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EXPLICIT MULTI-MODEL PREDICTIVE CONTROL OF A WASTE HEAT RANKINE BASED SYSTEM FOR HEAVY DUTY TRUCKS

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In nowadays heavy duty engines, a major part of the chemical energy contained in the fuel is released to the ambient through heat.
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Waste heat recovery based on the Rankine cycle is a promising technique to increase fuel efficiency.
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Waste heat recovery based on the Rankine cycle is a promising technique to increase fuel efficiency.

Dynamic models needed for concept optimization, fuel economy evaluation and control algorithm development.
Rankine process

- Liquid compression (1 → 2) from condensing to evaporating pressure by means of the pump power $\dot{W}_{in}$.
- Preheating (2 → 3a), vaporization (3a → 3b) and superheating (3b → 3c) by means of the input heat power $\dot{Q}_{in}$.
- Vapor expansion (3c → 4) from evaporating to condensing pressure creating power $\dot{W}_{out}$ on the expander shaft.
- Condensation (4 → 1) releasing heat $\dot{Q}_{out}$ in the heat sink.

Figure: Temperature-entropy diagram of the Rankine cycle
Recover heat from both EGR and exhaust in a serial configuration.

Working fluid: water ethanol mixture.

Focus on the control of the working fluid superheat at the expansion machine inlet.

Even more critical when using a kinetic expander.

Control issue: Reduce the deviation of the superheat around its set point to have safe and efficient operation.
Nonlinear evaporator detailed model

Model representation

\[ \dot{x}_i = f_i(x_i, u), \quad (1) \]

\[ u^T = [\dot{m}_f, P_f, h_f, \dot{m}_g, T_g], \quad x_i^T = [\dot{m}_f, h_f, T_{w_{int}}, T_g, T_{w_{ext}}] \quad (2) \]

\[ f_i(x_i, u) = \]
\[ \begin{bmatrix}
\dot{m}_f \left( \frac{h_f}{T_{g_{i-1}}} - \frac{\rho_f h_f}{T_{g_{i-1}}} \right) + \frac{1}{\rho_f} \frac{\partial \rho_f}{\partial h_f} \alpha_f A_{\text{exch}_{int}} \left( T_f - T_{w_{int}} \right) - \dot{m}_f \\
1 - \frac{h_f}{\rho_f} \frac{\partial \rho_f}{\partial h_f} \\
\left( \dot{m}_f h_f - \dot{m}_f h_f \right) - \alpha_f A_{\text{exch}_{int}} \left( T_f - T_{w_{int}} \right) \\
\rho_f V_f f_i \left( T_f - T_{w_{int}} \right) + \alpha_g A_{\text{exch}_{int}} \left( T_g - T_{w_{int}} \right) \\
\dot{m}_g c_{pg} \left( T_{g_{i-1}} - T_{g_i} \right) - \alpha_g \left[ A_{\text{exch}_{int}} \left( T_g - T_{w_{int}} \right) - A_{\text{exch}_{ext}} \left( T_g - T_{w_{ext}} \right) \right] \\
\rho_{g_{i-1}} V_g c_{pg} \left( T_{g_{i-1}} \right) + \alpha_g A_{\text{exch}_{ext}} \left( T_g - T_{w_{ext}} \right) \\
\rho_{w_{int}} V_{w_{int}} \left( T_{g_{i-1}} - T_{g_i} \right) \\
\rho_{w_{ext}} V_{w_{ext}} \left( T_{g_{i-1}} - T_{g_i} \right) \\
\alpha_{amb} A_{\text{exch}_{ext}_{amb}} \left( T_{amb} - T_{w_{ext}} \right) + \alpha_g A_{\text{exch}_{ext}} \left( T_g - T_{w_{ext}} \right)
\end{bmatrix} \]
Dynamic relation between $u$ (working fluid mass flow) and $y$ (working fluid superheat) can be described around an operating point by a first order plus time delay (FOPTD) model:

$$
\frac{y(s)}{u(s)} = \frac{G}{1 + \tau s} e^{-Ls}, \quad (3)
$$

High variation in FOPTD parameters shows high nonlinearity.

Linear time invariant controller will hardly achieve the control objective with good performance under transient driving cycle.

Identification

- Dynamic relation between $u$ (working fluid mass flow) and $y$ (working fluid superheat) can be described around an operating point by a first order plus time delay (FOPTD) model:

$$
\frac{y(s)}{u(s)} = \frac{G}{1 + \tau s} e^{-Ls}, \quad (3)
$$

- High variation in FOPTD parameters shows high nonlinearity.

- Linear time invariant controller will hardly achieve the control objective with good performance under transient driving cycle.
Multi linear model approach consists into identifying a bank of $N$ linear models and combine them by means of a weighting scheme.

Global model output is (at time $t_k$):

$$y_k = \sum_{i=1}^{N} y_{i,k} W_{i,k}$$

(4)

Key design issues are: 1/ the selection of the good model(s) in the bank. 2/ linear models mixing.

Modeling error of the $i^{th}$ model at the current time $t_k$ is defined by:

$$\epsilon_{i,k} = y_{p,k} - y_{i,k}.$$  

(5)
## Weighting scheme

**Bayesian recursive scheme**

\[
p_{i,k} = \frac{\exp(-\frac{1}{2} \epsilon_{i,k} K \epsilon_{i,k}^T p_{i,k-1})}{\sum_{m=1}^{N} \left(\exp(-\frac{1}{2} \epsilon_{m,k} K \epsilon_{m,k}^T p_{m,k-1})\right)} \tag{6}
\]

\[
W_{i,k} = \begin{cases} 
\frac{p_{i,k}}{\sum_{m=1}^{N} p_{m,k}} & \text{for } p_{i,k} > \delta \\
0 & \text{for } p_{i,k} < \delta 
\end{cases} \tag{7}
\]

where \(K\) is a vector and \(\delta\) a scalar.

**New proposed scheme**

\[
\tilde{\epsilon}_{i,k} = \frac{\epsilon_{i,k}^2}{\sum_{m=1}^{N} \epsilon_{m,k}^2} \tag{8}
\]

\[
X_{i,k} = (1 - \tilde{\epsilon}_{i,k}) \prod_{j \neq i, j=1}^{N} \tilde{\epsilon}_{j,k} \tag{9}
\]

\[
\tilde{X}_{i,k} = \frac{X_{i,k}}{\sum_{m=1}^{N} X_{m,k}} \tag{10}
\]

\[
W_{i,k} = \frac{1}{1 + Ts \tilde{X}_{i,k}} \tag{11}
\]

where \(T\) is a scalar.
Optimization problem

Model Predictive Control cost function for set-point tracking

\[
\begin{cases}
\min_{u_{\inf} \leq u_k \leq u_{\sup}} J(u_k) = \int_{t_k}^{t_k + t_p} (y_p(t) - y^{sp})^2 + w_u \Delta u_k^2 \, dt,
\end{cases}
\]  

(12)

where \( w_u \) is a scaling factor and a penalty weight.

Modeling error

\[
e_k = y_{p,k} - y_k.
\]

(13)

Output prediction \( y_p(t) \) in (12) can be written based on the \( N \) models and feedback:

\[
y_p(t) = y(t) + e_k.
\]

(14)

Output response of a FOPTD model

\[
y_i(t) = y_{p,k} e^{-\frac{(t-t_k)}{\tau_i}} + \int_{t_k}^{t} e^{-\frac{(t-s)}{\tau_i}} \frac{G_i}{\tau_i} u(s - L_i) \, ds.
\]

(15)
A model response (15) can be developed as:

\[ y_i(t) = y_{p,k} e^{\frac{-(t-t_k)}{\tau_i}} + \frac{G_i}{\tau_i} e^{\frac{-t}{\tau_i}} \int_{t_k}^{t} e^{\frac{s}{\tau_i} u(s - L_i)} ds, \tag{16} \]

Based on the time delay \( L_i \), let us define:

\[
\begin{align*}
\lambda_i &= \max(a_i \in \mathbb{N} | a_i \leq \frac{L_i}{T_s}) \\
\Delta L_i &= L_i - \lambda_i T_s, \in \mathbb{R}^+.
\end{align*}
\tag{17}
\]

Integration in (16) is done by parts, where the \( \lambda_i + 2 \) time intervals are:

<table>
<thead>
<tr>
<th>( s )</th>
<th>( s - L_i )</th>
<th>( u(s - L_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_k \rightarrow t_k + \Delta L_i )</td>
<td>( t_k - L_i \rightarrow t_k - \lambda_i )</td>
<td>( u(t_k - \lambda_i - 1) )</td>
</tr>
<tr>
<td>( t_k + \Delta L_i \rightarrow t_k + \Delta L_i + T_s )</td>
<td>( t_k - \lambda_i \rightarrow t_k - \lambda_i + 1 )</td>
<td>( u(t_k - \lambda_i) )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( t_k + \Delta L_i + (\lambda_i - j) T_s \rightarrow t_k + \Delta L_i + (\lambda_i - j + 1) T_s )</td>
<td>( t_{k-j} \rightarrow t_{k-j+1} )</td>
<td>( u(t_{k-j}) )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( t_k + \Delta L_i + (\lambda_i - 1) T_s \rightarrow t_k + L_i )</td>
<td>( t_{k-1} \rightarrow t_k )</td>
<td>( u(t_{k-1}) )</td>
</tr>
<tr>
<td>( t_k + L_i \rightarrow t )</td>
<td>( t_k \rightarrow t - L_i )</td>
<td>( u(t_k) = u_k )</td>
</tr>
</tbody>
</table>
Explicit MMPC formulation

Once integrated, (16) is a linear expression in the optimization argument $u_k$:

$$
y_i(t) = y_{p,k}f_1_i(\tau_i, t_k, t) + f_2_i(T_s, G_i, \tau_i, \Delta L_i, \lambda_i, t_k, t, u(past)) + u_kf_3_i(G_i, \tau_i, L_i, t_k, t),
$$

(18)

where the $f_i$ may be explicitly defined offline and updated online at each time $t_k$. Hence the initial cost function is:

$$
J(u_k) = \int_{t_k}^{t_k+t_p} \left( \sum_{i=1}^{N} \left( w_{i,k}y_i(t) + e_k - y_{sp}^k \right)^2 + w_u\Delta u_k^2 \right) dt
$$

(19)

where the prediction horizon $t_p = max(t_{pi}) \forall i$ may be tuned as:

$$
\begin{align*}
t_{pi} &= \gamma_p \tau_i + L_i; \quad \gamma_p \in \mathbb{R}^+; \\
e.g.: \gamma_p &= 1 \text{ (63\% of the dynamics is predicted)} \\
or \gamma_p &= 3 \text{ (95\% of the dynamics is predicted)}. 
\end{align*}
$$

(20)
Explicit MMPC formulation

Based on the step response series (16) of the $N$ linear FOPTD models:

$$J(u_k) = \beta_{2,k}(N, G_i, \tau_i, L_i, t_p, w_u, w_i,k)u_k^2$$

$$+\beta_{1,k}(N, T_s, G_i, \tau_i, L_i, t_p, \Delta L_i, \lambda_i, w_u, y_p, y_{k,s}, e_k, u(past), w_i,k)u_k$$

$$+\beta_{0,k}(N, T_s, G_i, \tau_i, L_i, t_p, \Delta L_i, \lambda_i, w_u, y_p, y_{k,s}, e_k, u(past), w_i,k),$$

Minimization of (21) obtained with the first order optimality at each $t_k$:

$$\frac{\partial J}{\partial u_k} = 0 \text{ at } u_k = u_{min}^k.$$

(22)

Calculation of $u_{min}^k$ is then straightforward:

$$u_{min}^k = \frac{-\beta_{1,k}}{2\beta_{2,k}}$$

(23)

which leads to the explicit formulation of the solution $u_{k}^\star$:

$$\begin{cases}
\text{if } u_{inf} \leq u_{min}^k \leq u_{sup} : u_{k}^\star = u_{min}^k \\
\text{if } u_{min}^k \leq u_{inf} : u_{k}^\star = u_{inf} \\
\text{if } u_{sup} \leq u_{min}^k : u_{k}^\star = u_{sup}.
\end{cases}$$

(24)
Input disturbances

Gas mass flow vs Time

Gas Temperature vs Time

Egr Mass Flow
Exhaust Mass Flow

Egr Temperature
Exhaust Temperature
Tracking error and manipulated variable

![Graph showing tracking error and manipulated variable over time.](image)
Conclusion

- New modeling weighting scheme based on a piecewise linear approach has been developed and validated.
- New scheme has less tuning parameters than the Bayesian scheme.
- Explicit MMPC strategy for Rankine cycle based heat recovery system is presented.
- MMPC is compliant with classical automotive integration constraints (i.e., basic CPU and fast sampling time).

Next steps

- Experimental validation.
- Robustness study.
Contacts and discussion

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\[ \beta_{1,k} = \sum_{i=1}^{N} \left[ \beta_{1,k}^{'} + \beta_{1,k}^{''} + \beta_{1,k}^{'''} + \beta_{1,k}^{'''} + \beta_{1,k}^{''''} \right] - 2w_{u}u_{k-1}t_{p}, \quad (25) \]

\[ \beta_{1,k}^{'} = -G_{i} \tau_{i} w_{i,k}^{2} y_{p,k} \left( e^{\frac{L_{i}}{\tau_{i}}} + 2e^{\frac{-t_{p}}{\tau_{i}}} - e^{\frac{L_{i}-2t_{p}}{\tau_{i}}} - 2 \right), \quad (26) \]

\[ \beta_{1,k}^{''} = -G_{i}^{2} \tau_{i} u(t_{k-\lambda_{i}-1}) w_{i,k}^{2} e^{-\frac{2t_{p}}{\tau_{i}}} \left( e^{\frac{\Delta L_{i}}{\tau_{i}}} - 1 \right) \left( e^{\frac{t_{p}}{\tau_{i}}} - 1 \right) \ldots \]

\[ \left( e^{\frac{L_{i}}{\tau_{i}}} - 2e^{\frac{-t_{p}}{\tau_{i}}} + e^{\frac{L_{i}+t_{p}}{\tau_{i}}} \right) \quad (27) \]
\[ \beta_{1,k}''' = \sum_{j=1}^{\lambda_i} G_i^2 \tau_i u(t_{k-j}) w_{i,k}^2 e^{\frac{\Delta L_i - t_p - j T_s + \lambda_i T_s}{\tau_i}} \left( e^{\frac{T_s}{\tau_i}} - 1 \right) \left( e^{\frac{L_i - t_p}{\tau_i}} - 2 \right) \cdots \]

\[ - G_i^2 \tau_i u(t_{k-j}) w_{i,k}^2 e^{\frac{\Delta L_i - j T_s + \lambda_i T_s}{\tau_i}} \left( e^{\frac{L_i}{\tau_i}} - 2 \right) \left( e^{\frac{T_s}{\tau_i}} - 1 \right) \] (28)

\[ \beta_{1,k}'''' = 2 G_i w_{i,k} \left( e_k - y^{SP} \right) \left( t_p + \tau_i e^{\frac{L_i - t_p}{\tau_i}} - \tau_i e^{\frac{L_i}{\tau_i}} \right) \] (29)

\[ \beta_{2,k} = \frac{G_i^2 w_{i,k}^2 \left( 2 t_p + 4 \tau_i e^{\frac{L_i - t_p}{\tau_i}} - \tau_i e^{\frac{2 L_i - 2 t_p}{\tau_i}} - 4 \tau_i e^{\frac{L_i}{\tau_i}} + \tau_i e^{\frac{2 L_i}{\tau_i}} \right)}{2} + w_u t_p \] (30)
Input disturbances

Engine speed and torque

- Engine speed
  - Speed (rpm)
  - Time (s)

- Engine torque
  - Torque (N.m)
  - Time (s)
Tracking error and manipulated variable

Developed weighting scheme