad (20)$$

where $\epsilon > 0$ and $\delta > 0$ are arbitrarily small numbers and $R_i(s_1, s_2)$ denotes an information transmission rate achievable (Def. 1) with the configurations s_1 , and s_2 . Note that the utility is -1 when either the error probability or the energy outage probability is not arbitrarily small. This is meant to favor the action profiles in which there is no information transmission (information rate and error probability are zero) but there is energy transmission (probability of energy outage can be made arbitrarily close to zero), over the actions in which the information rate is zero but the energy constraint is not satisfied. Often, the information rate $R_i(s_1, s_2)$ is written as R_i for simplicity. However, every non-negative achievable information rate is associated with a particular transmit-receive configuration pair (s_1, s_2) that achieves it. It is worth noting that there might exist several transmit-receive configurations that achieve the same triplet (R_1, R_2, B) and distinction between the different transmit-receive configurations is made only when needed.

A class of transmit-receive configurations $\mathbf{s}^* = (s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$ that are particularly important in the analysis of this game are referred to as η -Nash equilibria (η -NE).

2.7 η -Nash Equilibrium

A transmit-receive configuration $\mathbf{s}^* = (s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$ that is an η -NE satisfies the following conditions:

Definition 3 (η -NE [18]). In the game $\mathcal{G}(b) = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$, an action profile (s_1^*, s_2^*) is an η -NE if for all $(s_1, s_2) \in \mathcal{A}_1 \times \mathcal{A}_2$, it holds that

$$u_1(s_1, s_2^*) \leq u_1(s_1^*, s_2^*) + \eta, \text{ and} \quad (21)$$

$$u_2(s_1^*, s_2) \leq u_2(s_1^*, s_2^*) + \eta. \quad (22)$$

From Def. 3, it becomes clear that if (s_1^*, s_2^*) is an η -NE, then none of the transmitters can increase its own information transmission rate by more than η bits per channel use by changing its own transmit-receive configuration and keeping the average error probability and the energy outage probability arbitrarily close to zero. Thus, at a given η -NE, every player achieves a utility that is η -close to its maximum achievable information rate given the transmit-receive configuration of the other players. Note that if $\eta = 0$, then the definition of NE is obtained [7]. The following investigates the set of information and energy rate triplets that can be achieved at an η -NE. This set of rate triplets is known as the η -NE information-energy region.

Definition 4 (η -NE Region). Let $\eta \geq 0$. An achievable information-energy rate triplet $(R_1, R_2, B) \in \mathcal{E}_b(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ is said to be in the η -NE region of the game $\mathcal{G}(b) = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$ if there exists a pair $(s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$ that is an η -NE and the following holds:

$$u_1(s_1^*, s_2^*) = R_1 \text{ and} \quad (23)$$

$$u_2(s_1^*, s_2^*) = R_2. \quad (24)$$

The following section studies the η -NE region of the game $\mathcal{G}(b)$, with $\eta \geq 0$ arbitrarily small, for several decoding strategies adopted by the receiver.

3 Main Results

This section describes the η -NE region of the game $\mathcal{G}(b)$ under a fixed decoding strategy. Three cases are examined: single-user decoding (SUD), successive interference cancellation (SIC), and any time-sharing between the previous decoding strategies.

3.1 η -NE Region with Single User Decoding

The η -NE region of the game $\mathcal{G}(b)$ when the receiver uses single-user decoding (SUD), denoted by $\mathcal{N}_{\text{SUD}}(b)$, is described by the following theorem.

Theorem 2 (η -NE Region of the Game $\mathcal{G}(b)$ with SUD). Let $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ and $\eta \geq 0$ arbitrarily small. Then, the set $\mathcal{N}_{\text{SUD}}(b)$ of η -NEs of the game $\mathcal{G}(b)$ contains all information-energy rate-triplets (R_1, R_2, B) which satisfy:

$$0 \leq R_1 \leq \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right), \quad (25a)$$

$$0 \leq R_2 \leq \frac{1}{2} \log_2 \left(1 + \frac{\beta_2 \text{SNR}_{12}}{1 + \beta_1 \text{SNR}_{11}} \right), \quad (25b)$$

$$b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (25c)$$

where $\beta_1 = \beta_2 = 1$ when $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ and (β_1, β_2) satisfy

$$b = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}} \quad (26)$$

when $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$.

Proof: The proof of Theorem 2 is provided in Section 4. ■

That is, when $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$, $\mathcal{N}_{\text{SUD}}(b)$ contains all information-energy rate triplets (R_1, R_2, B) such that

$$R_1 = \frac{1}{2} \log_2 \left(1 + \frac{\text{SNR}_{11}}{1 + \text{SNR}_{12}} \right), \quad (27a)$$

$$R_2 = \frac{1}{2} \log_2 \left(1 + \frac{\text{SNR}_{12}}{1 + \text{SNR}_{11}} \right), \quad (27b)$$

$$b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22}. \quad (27c)$$

Thus, any projection of $\mathcal{N}_{\text{SUD}}(b)$ over a plane $B = b_1$, with $b \leq b_1 \leq 1 + \text{SNR}_{21} + \text{SNR}_{22}$ reduces to a unique information rate point (See Fig. 2).

When $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$, there is an infinity of possible pairs (β_1, β_2) satisfying (26). For a given choice of (β_1, β_2) , the constraints (25) reduce to:

$$R_1 = \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right), \quad (28a)$$

$$R_2 = \frac{1}{2} \log_2 \left(1 + \frac{\beta_2 \text{SNR}_{12}}{1 + \beta_1 \text{SNR}_{11}} \right), \quad (28b)$$

$$B = b. \quad (28c)$$

Hence, at any η -NE the energy rate must be satisfied with equality in order to maximize the information rates (See Fig. 3).

3.2 η -NE Region with Successive Interference Cancellation

Let $\text{SIC}(i \rightarrow j)$ denote the case in which the receiver uses successive interference cancellation (SIC) with decoding order: transmitter i before transmitter j , with $i \in \{1, 2\}$. Then, the η -NE region of the game $\mathcal{G}(b)$ when the receiver uses $\text{SIC}(i \rightarrow j)$, denoted by $\mathcal{N}_{\text{SIC}(i \rightarrow j)}(b)$, is described by the following theorem.

Theorem 3 (η -NE Region of the Game $\mathcal{G}(b)$ with SIC). *Let $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ and $\eta \geq 0$ arbitrarily small. Then, the set $\mathcal{N}_{\text{SIC}(i \rightarrow j)}(b)$ contains all information-energy rate-triplets (R_1, R_2, B) satisfying:*

$$0 \leq R_i = \frac{1}{2} \log_2 \left(1 + \frac{\beta_i \text{SNR}_{1i}}{1 + \beta_j \text{SNR}_{1j}} \right), \quad (29a)$$

$$0 \leq R_j = \frac{1}{2} \log_2 (1 + \beta_j \text{SNR}_{1j}), \quad (29b)$$

$$b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (29c)$$

where $\beta_1 = \beta_2 = 1$ when $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ and (β_1, β_2) satisfy

$$b = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}} \quad (30)$$

when $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$.

Proof: The proof of Theorem 3 is provided in Section 5. ■

The observations in the previous case continue to hold here. The only difference with respect to the previous case is in the information rate constraint at transmitter j . Interference cancellation allows in particular the achievability of information sum-rate optimal points at an η -NE (points on the boundary of the information-energy capacity region) as shown in Fig. 2 and Fig. 3.

3.3 η -NE Region with Time-Sharing

Let $\mathcal{N}(b)$ denote the η -NE region of the game $\mathcal{G}(b)$ when the receiver uses any time-sharing between the previous decoding techniques. This region is described by the following theorem.

Theorem 4 (η -NE Region of the Game $\mathcal{G}(b)$). *Let $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$ and $\eta \geq 0$ arbitrarily small. Then, the set $\mathcal{N}(b)$ is defined as:*

$$\mathcal{N}(b) = \text{Convex hull} \left(\mathcal{N}_{\text{SUD}}(b) \cup \mathcal{N}_{\text{SIC}(1 \rightarrow 2)}(b) \cup \mathcal{N}_{\text{SIC}(2 \rightarrow 1)}(b) \right). \quad (31)$$

That is, if the receiver performs any time-sharing combination between SUD, SIC(1 \rightarrow 2), and SIC(2 \rightarrow 1) then the transmitters can use the same time-sharing combination between their corresponding η -Nash equilibria strategies to achieve any point inside $\mathcal{N}(b)$.

Proof: The proof is based on Theorem 2, Theorem 3, and a time-sharing argument. The details are omitted. \blacksquare

3.4 Example and Observations

Consider a symmetric G-MAC with $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22} = 10$ (EH and receiver are co-located). Note that for all $b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22}$, all transmitters use the whole available average power for information transmission as shown in Fig. 2. Alternatively, when $b > 1 + \text{SNR}_{21} + \text{SNR}_{22}$, both transmitters use the minimum energy needed to make the energy-outage probability arbitrarily close to zero and seek for the largest possible information transmission rate (See Fig. 3).

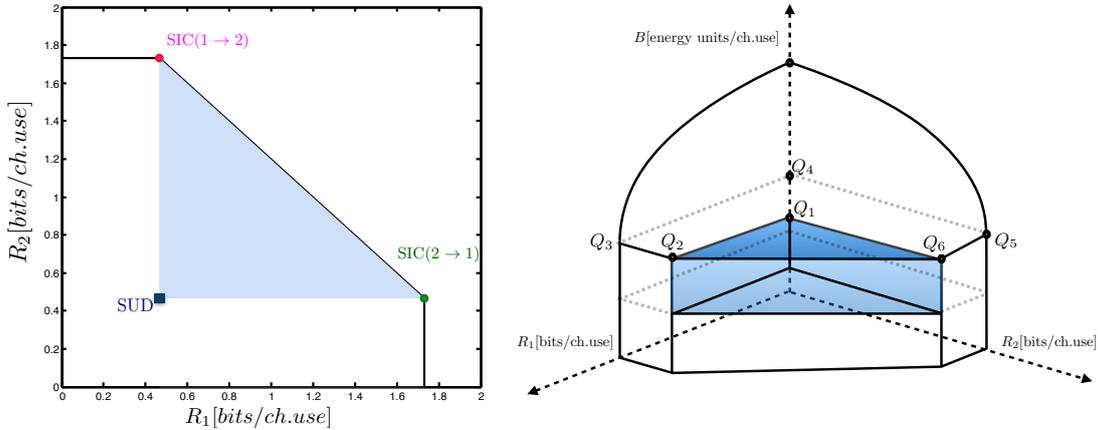


Figure 2: Left figure depicts the the projection of the sets $\mathcal{N}_{\text{SUD}}(b)$ (square point), $\mathcal{N}_{\text{SIC}(i \rightarrow j)}(b)$ (round points), and $\mathcal{N}(b)$ (blue region) over the R_1 - R_2 plane for $b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22}$. The information capacity region is also plotted as a reference (white region inside solid lines) for $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22} = 10$. Note that the information capacity region with and without energy transmission rate constraint are identical in this case. Right figure is a 3-D representation of $\mathcal{N}(b)$ (blue volume). The information-energy capacity region $\mathcal{E}_0(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ is also plotted as a reference (white volume inside solid lines) for $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22} = 10$.

Three main observations arising from Theorem 2, Theorem 3, and Theorem 4 are described in the sequel.

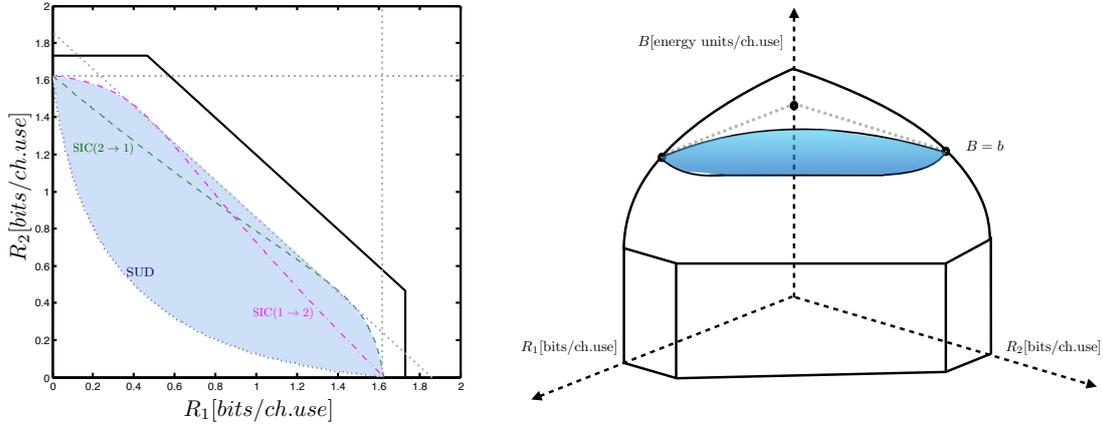


Figure 3: Left figure depicts the projection of the sets $\mathcal{N}_{\text{SUD}}(b)$ (dotted line), $\mathcal{N}_{\text{SIC}(i \rightarrow j)}(b)$ (dashed lines), and $\mathcal{N}(b)$ (blue region) over the R_1 - R_2 plane for $b = 0.7B_{\max} > 1 + \text{SNR}_{21} + \text{SNR}_{22}$. The information capacity region without energy transmission constraints (region inside solid lines) is plotted for $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22} = 10$ (Note that $B_{\max} \triangleq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$). Right figure is a 3-D representation of $\mathcal{N}(b)$ (blue volume). The information-energy capacity region $\mathcal{E}_0(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ is also plotted as a reference (white volume inside solid lines) for $\text{SNR}_{11} = \text{SNR}_{12} = \text{SNR}_{21} = \text{SNR}_{22} = 10$.

3.4.1 Existence of an η -NE

The first observation is that the existence of an η -NE, with η arbitrarily small, is always guaranteed as long as the SEIT problem is feasible, i.e., as long as $b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$. This statement follows immediately from the fact that $\mathcal{N}_{\text{SUD}}(b) \neq \emptyset$, $\mathcal{N}_{\text{SIC}}(b) \neq \emptyset$, and thus $\mathcal{N}(b) \neq \emptyset$, which ensures the existence of at least one action profile (s_1^*, s_2^*) that is an η -NE. Interestingly, when $b > 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$, the energy transmission cannot be performed reliably and thus, the information-energy capacity region is empty and so is the η -NE region. In this particular case, the problem is not well-posed since such an energy rate is outside the information-energy capacity region.

Remark: Note that for any given $b \geq 0$, the sets $\mathcal{N}_{\text{SUD}}(b)$, $\mathcal{N}_{\text{SIC}}(b)$, and $\mathcal{N}(b)$ include only the information-energy triplets (R_1, R_2, B) that satisfy $B \geq b$. That is, the η -NEs at which the energy constraint can be satisfied. However, this suggests that there might exist other η -NE that are not in these sets at which either one of the conditions, (12) or (13), is not met. Consider for instance a case in which $b \geq 1 + \max(\text{SNR}_{21}, \text{SNR}_{22})$ and both transmitters decide to use the strategies s_1 and s_2 at which none of the transmitters actually transmits, e.g., standby mode. Hence, none of the transmitters can unilaterally deviate and achieve a utility other than $u_1(s_1, s_2) = u_2(s_1, s_2) = -1$ which translates into an information-energy triplet $(0, 0, 0)$ which is also an η -NE but is not in any of the sets $\mathcal{N}_{\text{SUD}}(b)$ or $\mathcal{N}_{\text{SIC}(i \rightarrow j)}(b)$ as the energy constraint cannot be satisfied (Def. 4). More specifically, $(0, 0, 0) \notin \mathcal{E}_b(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$ for all $b > 0$.

3.4.2 Cardinality of the set of η -NE equilibria

The unicity of a given η -NE of the game $\mathcal{G}(b)$ is not ensured even in the case in which the cardinality of the η -NE information-energy region is one. Consider the case in which $\eta = 0$

and $b = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$. In this case, $\mathcal{N}(b) = \{(0, 0, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}})\}$ and for instance, using all the power budget for sending common randomness is an NE action profile. However, there is an infinite number of possible common random sequences that can be adopted by both transmitters independently of the action taken by the receiver as in this case $R_1 = R_2 = 0$. The cardinality of the set of η -NEs is an acceptable lower-bound for the number of equilibria. This suggests that if the cardinality of the η -NE set is infinity. Thus, the number of η -NE is also infinity as every information-energy rate triplet in $\mathcal{N}(b)$ is associated with at least one achievability scheme that is an η -NE (Def. 4).

3.4.3 Optimality of the η -NE

Probably the most interesting observation regarding Theorem 2, Theorem 3, and Theorem 4 is that some of the sum-rate optimal triples (R_1, R_2, B) given a minimum energy-rate b required at the EH are achievable at an η -NE. These η -NE sum-rate optimal triplets are Pareto optimal points of the information-energy capacity region $\mathcal{E}_b(\text{SNR}_{11}, \text{SNR}_{12}, \text{SNR}_{21}, \text{SNR}_{22})$. This suggests that, under the assumption that the players are able to properly choose the operating η -NE for instance via learning algorithms, there is no loss of performance in the decentralized SEIT case with respect to the fully centralized SEIT case.

4 Proof of Theorem 2

Consider the set of information-energy rate-triplets that can be achieved under the assumption that the receiver performs SUD to recover the messages M_1 and M_2 . This set is denoted by $\mathcal{C}_{\text{SUD}}(b)$ and is defined by the following lemma.

Lemma 1 (Achievable Information-Energy Region with SUD). *The set $\mathcal{C}_{\text{SUD}}(b)$ contains all non-negative information-energy rate-triplets (R_1, R_2, B) that satisfy*

$$0 \leq R_1 \leq \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right), \quad (32a)$$

$$0 \leq R_2 \leq \frac{1}{2} \log_2 \left(1 + \frac{\beta_2 \text{SNR}_{12}}{1 + \beta_1 \text{SNR}_{11}} \right), \quad (32b)$$

$$b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (32c)$$

where $(\beta_1, \beta_2) \in [0, 1]^2$.

Proof: The proof of achievability follows similar arguments to those in the proof of Theorem 1 when the decoder is restricted to use SUD to recover the messages M_1 and M_2 . ■

Let the subset $\mathcal{U}_{\text{SUD}}(b) \subseteq \mathcal{C}_{\text{SUD}}(b)$ contain all information-energy rate-triplets $(R_1, R_2, B) \in \mathcal{C}_{\text{SUD}}(b)$ satisfying

$$0 \leq R_1 = \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right), \quad (33a)$$

$$0 \leq R_2 = \frac{1}{2} \log_2 \left(1 + \frac{\beta_2 \text{SNR}_{12}}{1 + \beta_1 \text{SNR}_{11}} \right), \quad (33b)$$

$$b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (33c)$$

where $\beta_1 = \beta_2 = 1$ when $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ and $(\beta_1, \beta_2) \in [0, 1]^2$ are chosen to satisfy the following equality

$$b = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}} \quad (34)$$

when $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$.

Let also the subset $\mathcal{V}_{\text{SUD}}(b) \subseteq \mathcal{C}_{\text{SUD}}(b)$ be defined by $\mathcal{V}_{\text{SUD}}(b) \triangleq \mathcal{C}_{\text{SUD}}(b) \setminus \mathcal{U}_{\text{SUD}}(b)$. Note that for any $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$, the sets $\mathcal{V}_{\text{SUD}}(b)$ and $\mathcal{U}_{\text{SUD}}(b)$ form a partition of $\mathcal{C}_{\text{SUD}}(b)$.

To prove Theorem 2, the first step is to show that

$$\mathcal{N}_{\text{SUD}}(b) \subseteq \mathcal{U}_{\text{SUD}}(b). \quad (35)$$

That is, any achievable information-energy rate-triplet $(R_1, R_2, B) \in \mathcal{V}_{\text{SUD}}(b)$ cannot be an η -NE with η arbitrarily small, i.e., $\mathcal{V}_{\text{SUD}}(b) \cap \mathcal{N}_{\text{SUD}}(b) = \emptyset$. This is proved by Proposition 1.

Proposition 1. *Any information-energy rate-triplet $(R_1, R_2, B) \in \mathcal{V}_{\text{SUD}}(b)$ is not an η -NE with η arbitrarily small. That is,*

$$\mathcal{N}_{\text{SUD}}(b) \subseteq \mathcal{U}_{\text{SUD}}(b). \quad (36)$$

Proof: The proof of Proposition 1 is provided in Section 4.1. ■

The second step consists of showing that:

$$\mathcal{U}_{\text{SUD}}(b) \subseteq \mathcal{N}_{\text{SUD}}(b). \quad (37)$$

That is, all information-energy rate-triplets in $\mathcal{U}_{\text{SUD}}(b)$ are achievable at at least one η -NE with $\eta \geq 0$ arbitrarily small. This is proved by Proposition 2.

Proposition 2. *Any information-energy rate-triplet $(R_1, R_2, B) \in \mathcal{U}_{\text{SUD}}(b)$ is achievable at an η -NE with an arbitrarily small $\eta \geq 0$. That is,*

$$\mathcal{U}_{\text{SUD}}(b) \subseteq \mathcal{N}_{\text{SUD}}(b). \quad (38)$$

Proof: The proof of Proposition 2 is provided in Section 4.2. ■

This completes the proof of Theorem 2.

4.1 Proof of Proposition 1

Any information-energy triplet $(R_1, R_2, B) \in \mathcal{V}_{\text{SUD}}(b)$ satisfies at least one of the following conditions:

$$R_1 < \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right), \quad (39)$$

$$R_2 < \frac{1}{2} \log_2 \left(1 + \frac{\beta_2 \text{SNR}_{12}}{1 + \beta_1 \text{SNR}_{11}} \right), \quad (40)$$

$$B < b, \quad (41)$$

$$B > 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (42)$$

where $\beta_1 = \beta_2 = 1$ when $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ and (β_1, β_2) are chosen to satisfy the following equality

$$b = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}} \quad (43)$$

when $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$.

Before introducing the proof of Proposition 1, some necessary conditions for NE-action profiles are identified. These conditions are provided by Lemma 2 and Lemma 3. Under these necessary conditions, it is later shown that any rate triplet (R_1, R_2, B) that satisfies at least one of the conditions (39)-(42) is not an η -NE, with η arbitrarily small. This establishes the proof of Proposition 1.

4.1.1 Necessary Conditions on NE-Actions

Let (R_1^*, R_2^*, B^*) be an η -NE for any $\eta \geq 0$ arbitrarily small, achievable by an action profile $(s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$. Denote by $X_{i,1}^*, \dots, X_{i,n}^*$ the channel inputs generated by transmitter i , for $i \in \{1, 2\}$, with the equilibrium action s_i^* and denote by P_i^* their average power, that is

$$P_i^* \triangleq \frac{1}{n} \sum_{t=1}^n \mathbb{E}[(X_{i,t}^*)^2]. \quad (44)$$

From the assumption that (R_1^*, R_2^*, B^*) is achievable, $P_{\text{error}}^{(n)}(R_1^*, R_2^*)$ and $P_{\text{outage}}^{(n)}(B^*)$ can be made arbitrarily small. Thus, from (20) it follows that

$$u_1(s_1^*, s_2^*) = R_1^*, \quad (45a)$$

$$u_2(s_1^*, s_2^*) = R_2^*. \quad (45b)$$

Using this notation, the following can be stated.

Lemma 2 (Common Randomness). *A necessary condition for the action profile (s_1^*, s_2^*) to be an η -NE action is that, if the channel inputs $X_{i,t}^*$ are of the form $X_{i,t}^* = X_{i,1,t}^* + X_{i,2,t}^*$ where $X_{i,1,t}^*$ and $X_{i,2,t}^*$ are an information-carrying component and a non-information-carrying component, respectively, then, $X_{i,2,t}^*$ must exclusively be common randomness that is known to the receiver, for $i \in \{1, 2\}$.*

Proof: Without loss of generality, consider transmitter 1 whose utility is given by:

$$u_1(s_1^*, s_2^*) = R_1^*. \quad (46)$$

From the assumptions of the lemma, component $X_{i,2,t}^*$ does not increase the information rate. Let R_1 denote the information rate that can be achieved by transmitter 1 if the interference caused by its component $X_{i,2,t}^*$ can be completely canceled at the receiver before decoding the messages M_1 and M_2 .

Assume that in the action s_1^* , the component $X_{i,2,t}^*$ does not exclusively carry common randomness that is known to the receiver. Hence, the receiver is not able to cancel the energy-carrying component before decoding it. This additional interference reduces the information rate of transmitter 1. Let $\delta > 0$ denote the penalty on the information rate of transmitter 1 that is caused by this additional interference. That is,

$$R_1^* = R_1 - \delta. \quad (47)$$

Regardless of the action s_2^* , transmitter 1 can use an alternative action s_1 in which the component $X_{i,2,t}^*$ carries only common randomness known to the receiver. Thus, its effect can be completely canceled and the information transmission can be performed at rate R_1 . The corresponding utility is

$$u_1(s_1, s_2^*) = R_1. \quad (48)$$

From (46), (47), and (48), it holds that

$$u_1(s_1, s_2^*) - u_1(s_1^*, s_2^*) = R_1 - R_1^* = \delta > 0. \quad (49)$$

The utility improvement is bounded away from zero, thus the action profile (s_1^*, s_2^*) cannot be η -NE (Def. 3), with an arbitrarily small $\eta \geq 0$. \blacksquare

Remark 1. Since the messages M_1 and M_2 are independent, the only possible source of correlation between the time- t channel inputs $X_{1,t}$ and $X_{2,t}$ is the common randomness that is known non-causally to the transmitters and to the receiver.

Lemma 3 (IID Gaussian Inputs with Maximum Power). *A necessary condition for the action profile (s_1^*, s_2^*) to be an η -NE action is that the input symbols $X_{i,t}^*$, with $i \in \{1, 2\}$, must be generated i.i.d. following a zero-mean Gaussian distribution with variance $P_i^* = P_{i,\max}$.*

Proof: Without loss of generality, consider transmitter 1 and let \tilde{R}_1 denote the information rate that can be achieved by transmitter 1 when the input symbols are generated i.i.d. following a Gaussian distribution with maximum power $P_{1,\max}$ and where the information-carrying components of both transmitters are uncorrelated.

Assume that in the action s_1^* , the input symbols are not generated i.i.d. following a Gaussian distribution with variance P_1^* . Since Gaussian distribution maximizes the entropy and since the information rates are increasing in the input power, using non-Gaussian inputs or using less power result in a loss in the achievable information rate. Thus in the action s_1^* the utility of transmitter 1 is

$$u_1(s_1^*, s_2^*) = R_1^* = \tilde{R}_1 - \zeta, \quad (50)$$

where $\zeta > 0$ quantifies the loss in information rate.

From the assumption that the receiver implements SUD, independently of the action s_2^* of transmitter 2, there always exists an alternative action s_1 in which transmitter 1 uses i.i.d. Gaussian codebooks with variance $P_1^* = P_{1,\max}$, which achieves an information rate (and thus a utility)

$$u_1(s_1, s_2^*) = \tilde{R}_1. \quad (51)$$

From (50) and (51), it follows that

$$u_1(s_1, s_2^*) - u_1(s_1^*, s_2^*) = \zeta > 0. \quad (52)$$

The utility improvement is bounded away from zero, thus the action profile (s_1^*, s_2^*) cannot be η -NE (Def. 3), with an arbitrarily small $\eta \geq 0$. ■

4.1.2 Proof of (39) and (40)

Without loss of generality, consider transmitter 1 and assume that in the action profile (s_1^*, s_2^*) , the information rate R_1^* satisfies (39). From Lemmas 2 and 3, a necessary condition for the action s_1^* to be an η -NE action is to have i.i.d. Gaussian channel inputs with maximum power $P_{1,\max}$ in which the energy-carrying component exclusively carries common randomness known to the receiver. This condition implies that any information rate R_1 satisfying

$$0 \leq R_1 \leq \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right), \quad (53)$$

can be achieved with arbitrarily small probability of error.

Assume that in the action s_1^* transmitter 1's utility (information rate) satisfies

$$u_1(s_1^*, s_2^*) = R_1^* = \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right) - \xi, \quad (54)$$

with $\xi > 0$. Regardless of the action of transmitter 2, transmitter 1 can always choose an alternative action \tilde{s}_1 in which it has a utility

$$u_1(\tilde{s}_1, s_2^*) = \tilde{R}_1 = \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right). \quad (55)$$

From (54) and (55), it holds that

$$u_1(\tilde{s}_1, s_2^*) - u_1(s_1^*, s_2^*) = \xi > 0. \quad (56)$$

The utility improvement is bounded away from zero, thus the action profile (s_1^*, s_2^*) cannot be η -NE (Def. 3), with an arbitrarily small $\eta \geq 0$.

The same reasoning holds for transmitter 2 and thus an action profile (s_1^*, s_2^*) which induces an information-energy rate triplet (R_1^*, R_2^*, B^*) for which either

$$R_1^* < \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right)$$

or

$$R_2^* < \frac{1}{2} \log_2 \left(1 + \frac{\beta_2 \text{SNR}_{12}}{1 + \beta_1 \text{SNR}_{11}} \right)$$

cannot an η -NE, with η arbitrarily small.

4.1.3 Proof of (41)

This is trivial since, $B^* < b$ implies that the triplet (R_1^*, R_2^*, B^*) is not achievable (Def. 1).

4.1.4 Proof of (42)

Assume that there exists an energy-information triplet (R_1^*, R_2^*, B^*) that is achievable at an η -NE by the action profile $(s_1^*, s_2^*) \in \mathcal{A}_1 \times \mathcal{A}_2$ in which

$$B^* > 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (57)$$

where $\beta_1 = \beta_2 = 1$ when $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ and (β_1, β_2) are chosen to satisfy the following equality

$$b = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}} \quad (58)$$

when $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$. That is, (57) can equivalently be written as

$$B^* > 1 + \text{SNR}_{21} + \text{SNR}_{22}, \quad (59)$$

when $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$ and

$$B^* > b, \quad (60)$$

when $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$. More concisely, this can be written as

$$B^* > \max\{b, 1 + \text{SNR}_{21} + \text{SNR}_{22}\}. \quad (61)$$

Assume that in the action profile (s_1^*, s_2^*) , the energy rate B^* satisfies

$$B^* > \max\{b, 1 + \text{SNR}_{21} + \text{SNR}_{22}\}. \quad (62)$$

From the previous parts of the proof (Lemma 2 and Lemma 3), a necessary condition at any η -NE with η arbitrarily small is to have the transmitters use Gaussian codebooks in which the channel inputs $\{(X_{1,t}^*, X_{2,t}^*)\}_{t=1}^n$ are generated i.i.d. according to a Gaussian distribution with maximum powers $P_{1,\max}$ and $P_{2,\max}$, respectively.

The fact that the energy rate cannot exceed the maximum feasible value given the constrained power budget at the transmitters, together with assumption (62) lead the following constraints on B^* :

$$\max\{b, 1 + \text{SNR}_{21} + \text{SNR}_{22}\} < B^* \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}. \quad (63)$$

Using continuity arguments, the energy rate B^* can be written as:

$$B^* = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}\lambda. \quad (64)$$

where $\lambda \in [0, 1]$ is given by

$$\lambda \triangleq \frac{B^* - (1 + \text{SNR}_{21} + \text{SNR}_{22})}{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}}. \quad (65)$$

Assumption (62) requires in particular that the energy rate is larger than $1 + \text{SNR}_{21} + \text{SNR}_{22}$ which is the maximum energy rate that can be achieved at the input of the EH when the channel inputs $X_{1,t}^*$ and $X_{2,t}^*$ are independent. The strict inequality in (62) implies that $X_{1,t}^*$ and $X_{2,t}^*$ must be correlated, i.e., $\lambda > 0$. The parameter λ can thus be interpreted as the Pearson correlation factor between $X_{1,t}^*$ and $X_{2,t}^*$ which is the same at each time t since the inputs are i.i.d. (Lemma 3), i.e., $\forall t \in \{1, \dots, n\}$,

$$\lambda \triangleq \frac{1}{n} \sum_{t=1}^n \frac{\mathbb{E}[X_{1,t}^* X_{2,t}^*]}{\sqrt{P_{1,\max} P_{2,\max}}} = \frac{\mathbb{E}[X_{1,t}^* X_{2,t}^*]}{\sqrt{P_{1,\max} P_{2,\max}}}. \quad (66)$$

Assumption (62) implies the following lower bound on λ :

$$\lambda > \frac{\max\{b, 1 + \text{SNR}_{21} + \text{SNR}_{22}\} - (1 + \text{SNR}_{21} + \text{SNR}_{22})}{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}}. \quad (67)$$

Since the only source of correlation is common randomness whose effect is canceled before decoding the messages (See Remark 1), for any values of the utilities (R_1^*, R_2^*) , one can always find β_1 and β_2 satisfying

$$\lambda = \sqrt{(1 - \beta_1)(1 - \beta_2)} \quad (68)$$

such that the utilities at the equilibrium (Recall the necessary condition at any η -NE with η arbitrarily small) can be written as

$$u_1(s_1^*, s_2^*) = R_1^* = \frac{1}{2} \log_2 \left(1 + \frac{\beta_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right) - \epsilon_1, \quad (69)$$

$$u_2(s_1^*, s_2^*) = R_2^* = \frac{1}{2} \log_2 \left(1 + \frac{\beta_2 \text{SNR}_{12}}{1 + \beta_1 \text{SNR}_{11}} \right) - \epsilon_2, \quad (70)$$

with $\epsilon_1, \epsilon_2 > 0$ arbitrarily small.

The corresponding energy rate is given by

$$B^* = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}. \quad (71)$$

One way to construct the channel inputs $X_{i,t}^*$ is

$$X_{i,t}^* = \sqrt{(1 - \beta_i)P_{i,\max}}U_t^* + \sqrt{\beta_i P_{i,\max}}V_{i,t}^*, \quad i \in \{1, 2\}, \quad (72)$$

where U^* , V_1^* , and V_2^* are zero-mean unit-variance Gaussian RVs that are mutually independent. The variable U^* depends on the common randomness Ω and the variable V_i^* depends on the message M_i for $i \in \{1, 2\}$.

Case 1: $b \in [0, 1 + \text{SNR}_{21} + \text{SNR}_{22}]$

In this case, the lower bound (67) is simply $\lambda > 0$ which implies that $\beta_1 < 1$ and $\beta_2 < 1$ in (69) and (70), respectively.

Without loss of generality consider transmitter 1 and assume that in the action s_1^* , transmitter 1 uses a power-split $\beta_1 < 1$, which achieves a utility as in (69). Regardless of the action s_2^* , transmitter 1 can use an action s_1 in which it uses a power-split $\beta_1 = 1$ and which achieves

$$R_1 = \frac{1}{2} \log_2 \left(1 + \frac{\text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right), \quad (73a)$$

$$B = 1 + \text{SNR}_{21} + \text{SNR}_{22} \geq b, \quad (73b)$$

and thus, the resulting utility of transmitter 1 is

$$u_1(s_1, s_2^*) = \frac{1}{2} \log_2 \left(1 + \frac{\text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right). \quad (74)$$

From (69) and (74), it follows that

$$u_1(s_1, s_2^*) - u_1(s_1^*, s_2^*) = \frac{1}{2} \log_2 \left(1 + \frac{(1 - \beta_1) \text{SNR}_{11}}{1 + \beta_1 \text{SNR}_{11} + \beta_2 \text{SNR}_{12}} \right) + \epsilon_1 > 0, \quad (75)$$

which contradicts the assumption that (s_1^*, s_2^*) is an η -NE (Def. 3), with an $\eta \geq 0$ arbitrarily small. Hence, a necessary condition for an action profile to be an η -NE with $\eta \geq 0$ is $\beta_1 = 1$.

A similar argument applies to transmitter 2, i.e., and thus any action profile for which $\beta_1 \neq 1$ or $\beta_2 \neq 1$ cannot be an η -NE with $\eta \geq 0$. As a consequence, any action profile with

$$B^* > 1 + \text{SNR}_{21} + \text{SNR}_{22}. \quad (76)$$

cannot be an η -NE with $\eta \geq 0$.

Case 2: $b \in (1 + \text{SNR}_{21} + \text{SNR}_{22}, 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}]$

In this case, the lower bound (67) is

$$\lambda > \frac{b - (1 + \text{SNR}_{21} + \text{SNR}_{22})}{2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}} > 0, \quad (77)$$

and thus (β_1, β_2) must satisfy

$$b < 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (78)$$

with strict inequality.

Without loss of generality, consider transmitter 1 whose utility is given by (69) where (β_1, β_2) satisfy (78).

Independently of the action s_2^* of transmitter 2, transmitter 1 can use an alternative action s_1 in which it uses more correlation by choosing a parameter β'_1 that satisfies

$$b = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta'_1)\text{SNR}_{21}(1 - \beta_2)\text{SNR}_{22}}, \quad (79)$$

and achieves

$$R'_1 = \frac{1}{2} \log_2 \left(1 + \frac{\beta'_1 \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right) - \epsilon, \quad (80)$$

$$B' = b, \quad (81)$$

with $0 < \epsilon < \epsilon_1$ arbitrarily small. Hence its utility is given by

$$u_1(s_1, s_2^*) = \frac{1}{2} \log_2 \left(1 + \frac{\beta_1' \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12}} \right) - \epsilon. \quad (82)$$

From (78) and (79), it holds that $\beta_1' > \beta_1$. Hence, the utility improvement ((69) and (82)) induced by the alternative action s_1 is given by

$$u_1(s_1, s_2^*) - u_1(s_1^*, s_2^*) = \frac{1}{2} \log_2 \left(1 + \frac{(\beta_1' - \beta_1) \text{SNR}_{11}}{1 + \beta_2 \text{SNR}_{12} + \beta_1 \text{SNR}_{11}} \right) - \epsilon + \epsilon_1 > 0 \quad (83)$$

which contradicts the assumption that (s_1^*, s_2^*) is an η -NE (Def. 3), with an $\eta \geq 0$ arbitrarily small. This completes the proof of Proposition 1.

4.2 Proof of Proposition 2

Let $\eta \geq 0$ be arbitrarily small and assume that the decoder performs SUD.

Case 1: $0 \leq b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22}$:

Consider the rate triplet (R_1^*, R_2^*, B^*) satisfying

$$R_1^* = \frac{1}{2} \log_2 \left(1 + \frac{\text{SNR}_{11}}{1 + \text{SNR}_{12}} \right), \quad (84a)$$

$$R_2^* = \frac{1}{2} \log_2 \left(1 + \frac{\text{SNR}_{12}}{1 + \text{SNR}_{11}} \right), \quad (84b)$$

$$B^* = 1 + \text{SNR}_{21} + \text{SNR}_{22}. \quad (84c)$$

The targeted energy rate b is less than what is strictly necessary to guarantee reliable communication at maximum information sum-rate. Thus, the energy rate constraint is vacuous and the transmitters can exclusively use all their available power budget to send information, i.e., $\beta_1 = \beta_2 = 1$.

To achieve (R_1^*, R_2^*, B^*) , transmitters 1 and 2 can use the action profile (s_1^*, s_2^*) described in the sequel. Transmitters 1 and 2 use independent Gaussian codebooks with powers $P_{1,\max}$ and $P_{2,\max}$, respectively, as in [16] or [17]. The messages M_1 and M_2 are encoded at the information rates R_1^* and R_2^* , respectively. The resulting average energy rate at the input of the EH is given by $B^{(n)} = 1 + \text{SNR}_{21} + \text{SNR}_{22}$, which ensures that the energy outage probability $P_{\text{outage}}^{(n)}(B^*)$ can be made arbitrarily small as the blocklength tends to infinity. From the assumption that the receiver performs SUD, probability of error $P_{\text{error}}^{(n)}(R_1^*, R_2^*)$ can be made arbitrarily small as the blocklength tends to infinity. Hence the resulting utilities are given by:

$$u_1(s_1^*, s_2^*) = R_1^*, \quad (85a)$$

$$u_2(s_1^*, s_2^*) = R_2^*. \quad (85b)$$

Assume that the action profile (s_1^*, s_2^*) is not an η -NE. Then, from Def. 3, there exist at least one player $i \in \{1, 2\}$ and at least one strategy $\tilde{s}_i \in \mathcal{A}_i$ such that the utility u_i is improved by at least η bits per channel use when player i deviates from s_i^* to \tilde{s}_i .

Without loss of generality, let transmitter 1 be the deviating player and denote by \tilde{R}_1 its new information rate. Hence,

$$u_1(\tilde{s}_1, s_2^*) = \tilde{R}_1 > u_1(s_1^*, s_2^*) + \eta. \quad (86)$$

From (84a), (85a), and (86), it holds that

$$\tilde{R}_1 > \frac{1}{2} \log_2 \left(1 + \frac{\text{SNR}_{11}}{1 + \text{SNR}_{12}} \right) + \eta. \quad (87)$$

Since the information rate-pair (R_1^*, R_2^*) already saturates the decoding capability of the receiver, the new information rate pair (\tilde{R}_1, R_2^*) cannot be achieved and will result in a probability of error bounded away from zero and consequently the corresponding utility will be:

$$u_1(\tilde{s}_1, s_2^*) = -1, \quad (88)$$

which contradicts the initial assumption (86) and establishes that the action profile (s_1^*, s_2^*) is an η -NE. For the same information rates (R_1^*, R_2^*) , for any energy rate B with $b \leq B \leq 1 + \text{SNR}_{21} + \text{SNR}_{22}$ the information-energy rate-triplet (R_1^*, R_2^*, B) is also achievable by the same action profile (s_1^*, s_2^*) . Note that (R_1^*, R_2^*, B) is also achievable at an η -NE.

Case 2: $1 + \text{SNR}_{21} + \text{SNR}_{22} < b \leq 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$:

Consider the information-energy rate-triplet (R_1^*, R_2^*, B^*) such that:

$$R_1^* = \frac{1}{2} \log_2 \left(1 + \frac{\beta_1^* \text{SNR}_{11}}{1 + \beta_2^* \text{SNR}_{12}} \right), \quad (89a)$$

$$R_2^* = \frac{1}{2} \log_2 \left(1 + \frac{\beta_2^* \text{SNR}_{12}}{1 + \beta_1^* \text{SNR}_{11}} \right), \quad (89b)$$

$$B^* = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1^*)\text{SNR}_{21}(1 - \beta_2^*)\text{SNR}_{22}} \quad (89c)$$

where (β_1^*, β_2^*) are chosen to satisfy

$$b = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1^*)\text{SNR}_{21}(1 - \beta_2^*)\text{SNR}_{22}}. \quad (90)$$

To achieve (R_1^*, R_2^*, B^*) , transmitters 1 and 2 can use the action profile (s_1^*, s_2^*) described in the sequel. Transmitters 1 and 2 use power fractions β_1^* and β_2^* of their power budgets $P_{1,\max}$ and $P_{2,\max}$ to send information using independent Gaussian codebooks as in [16] or [17], and use the remaining power $((1 - \beta_1^*)P_{1,\max}$ and $(1 - \beta_2^*)P_{2,\max}$) to send common randomness that is known to both transmitters and the receiver. This common randomness does not carry any information and does not produce any interference to the information-carrying signals. The messages M_1 and M_2 are encoded at the information rates R_1^* and R_2^* chosen by transmitters 1 and 2, respectively. The receiver first subtracts the common randomness and then performs SUD to recover the messages M_1 and M_2 .

The resulting average energy rate at the input of the EH is given by $B^{(n)} = 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{(1 - \beta_1^*)\text{SNR}_{21}(1 - \beta_2^*)\text{SNR}_{22}}$. This ensures that $B^* \geq b$ and that the energy outage probability $P_{\text{outage}}^{(n)}(B^*)$ can be made arbitrarily small as the blocklength tends to infinity.

Assume that the action profile (s_1^*, s_2^*) is not an η -NE. Then, from Def. 3, there exist at least one player $i \in \{1, 2\}$ and at least one strategy $\tilde{s}_i \in \mathcal{A}_i$ such that the utility u_i is improved by at least η bits per channel use when player i deviates from s_i^* to \tilde{s}_i .

Without loss of generality, let transmitter 1 be the deviating player and denote by \tilde{R}_1 its new information rate. Hence,

$$u_1(\tilde{s}_1, s_2^*) = \tilde{R}_1 > u_1(s_1^*, s_2^*) + \eta. \quad (91)$$

The new information-energy rate-triplet $(\tilde{R}_1, R_2^*, B^*)$ is outside the information-energy capacity region and will result in a utility

$$u_1(\tilde{s}_1, s_2^*) = -1 \quad (92)$$

which contradicts the assumption (91) and establishes that (s_1^*, s_2^*) is an η -NE.

5 Proof of Theorem 3

The proof of Theorem 3 follows the same lines as the proof of Theorem 2 when considering the set of information-energy rate-triplets which can be achieved if the receiver performs SIC($i \rightarrow j$) to recover the messages M_1 and M_2 . This set is denoted by $\mathcal{C}_{\text{SIC}(i \rightarrow j)}(b)$ and is defined as the set of (R_1, R_2, B) that satisfy

$$0 \leq R_i \leq \frac{1}{2} \log_2 \left(1 + \frac{\beta_i \text{SNR}_{1i}}{1 + \beta_j \text{SNR}_{1j}} \right), \quad (93a)$$

$$0 \leq R_j \leq \frac{1}{2} \log_2 (1 + \beta_j \text{SNR}_{1j}), \quad (93b)$$

$$b \leq B \leq 1 + \text{SNR}_{2i} + \text{SNR}_{2j} + 2\sqrt{(1 - \beta_i)\text{SNR}_{2i}(1 - \beta_j)\text{SNR}_{2j}}, \quad (93c)$$

with $(\beta_i, \beta_j) \in [0, 1]^2$ are feasible power-splits, i.e.,

$$b \leq 1 + \text{SNR}_{2i} + \text{SNR}_{2j} + 2\sqrt{(1 - \beta_i)\text{SNR}_{2i}(1 - \beta_j)\text{SNR}_{2j}}. \quad (94)$$

6 Conclusion

In this paper, the fundamental limits of decentralized SEIT in the two-user G-MAC with minimum received energy rate constraint were derived in terms of η -NE regions, with $\eta \geq 0$ arbitrarily small. The results presented here for the two-user G-MAC can be easily extended to an arbitrary number of users. A key observation in this work is the fact the decentralization induces no loss of performance for SEIT as long as the players are able to properly choose the operating η -NE for instance via learning algorithms. Recently, Belhadj Amor *et al.* have shown that channel output feedback enhances SEIT as it provides additional cooperation among the users. An interesting question to look at is whether feedback may help in the decentralized case. Also, the knowledge given to each player and the order in which actions can be taken substantially change the nature of the game and the corresponding stable region. Furthermore, such a region varies depending on the associated notion of equilibrium, e.g., η -Nash equilibrium (η -NE) [7], Stackelberg equilibrium [19], correlated equilibrium [20], satisfaction equilibrium [21], etc.

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