Phasor estimation using conditional maximum likelihood: Strengths and limitations

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PHASOR ESTIMATION USING CONDITIONAL MAXIMUM LIKELIHOOD: STRENGTHS AND LIMITATIONS

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ABSTRACT

This paper focuses on the estimation of the phasor parameters in three-phase power systems for smart grid monitoring. Specifically, it investigates the use of the Conditional Maximum Likelihood (ML) for phasor parameter estimation. The contribution of this paper is twofold. First, it presents the condition on the signal model for identifiability of the phasor parameters. Then, it shows that the Conditional Maximum Likelihood estimator has a simple closed form expression, which can be determined from simple geometrical properties. Simulation results illustrate the effectiveness of the proposed approach for the estimation of the phasor amplitude and angle shift under dynamic conditions.

Index Terms— Phasor Measurement Units, Smart Grid, Maximum Likelihood, Condition Monitoring.

1. INTRODUCTION

For practical and economical reasons, electrical grids use three-phase electric power at the generation, transmission and distribution sides. Under nominal conditions, three-phase systems are composed of three conductors, each carrying a 50 Hz (or 60 Hz) sine wave. In balanced systems, these sine waves have the same amplitude but are phase-rotated from each other by an angle of $2\pi/3$. In practice, many phenomena can introduce deviations from the nominal conditions. For example, a mismatch between energy generation and load introduces frequency deviations [1]. Similarly, abnormal grid events, such as voltage sags, introduce three-phase unbalance [2]. These deviations must be detected at an early stage to avoid severe consequences. For these reasons, electrical signal monitoring is of main importance for improving the power grid reliability and enabling energy management systems.

To monitor the state of the grid, electronic devices called Phasor Measurement Units (PMUs) are dispersed throughout the power grid [3, 4]. From a signal processing viewpoint, these devices compute for each phase a complex number $d_k e^{j\varphi_k}$ called the phasor, where d_k and φ_k correspond to the amplitude and phase estimates of the electrical signal evaluated at the fundamental frequency. The IEEE Standard C37.118.1 specifies the performance requirements for phasor estimation [5]. To be compliant with this standard, the phasor estimator must meet certain requirements in terms of accuracy and ability to track fast variations. Several techniques have been proposed in the literature for estimating the phasor. Under stationary conditions, classical techniques include the Root Mean Square and DFT estimators [2, 6]. Under non-stationary conditions, more sophisticated algorithms have been proposed. A complete state-of-art can be found in [7]. From an estimation perspective, it should be mentioned that the most challenging requirement is the ability to track fast variations. Recently, a new estimator based on the Maximum Likelihood has been proposed for the estimation of the phasor amplitude [8]. The advantage of this estimator is twofold. First, by considering the modulated signals as unknown parameters, it is well suited for signals with amplitude and or phase modulation. Then, as it is based on the Maximum Likelihood framework, the proposed estimator has the attractive property to be asymptotically optimal under some conditions. Nevertheless, the main limitation of this estimator is the fact that it assumes a perfect $2\pi/3$ angle shift between phases. In practice, this assumption limits its use for phasor estimation.

In this paper, we extend the Conditional Maximum Likelihood Technique in [8] by considering a more general setting, where both the amplitude and phase of the phasors are unknown. The contribution of this paper is twofold. First, we provide the conditions for identifiability of the phasor parameters. Then, we show that the phasor parameters can be simply estimated using basic geometrical properties. This paper is organized as follows. Section 2 presents the signal model and section 3 addresses the phasor estimation problem. Finally, section 4 provides some simulation results.

2. SIGNAL MODEL

In unbalanced three-phase systems, the electrical waveform on the k^{th} phase can be modeled as ($k = 0, 1, 2$)

$$x_k[n] = d_k a[n] \cos(\phi[n] + \varphi_k) + b_k[n], \tag{1}$$

where $a[n]$ and $\phi[n]$ correspond to the instantaneous scaling factor and instantaneous phase offset, respectively, and

$b_k[n]$ is the additive noise. Note that this model is quite general and includes all the dynamic scenarios of the IEEE standard C37.118.1 [5, 5.5.6-5.5.8]. The parameters $d_k e^{j\varphi_k}$ corresponds to the phasor on the k^{th} phase. Let us define $\mathbf{c} = [d_0 e^{j\varphi_0}, d_1 e^{j\varphi_1}, d_2 e^{j\varphi_2}]^T$ the 3×1 column vector containing the phasors. Then, let us use the notation $\mathbf{c}(\boldsymbol{\theta})$ to stress that this vector depends on several unknown parameters, $\boldsymbol{\theta}$. Using matrix notations, the three-phase signals can be expressed as

$$\mathbf{x}[n] = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}[n] + \mathbf{b}[n] \tag{2}$$

where

- $\mathbf{x}[n]$ et $\mathbf{b}[n]$ are column matrices which are defined as

$$\mathbf{x}[n] \triangleq \begin{bmatrix} x_0[n] \\ x_1[n] \\ x_2[n] \end{bmatrix} \qquad \mathbf{b}[n] = \begin{bmatrix} b_0[n] \\ b_1[n] \\ b_2[n] \end{bmatrix}, \tag{3}$$

- $\mathbf{A}(\boldsymbol{\theta})$ is a 3×2 matrix defined as

$$\mathbf{A}(\boldsymbol{\theta}) \triangleq \begin{bmatrix} \Re e[\mathbf{c}(\boldsymbol{\theta})] & -\Im m[\mathbf{c}(\boldsymbol{\theta})] \end{bmatrix}, \tag{4}$$

- $\mathbf{s}[n]$ is a 2×1 column vector containing the direct and quadrature components i.e.

$$\mathbf{s}[n] \triangleq \begin{bmatrix} a[n]\cos(\phi[n]) \\ a[n]\sin(\phi[n]) \end{bmatrix}. \tag{5}$$

In this paper, we assume that the noise is a zero-mean, white Gaussian noise with covariance matrix $\sigma^2\mathbf{I}$. Under this assumption, the goal of this study is to estimate the phasor parameters, $\boldsymbol{\theta}$, from the three-phase signals without knowing $\mathbf{S} \triangleq [\mathbf{s}[0], \cdots, \mathbf{s}[N-1]]$.

3. CONDITIONAL MAXIMUM LIKELIHOOD

The Conditional Maximum Likelihood (CML) models the in-phase and quadrature components as unknown deterministic parameters. This is a reasonable assumption when no a-priori information is available about the direct and quadrature components. However, as the in-phase and quadrature components are treated as unknown quantities, there is no guarantee that the unique identification of the parameters is enabled. For this reason, we investigate the conditions for parameter identifiability before addressing the estimation problem.

3.1. Conditions for parameter identifiability

The parameter identifiability requires that

$$\mathbf{A}(\boldsymbol{\theta})\mathbf{S} = \mathbf{A}(\boldsymbol{\theta}_2)\mathbf{S}_2 \Rightarrow \boldsymbol{\theta} = \boldsymbol{\theta}_2 \tag{6}$$

where $\mathbf{S}$ and $\mathbf{S}_2$ are arbitrary $2 \times N$ full rank matrix.

As $\mathbf{A}(\boldsymbol{\theta})$ is a full rank 3×2 matrix, the orthogonal projector onto the range of $\mathbf{A}(\boldsymbol{\theta})$ can be decomposed as [9, p266]

$$\mathbf{P}_{\mathbf{A}}(\boldsymbol{\theta}) = \mathbf{A}(\boldsymbol{\theta})(\mathbf{A}^T(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta}))^{-1}\mathbf{A}^T(\boldsymbol{\theta}) \tag{7}$$

$$= \mathbf{I} - \mathbf{u}(\boldsymbol{\theta})\mathbf{u}^T(\boldsymbol{\theta}) \tag{8}$$

where $\mathbf{u}(\boldsymbol{\theta})$ is the unit-norm eigenvector of $\mathbf{A}(\boldsymbol{\theta})\mathbf{A}^T(\boldsymbol{\theta})$ associated with the zero eigenvalue. If $\mathbf{u}(\boldsymbol{\theta}) = \mathbf{u}(\boldsymbol{\theta}_2)$, it can be checked that the orthogonal projector onto the range of $\mathbf{A}(\boldsymbol{\theta})$ and $\mathbf{A}(\boldsymbol{\theta}_2)$ are equal i.e. $\mathbf{P}_{\mathbf{A}}(\boldsymbol{\theta}) = \mathbf{P}_{\mathbf{A}}(\boldsymbol{\theta}_2)$. Therefore, if it exists $\boldsymbol{\theta}_2 \neq \boldsymbol{\theta}$ such as $\mathbf{u}(\boldsymbol{\theta}) = \mathbf{u}(\boldsymbol{\theta}_2)$, then it is possible to find a matrix $\mathbf{S}_2 = (\mathbf{A}^T(\boldsymbol{\theta}_2)\mathbf{A}(\boldsymbol{\theta}_2))^{-1}\mathbf{A}^T(\boldsymbol{\theta}_2)\mathbf{A}(\boldsymbol{\theta})\mathbf{S}$ such as the condition in (6) is not satisfied. Therefore, we obtain the following lemma.

Lemma 1. *To enable unique identification of the parameter, it must exist a one-to-one mapping between $\mathbf{u}(\boldsymbol{\theta})$ and $\boldsymbol{\theta}$.*

By definition, there is a one-to-one mapping between $\mathbf{u}(\boldsymbol{\theta})$ and $\boldsymbol{\theta}$ if the value of $\boldsymbol{\theta}$ satisfying $\mathbf{u}^T\mathbf{A}(\boldsymbol{\theta}) = \mathbf{0}$ is unique. As $\mathbf{A}(\boldsymbol{\theta})$ is a 3×2 matrix and $\mathbf{u}$ a 3×1 column vector, the system $\mathbf{u}^T\mathbf{A}(\boldsymbol{\theta}) = \mathbf{0}$ is composed of two equations. Therefore, a unique solution requires that $\boldsymbol{\theta}$ contains at most 2 (real) unknown parameters. Note that this condition does not guarantee uniqueness of the solution. Specifically, for particular phasor models $\mathbf{c}(\boldsymbol{\theta})$, it is possible to show that $\mathbf{u}(\boldsymbol{\theta}) = \mathbf{u}$ for all $\boldsymbol{\theta}$. For example, let us consider the phasor model $\mathbf{c}(\boldsymbol{\theta})$ satisfying $[1\ 1\ 1]\mathbf{c}(\boldsymbol{\theta}) = 0$ for all $\boldsymbol{\theta}$. This condition implies that $[1\ 1\ 1]\mathbf{A}(\boldsymbol{\theta}) = \mathbf{0}$. By identification, we conclude that the unit-norm eigenvector is equal to $\mathbf{u} = 1/\sqrt{3}[1\ 1\ 1]^T$ regardless the value of $\boldsymbol{\theta}$.

Proposition 1. *To enable unique identification of the parameters $\boldsymbol{\theta}$, the following conditions are necessary*

- *$\mathcal{C}_1$: $\boldsymbol{\theta}$ contains at most 2 (real) unknown parameters,*
- *$\mathcal{C}_2$: the so-called zero sequence component is non-zero for all $\boldsymbol{\theta}$ i.e.*

$$[1\ 1\ 1]\boldsymbol{c}(\boldsymbol{\theta}) \neq 0 \tag{9}$$

To illustrate proposition 1, we provide several examples where the parameter $\boldsymbol{\theta}$ cannot be identified.

Example 1. *Let us consider the following phasor model*

$$\boldsymbol{c}(\alpha, \beta) = \alpha e^{j\beta} \begin{bmatrix} 1 \\ e^{2j\pi/3} \\ e^{4j\pi/3} \end{bmatrix}, \tag{10}$$

As $\boldsymbol{\theta} = \{\alpha, \beta\}$ contains two parameters, condition $\mathcal{C}_1$ is satisfied. However, as $[1\ 1\ 1]\boldsymbol{c}(\boldsymbol{\theta}) = 0$, the second condition does not hold and $\boldsymbol{\theta}$ is not uniquely identifiable.

Example 2. *The type-C phasor model of the ABC classification is given by [2, Table 6.2]*

$$\boldsymbol{c}(\alpha) = \begin{bmatrix} 1 \\ -\frac{1}{2} + j\alpha \\ -\frac{1}{2} - j\alpha \end{bmatrix}, \tag{11}$$

As $[1\ 1\ 1]\boldsymbol{c}(\alpha) = 0$, α is not identifiable. In the ABC classification, this problem occurs for sags of type A, C, D, F and G.

3.2. Estimation of the phasor parameters.

Assuming that conditions $\mathcal{C}_1$ and $\mathcal{C}_2$ hold, we show in this subsection that the Maximum Likelihood estimator has a simple expression. Using the CML, the estimate of the unbalance parameters is given by [10]

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \operatorname{Tr}\left[\mathbf{P}_{\mathbf{A}}^{\perp}(\boldsymbol{\theta})\widehat{\mathbf{R}}\right] \tag{12}$$

where

- $\widehat{\mathbf{R}}$ is the sample covariance matrix, which is defined as

$$\widehat{\mathbf{R}} \triangleq \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{y}[n]\mathbf{y}^T[n] \tag{13}$$

- $\mathbf{P}_{\mathbf{A}}^{\perp}(\boldsymbol{\theta})$ is the orthogonal projector onto the kernel of $\mathbf{A}^T(\boldsymbol{\theta})$, which is defined as

$$\mathbf{P}_{\mathbf{A}}^{\perp}(\boldsymbol{\theta}) = \mathbf{I} - \mathbf{A}(\boldsymbol{\theta})\left(\mathbf{A}^T(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta})\right)^{-1}\mathbf{A}^T(\boldsymbol{\theta}) \tag{14}$$

Using (8) and (14), $\mathbf{P}_{\mathbf{A}}^{\perp}(\boldsymbol{\theta})$ can be decomposed as

$$\mathbf{P}_{\mathbf{A}}^{\perp}(\boldsymbol{\theta}) = \mathbf{u}(\boldsymbol{\theta})\mathbf{u}^T(\boldsymbol{\theta}) \tag{15}$$

where $\mathbf{u}(\boldsymbol{\theta})$ is the unit eigenvector associated with the null eigenvalue of $\mathbf{A}(\boldsymbol{\theta})\mathbf{A}^T(\boldsymbol{\theta})$. Using this decomposition, the cost function in (12) can be simplified as follows

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \operatorname{Tr}\left[\mathbf{u}(\boldsymbol{\theta})\mathbf{u}^T(\boldsymbol{\theta})\widehat{\mathbf{R}}\right] \tag{16}$$

$$= \arg\min_{\boldsymbol{\theta}} \mathbf{u}^T(\boldsymbol{\theta})\widehat{\mathbf{R}}\mathbf{u}(\boldsymbol{\theta}) \tag{17}$$

To determine $\boldsymbol{\theta}$ analytically, we resort to the invariance property of the Maximum Likelihood. First, we estimate $\mathbf{u}$ from $\widehat{\mathbf{R}}$, then we estimate $\boldsymbol{\theta}$ from $\widehat{\mathbf{u}}$.

3.2.1. *Estimation of* $\mathbf{u}$

In the first step, we find the value of $\mathbf{u}$ that minimizes the cost function in (17). This problem can be formalized as

$$\widehat{\mathbf{u}} = \arg\min_{\mathbf{u}} \mathbf{u}^T\widehat{\mathbf{R}}\mathbf{u}, \text{ subject to } \mathbf{u}^T\mathbf{u} = 1. \tag{18}$$

As the scalar $\mathbf{u}^T\widehat{\mathbf{R}}\mathbf{u}$ is a Rayleigh quotient, the minimum is reached at $\widehat{\mathbf{u}} = \mathbf{g}$, where $\mathbf{g}$ is the eigenvector associated with the smallest eigenvalue of the sample covariance matrix $\widehat{\mathbf{R}}$ [11, Theorem 4.2.2].

3.2.2. *Estimation of* $\boldsymbol{\theta}$

In the second step, we extract the value of $\boldsymbol{\theta}$ from $\mathbf{g}$. As $\mathbf{u}(\boldsymbol{\theta})$ is the eigenvector of $\mathbf{A}(\boldsymbol{\theta})\mathbf{A}^T(\boldsymbol{\theta})$ associated with the null eigenvalue, then $\mathbf{u}^T(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta}) = \mathbf{0}$. Replacing $\boldsymbol{\theta}$ and $\mathbf{u}^T(\boldsymbol{\theta})$ by their estimates, we obtain

$$\mathbf{g}^T\mathbf{A}(\widehat{\boldsymbol{\theta}}) = \mathbf{0}. \tag{19}$$

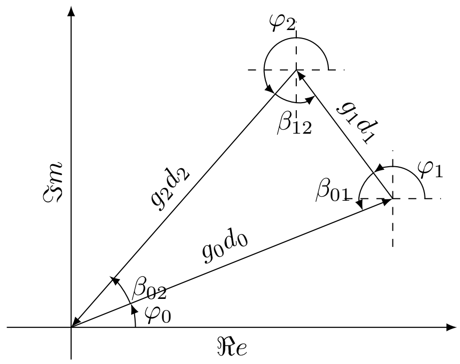

Fig. 1: Geometrical interpretation of (22) .

It should be noted that this system is the basis of subspace algorithms such as MUSIC. This system is composed of two equations involving real numbers. By introducing the complex number j, this equation can be compacted as

$$\mathbf{g}^T\mathbf{A}(\widehat{\boldsymbol{\theta}})\begin{bmatrix} 1 \\ -j \end{bmatrix} = 0. \tag{20}$$

Using the definition of $\mathbf{A}(\boldsymbol{\theta})$, we obtain Theorem 1.

Theorem 1. *The ML estimate of $\boldsymbol{\theta}$, denoted $\widehat{\boldsymbol{\theta}}$, is obtained by finding the value of $\boldsymbol{\theta}$ for which the phasors are orthogonal to the eigenvector $\mathbf{g}$ i.e.*

$$\mathbf{g}^T\boldsymbol{c}(\widehat{\boldsymbol{\theta}}) = 0. \tag{21}$$

The orthogonality condition in Theorem 1 has a simple geometric interpretation. Let us decompose $\mathbf{g}$ and $\mathbf{c}(\boldsymbol{\theta})$ as $\mathbf{g} = [g_0, g_1, g_2]^T$ and $\mathbf{c}(\boldsymbol{\theta}) = [d_0e^{j\varphi_0}, d_1e^{j\varphi_1}, d_2e^{j\varphi_2}]^T$. Note that for convenience, we write d_k and φ_k instead of $d_k(\boldsymbol{\theta})$ and $\varphi_k(\boldsymbol{\theta})$. Using this decomposition, it follows that

$$\sum_{k=0}^{2} g_kd_ke^{j\varphi_k} = 0. \tag{22}$$

In the complex plane, this equality corresponds to a triangle with side lengths d_kg_k and inner angles given by

$$\beta_{02} = \varphi_2 - \varphi_0 - \pi \tag{23a}$$

$$\beta_{01} = \pi - \varphi_1 + \varphi_0 \tag{23b}$$

$$\beta_{12} = \pi - \varphi_2 + \varphi_1. \tag{23c}$$

This triangle is presented in Fig.1. Using this representation, the estimation problem can be treated as a geometrical problem.

3.3. Closed form estimators

In this subsection, we address the two following estimation problems: i) the estimation of the amplitude parameters d_1 and d_2 and ii) the estimation of the phase-shift parameters φ_1, and φ_2. In each scenario, the first phase is taken as the reference i.e. $d_0 = 1$ and $\varphi_0 = 0$

3.3.1. Estimation of the amplitude parameters

In the first estimation problem, the unknown parameters are $\boldsymbol{\theta} = \{d_1, d_2\}$. To estimate d_1 and d_2, we use the sine rule in the triangle of Fig.1. After some simplifications, we obtain

$$\widehat{d}_1 = -\frac{g_0 \sin(\varphi_2)}{g_1 \sin(\varphi_2 - \varphi_1)} \tag{24a}$$

$$\widehat{d}_2 = \frac{g_0 \sin(\varphi_1)}{g_2 \sin(\varphi_2 - \varphi_1)}. \tag{24b}$$

Note that when the angle shift between phases is equal to $2\pi/3$, the triangle in Fig.1 is equilateral. It follows that $\widehat{d}_k = g_0/g_k$, which corresponds to the particular solution in [8].

3.3.2. Estimation of the phase parameters

In the second estimation problem, the unknown parameters are $\boldsymbol{\theta} = \{\varphi_1, \varphi_2\}$. Using the cosine law, we obtain

$$g_2^2 d_2^2 = g_0^2 + g_1^2 d_1^2 - 2g_0 g_1 d_1 \cos(\beta_{01}) \tag{25a}$$

$$g_1^2 d_1^2 = g_0^2 + g_2^2 d_2^2 - 2g_0 g_2 d_2 \cos(\beta_{02}). \tag{25b}$$

Therefore, the angle-shift estimators are given by

$$\widehat{\varphi}_1 = \arccos\left(\frac{g_2^2 d_2^2 - g_0^2 - g_1^2 d_1^2}{2g_0 g_1 d_1}\right) \tag{26a}$$

$$\widehat{\varphi}_2 = \arccos\left(\frac{g_2^2 d_2^2 + g_0^2 - g_1^2 d_1^2}{2g_0 g_2 d_2}\right) + \pi. \tag{26b}$$

4. SIMULATION RESULTS

In this section, the performances of the amplitude and angle shift estimators are accessed through simulations using Python/Numpy. The phasor parameters are set to $d_0 = 1$, $d_1 = 1.2$, $d_2 = 0.2$, $\phi_0 = 0$, $\phi_1 = 2.29$ rad, and $\phi_2 = 4.68$ rad. The instantaneous scaling factor and phase offset are fixed according to the worst condition of the IEEE Standard C37.118.1 [5, 5.5.6]

$$a[n] = 1 + 0.1\cos(2\pi f_m n/F_e) \tag{27a}$$

$$\phi[n] = 2\pi f_0 n/F_e + 0.1\cos(2\pi f_m n/F_e - \pi) \tag{27b}$$

where $f_m = 5$Hz, $f_0 = 50$Hz, $F_e = 1000$Hz. For each estimator, the Mean Square Error (MSE) is evaluated through 10000 Monte Carlo simulations. The MSE is analyzed and compared (asymptotically) with the Cramér Rao bounds for different data lengths and signal-to-noise ratios (SNRs), where the SNR is defined as

$$SNR = 10\log\left(\frac{\text{trace}(\mathbf{A}(\theta)\mathbf{S}\mathbf{S}^T\mathbf{A}^T(\theta))}{3N\sigma^2}\right). \tag{28}$$

In the following simulations, the SNR ranges from -20 to 50dB. It should be mentioned that low values of the SNR are unlikely in power system applications. Figure 2 presents the performance of the amplitude estimator described in (24). We observe that the MSE and CRB of $\widehat{d}_2$ are lower than that of $\widehat{d}_1$. As reported in [8], this behavior comes from the fact that $d_2 < d_1$. A constant offset between the MSE also suggests that the ratio $MSE[\widehat{d}_1]/MSE[\widehat{d}_2]$ is constant for large values of SNR or N. Furthermore, we note that the proposed estimator seems to reach the CRB for SNR$\to \infty$ but not for $N \to \infty$, which is compliant with the properties of the Conditional ML [12].

Figure 3 reports on the performance of the angle shift estimator described in (26). We note that the MSE of $\widehat{\phi}_1$ is lower than that of $\widehat{\phi}_2$, except for low N or SNR. Additional simulations suggest that the performances of $\widehat{\phi}_1$ and $\widehat{\phi}_2$ are equal when $\phi_1 = 2\pi/3$ and $\phi_2 = 4\pi/3$. Concerning the CRB, the proposed estimator is biased for low SNRs, making the comparison non-relevant in this asymptotic region. For moderate and large SNRs, we note that the angle shift estimator has the same behavior than the amplitude estimator (efficient for SNR$\to \infty$ but not efficient for $N \to \infty$).

5. CONCLUSION

This paper has presented a technique based on the Conditional Maximum Likelihood (ML) for the estimation of the phasor parameters in three-phase systems. When the identification is enabled, it has been demonstrated that the ML estimation of the phasor parameters can be determined by finding a phasor vector orthogonal to an eigenvector of the sample covariance matrix. Furthermore, this problem is transformed into the determination of the geometrical properties, such as side lengths and inner angles, of a triangle. This paper has also demonstrated that the Conditional ML can estimate at most two parameters. This limitation suggests that additional information must be incorporated in the signal model for the identification of all the phasor parameters.

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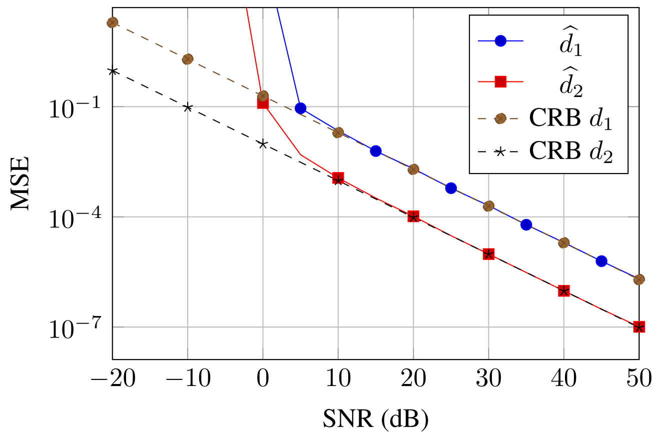
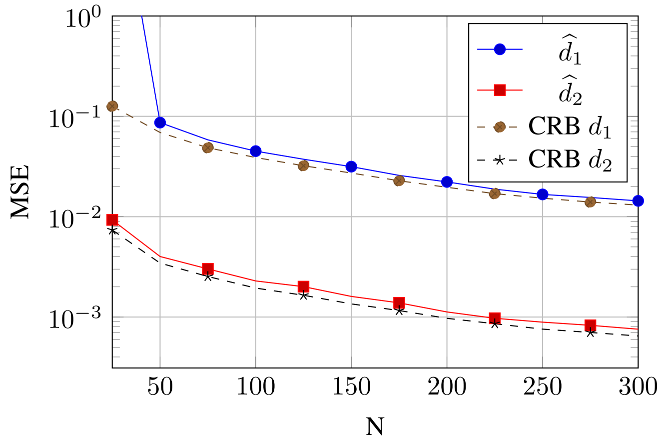
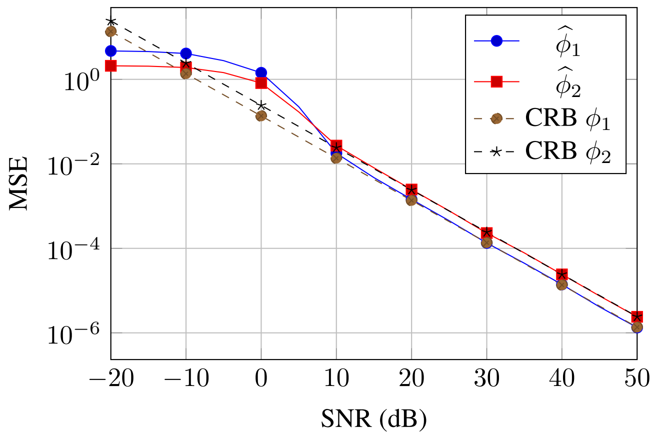


(a) MSE versus SNR ($N = 200$).

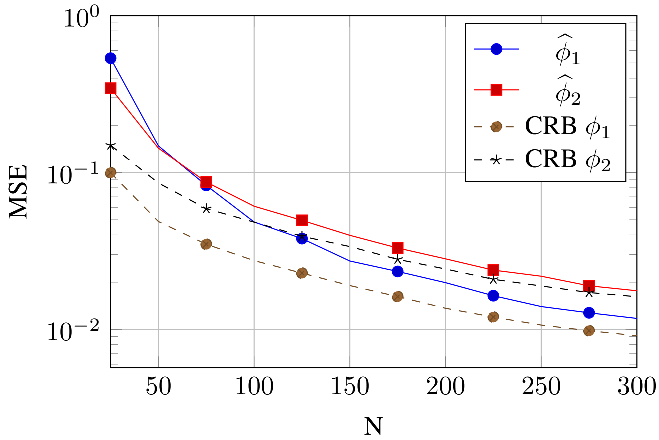


(b) MSE versus sample length ($SNR = 10$dB).

Fig. 2: Estimation of the amplitude parameters d_1 and d_2.



(a) MSE versus SNR ($N = 200$).



(b) MSE versus N ($SNR = 10$dB).

Fig. 3: Estimation of the angle shift parameters φ_1 and φ_2.

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