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Predictive control of disturbed systems with input delay: experimental validation on a DC motor

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Abstract: This paper deals with the speed control of a DC motor with known input delay and unknown disturbances. A recent predictive technique is used in order to design two controllers that will be able to satisfactorily reject disturbances in spite of the time delay. A comparison between the Artstein reduction method and this new one is performed throughout the article. The control laws are validated first by simulation and then by a practical implementation. The paper presents the very first application of this new predictive method.

Keywords: DC motor, predictive control, input delay, disturbance attenuation.

1. INTRODUCTION

Delays in the input can be caused by the physical nature of the plant itself (fluid transportation, biological system) or by the controller (computation or communication delays). Nowadays, the interest towards input delay systems has been accentuated by the fast development of remote controlled systems such as networked control systems and teleoperation (see Chiasson and Loiseau (2007) and Lafay (2014) for further applications).

The standard approach to control such a class of systems is the well-known Smith predictor introduced in Smith (1959). Then, the finite spectrum assignment technique and the Artstein reduction method respectively in Manitius and Olbrot (1979) and Artstein (1982) have extended Smith’s result. However, these methods have a major problem: they are not robust with respect to disturbances. In order to make the prediction more accurate in spite of possible perturbations, a new predictive method has been presented in Léchappé et al. (2015a). This new method works for disturbed MIMO LTI systems with a constant and known delay in the input.

DC motors are commonly used in many areas such as robotics or industry (robotic arms, lathes, drills, elevators, cranes...). Furthermore, the simple modeling facilitates its use as a benchmark system for evaluation of new control laws. Numerous techniques have been applied for driving DC motors: for example, sliding mode control in Utkin (1993), optimal control in Pelczewski and Kunz (1990). However, they do not consider delays.

Among the few existing works concerning DC motors with retarded inputs, one can cite for example Matsuo et al. (2006), which shows the influence of the delay time distribution on the stability of a DC motor with PI controller. In Luck and Ray (1994), they designed an observer-based delay compensator in the discrete framework associated with buffers to reduce the unknown delay variations. An adaptive controller is used in Tipsuwan and Chow (2003) to follow the "Quality-of-Service" (QoS) variations of the network. These works have two weak points: they do not take into account perturbations and they deal with very small delays (less than the sampling period). The present paper overcomes these difficulties. The contribution is the application of a new predictive method to a DC motor with large input delay.

The article is organized as follows. In Section 2, the new predictive scheme is recalled for a general class of systems. Then, it is applied to the DC motor in Section 3.1 and two robust controllers are designed in Section 3.2. Some simulations illustrate previous results in Section 4.2. The feasibility of the predictive scheme is demonstrated on a real DC motor and an extensive analysis of the result is performed in Section 4.3. Finally, the conclusion and some future developments are outlined in Section 5.

2. PRESENTATION OF THE PREDICTIVE SCHEMES

In this section, the new predictive scheme presented in Léchappé et al. (2015a) is briefly recalled. The control design is based on the computation of a new prediction for LTI systems
\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t - \tau) + d \\
u(t) &= \phi(t) \quad \text{for all } t \in [-\tau, 0[ \\
x(0) &= x_0
\end{align*}
\] (1)

with \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^m \), \( d(t) \in \mathbb{R}^n \), \( A \in \mathbb{R}^{n \times n} \) and \( B \in \mathbb{R}^{n \times m} \). The following assumptions are made:

**Hypothesis 1.** \((A, B)\) is controllable.

**Hypothesis 2.** The state \( x(t) \) is measurable.

**Hypothesis 3.** The delay \( \tau \) is constant and known.

**Hypothesis 4.** The disturbance \( d \) is constant and unknown.

Similar results to those described in this article exist for time-varying perturbation (see Léchappé et al. (2015a)). Nevertheless, the assumption of a constant disturbance is imposed by experimental constraint.

**Hypothesis 5.** \( u \) is a locally integrable function from \([-\tau, +\infty[\) to \( \mathbb{R}^m \).

A common way to control systems with large input delay is to use a predictive feedback (see Smith (1959), Krstic (2009)). To design such feedbacks, the computation of a predictive state is needed:

\[
x_p(t) = e^{A\tau}x(t) + \int_0^t e^{A(t-s)}Bu(s)ds
\] (2)

for all \( t \geq 0 \). The term \( x_p \) will be referred as the standard prediction in the sequel because it is the most widely used prediction in the literature of input delay systems. The advantage of using a state prediction is that (1) can be transformed into a delay-free system:

\[
\dot{x}_p(t) = Ax_p(t) + Bu(t) + e^{A\tau}d.
\] (3)

This transformation is called the Artstein reduction given in Artstein (1982). The idea is then to design a controller \( u \) that stabilizes (3) to zero. In particular, when \( d \neq 0 \), it will be interesting to design a robust controller than could attenuate the perturbation \( e^{A\tau}d \) such that \( x_p \) tends to zero. However, this will not make \( x \) converge to zero. Indeed, in the disturbance-free case, \( x_p \) is the exact prediction at instant \( t \) of the state \( x \) at time \( t + \tau \). However, when \( d \neq 0 \), there is an inevitable error

\[
x(t + \tau) - x_p(t) = d \int_0^t e^{A(t-s)}ds
\] (4)

This leads to the following proposition:

**Proposition 6.** Léchappé et al. (2015a)

The convergence of \( x_p \) to zero implies the convergence of \( x \) to \( d \int_{t-\tau}^t e^{A(t-s)}ds \) when \( t \) tends to infinity.

This underlines an inherent problem of the standard prediction: it does not take into account any disturbance information. Consequently, a new state prediction is introduced:

**Definition 7.** Léchappé et al. (2015a)

The new prediction is defined by

\[
X_p(t) = x_p(t) + x(t) - x_p(t - \tau)
\] (5)

for all \( t \geq \tau \), with \( x_p \) given by (2).

**Remark 8.** Note that \( X_p \) can be computed without any knowledge of \( d \).

Like for the standard prediction, system (1) can be rewritten as a delay-free system using \( X_p \):

\[
X_p(t) = AX_p(t) + Bu(t) + d
\] (6)

Again, the idea is to design a controller \( u \) on (6) to use all the control techniques from the delay-free literature.

**Proposition 9.** Léchappé et al. (2015a)

The convergence of \( X_p \) to zero implies the convergence of \( x \) to zero when \( t \) tends to infinity.

This is an interesting result because it means that if it is possible to design a robust controller that reject perfectly \( d \) in (6), the robust convergence of \( x \) to zero is guaranteed by Proposition 9. Consequently, the problem of disturbance rejection for input delay systems is reduced to a problem of disturbance rejection for delay-free systems.

In the next section, the new predictive scheme (prediction computation, reduction, control design) is applied to control the angular velocity of a DC motor.

### 3. APPLICATION TO A DC MOTOR

#### 3.1 Model presentation and reduction

The simplified transfer function of the DC motor with a retarded input \( u \) is:

\[
\frac{\Omega(s)}{U(s)} = \frac{K}{1 + sT}e^{-\tau s}.
\] (7)

where \( \Omega \) and \( U \) are the Laplace transforms of the angular velocity \( \omega \) and the input voltage \( u \) respectively. The steady-state gain \( K \), the time constant \( T \) and the input delay \( \tau \) are known. The transfer function \( \frac{K}{1 + sT}e^{-\tau s} \) is a classical simplified model for small DC motors where the inductance term can be neglected. The term \( e^{-\tau s} \) accounts for the delayed input. This delay is not internal to the motor model it comes from the delay in the input that can be introduced, for example, by the remote control over a network or the computational time needed to compute the control law. It does not take into account any nonlinear phenomenon such as friction or play. To apply above results, (7) has to be turned into its state-space representation. Adding the disturbance, one obtains

\[
\dot{\omega} = a\omega + bu(t - \tau) + d
\] (8)

with \( a = -\frac{K}{T} \), \( b = \frac{K}{T} \) and \( d \) an unknown but constant disturbance. Consequently, Assumptions 1, 3 and 4 are satisfied. Denoting \( \omega_p \) the standard prediction of system (8) and \( W_p \) the new prediction, it follows

\[
\omega_p(t) = e^{a\tau}\omega(t) + \int_{t-\tau}^t e^{a(t-s)}bu(s)ds
\]

and

\[
W_p(t) = \omega_p(t) + \omega(t) - \omega_p(t - \tau).
\]

From a practical point of view, the full state knowledge is required to compute both predictions (2) and (5). However, an extension to partial state knowledge gives similar result Léchappé et al. (2015b). In this scalar case, the state is reduced to \( \omega \) that is measured during the experimentation: Assumption 2 is verified.

**Remark 10.** To compute \( \omega_p \), the integral is discretized in a finite number of points; so the value of \( u \) is only needed.
for these points. For further details on the implementation of memory feedbacks see Mondie and Michiels (2003).

Finally, the reduced input delay-free models read as
\[ \dot{\chi}(t) = a\chi(t) + bu(t) \]  
and
\[ \ddot{\chi}(t) = aW\dot{\chi}(t) + bu(t) + d. \]

Note that (9) and (10) have the same disturbance-free form, i.e.
\[ \chi(t) = \omega_\phi(t) - d. \]

with \( \chi = \omega_\phi \) or \( \chi = W_\phi \). The objective of the next subsection is to design controllers \( u(\omega_\phi) \) and \( u(W_\phi) \) that stabilize respectively (9) and (10) in spite of the disturbance and to analyze the behavior induced on the state \( x \).

3.2 Design of robust controllers

The advantage of predictive schemes is that it is possible to use whatever controllers available for delay-free systems and just "plug" the prediction \( x_\phi \) or \( X_\phi \) instead of the normal state \( x \). To test and compare the schemes, a PID controller and a Super Twisting algorithm will be designed. These two controllers comply with Hypothesis 5. It is important to keep in mind that the objective is rather to compare both schemes and illustrate that they can be used with any controllers than to compare PID ans ST algorithm.

**PID controller.** The PID controller for (11) is
\[ u(\chi) = -k_p\chi(t) - k_d\dot{\chi}(t) - k_i \int_0^t \chi(s)ds \]
where \( k_p, k_d \) and \( k_i \) are positive constants.

**Proposition 11.** Consider system (9) (resp. system (10)) in closed-loop with PID controller (12) with \( \chi = \omega_\phi \) (resp. \( \chi = W_\phi \)). Then, the state prediction \( \omega_\phi \) (resp. \( W_\phi \)) converges to zero as \( t \) tends to infinity. □

**Remark 12.** It is well-known that an integrator is able to reject perfectly constant disturbances for scalar systems (11). The proof is based on the final value theorem.

**Remark 13.** The differential part of the PID is not required in this case since this is a first order plant.

**Super Twisting Algorithm (STA).** (Levant (1993)) For system (11), the Super Twisting controller is given by
\[ u(t) = \frac{1}{b}[ -a\chi - k_1\sqrt{|\chi|}\text{sign}(\chi) + \nu(t) \]  
where \( k_1, k_2 > 0 \) and with the dynamics of \( \nu \) governed by the following equation
\[ \dot{\nu} = -k_2\text{sign}(\chi). \]

**Proposition 14.** Consider system (9) (resp. system (10)) in closed-loop with STA controller (13) with \( \chi = \omega_\phi \) (resp. \( \chi = W_\phi \)). Then, the state prediction \( \omega_\phi \) (resp. \( W_\phi \)) converges to zero as \( t \) tends to infinity provided that
\[ k_1 > \sqrt{2k_2} \quad \text{with} \quad k_2 > 0. \]

**Proof.** The control \( u \) is divided in two parts such as
\[ u(t) = \frac{1}{b}[u_1(t) + u_2(t)] \]
with \( u_1(t) = -a\chi \) and \( u_2 = -k_1\sqrt{|\chi|}\text{sign}(\chi) + \nu(t) \). With this notation, \( \frac{1}{b}u_1 \) corresponds to a simple state feedback and \( u_2 \) is the Super Twisting contribution. Substituting (13) and (14) in (9), one obtains
\[ \begin{bmatrix} \dot{\chi} = u_2 + e^{\alpha t}d \\ \nu = -k_2\text{sign}(\chi) \end{bmatrix} \]

A direct application of the proof in Levant (1993) leads to conditions (15). Furthermore, \( k_1 \) and \( k_2 \) do not depend on the size of the perturbation because it is constant so the same condition is obtained when controller (13)-(14) is applied to (10). □

4. VALIDATION OF THE THEORETICAL RESULTS

4.1 Experimental setup and model validation

The experimental setup is a Modular Servo System (MSS) platform from Inteco (see Figure 1). It is composed of a servo motor with various modules, assembled on a metal rail and coupled with small clutches. In the experiments carried out for this paper, only the inertial load and the tachogenerator modules have been used. The measurement system is based on a RTDAC/PCI acquisition board equipped with A/D converters. A tacho-generator measures the angular velocity \( \omega \) with a 5% accuracy. The PC communicates with the sensors and motor by the I/O board and the power interface. The I/O board is controlled by the real-time software which operates in the MATLAB/Simulink RTW/RTWT environment. The armature voltage of the DC motor is controlled by a 12V PWM signal. However, the dimensionless control signal is the scaled input voltage \( u(t) = \frac{u(t)}{v_{\text{max}}} \). The admissible controls satisfy \( |u(t)| \leq 1 \) and \( v_{\text{max}} = 12V \).

The identification of model (7) by the surface method leads to parameters \( K \) and \( \tau \) given in Table 1. One can refer to the user’s manual Inteco (2006) for more details on the identification technique. Note that the time constant \( T = 1.14 \) s is quite large because it includes the dynamics of the motor and the load. The delay is artificially introduced in the loop by adding a "delay block" in the Simulink environment: it is perfectly known. It has been decided to take \( \tau = 1 \) s in order to have the same order of magnitude for the delay and the dynamics of the system (represented by \( T \)). In addition, a constant perturbation \( d = 24 \) rad.s\(^{-2} \) is introduced between 10 s to 30 s. This value is not used in the controller design because in practice it is unknown. Finally, the objective of the control laws (12) and (13) is to stabilize (8) to a reference velocity \( \omega_{\text{ref}} = 150 \) rad.s\(^{-1} \).

To illustrate the accuracy of the model, simulation and experimental results have been plotted on Figures 2 and 3.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$K$ (V^{-1} s^{-2}) & $T$ (s) & $\tau$ (s) \\
\hline
1/1.75 & 1.14 & 1 \\
\hline
\end{tabular}
\caption{DC Motor parameters}
\end{table}

On each figure, the angular velocity $\omega(t)$ and the control input $u(t)$ are displayed for $\tau = 0$ and controllers (12) and (13). It can be observed that the identified model is accurate because experimental results fit simulation ones. The only difference comes from the noise that affects experimental measurement. Note that when the perturbation has an effect on the system between 10s and 30s, the inputs reach the saturation. A deeper analysis of this results will be given in the next subsection. In the sequel, the objective is

- to illustrate the theoretical results,
- to compare the experimental results with the simulation ones,
- to compare the delay-free results with delayed ones.

Remark 15. A brief comparison will be drawn between PID controller and STA but it is not the purpose of the paper. Indeed, these two controllers have only been used to show that the efficiency of the predictive scheme is independent of the kind of controllers.

The results are displayed on Figure 4 for PID controller and on Figure 5 for STA controller. For Figure 4,

- on Fig.4-Top, (8) is controlled by (12) with $\chi = W_p$;
- on Fig.4-Bottom, (8) is controlled by (12) with $\chi = \hat{\omega}_p$.

For Figure 5,

- on Fig.5-Top, (8) is controlled by (13) with $\chi = W_p$;
- on Fig.5-Bottom, (8) is controlled by (13) with $\chi = \hat{\omega}_p$.

Considering Figure 4, on both graphs (Top and Bottom), it is clear that predictions $W_p$ and $\hat{\omega}_p$ are stabilized to the reference velocity $\omega_{ref}$ in spite of the perturbation $d$. This confirms the robustness of the PID controller established in Proposition 11. Note that an overshoot also appears on Figure 4-Bottom when the perturbation kicks the system in Proposition 9 whereas on Figure 4-Bottom one can observe that...
Before commenting the results, some extra details are pointed out. The computational effort to implement the algorithms is less than 20 ms so it is negligible with respect to the 1 s delay-block artificially introduced. A first order low-pass filter, with a time constant equal to 0.05 s, is implemented to decrease the noise level of the measure. This introduces a small extra delay of about 50 ms. The angular velocity reference is fixed, as in the simulation case, to \( \omega_{ref} = 150 \text{ rad.s}^{-1} \). A friction torque is applied between 10 s and 30 s to disturbed the system. The same torque is applied for each test but its magnitude is not measured precisely. Finally, the sampling period is the same as in the simulation section: \( 10^{-3} \text{ s} \).

The results are shown on Figures 7, 8 and 9. They are very close to simulation ones. It can be seen that using the new prediction \( W_p \) in the design of a controller leads to the perfect disturbance rejection (Figures 7-Top). The rejection is almost perfect on Figure 7-Top. Note that even in the delay-free case, the PID controller could not reject perfectly \( d \) (Figure 2). This is due to the input saturation that is more detrimental for the noisy signal that is why this phenomenon was not observed in simulation.

On the contrary, using the standard prediction only makes \( \omega_p \) converge to \( \omega_{ref} \) but not the angular velocity \( \omega \) (Figures 7-Bottom and 8-Bottom). Consequently, it illustrates the advantage of the new predictive scheme over the standard one to control input delay system with perturbation. Figure 9 shows the convergence error.

In addition, one can notice on Figure 9 that, even when no perturbation affects the system, there is a steady state error when the controllers use the standard prediction: \( \omega \) does not exactly reach \( \omega_{ref} \). It is due to the small model errors that makes the computation of the prediction \( \omega_p \) inexact whereas these errors are indirectly included in \( W_p \) as if it was an external perturbation.

A difference between simulations and experiments is the presence of oscillations on Figure 7-Top: a possible explanation is the wind-up effect caused by the saturation of the noisy signal or by the extra delays of computation time and output filtering. Note that the noise on Figure 8 is due to measurement and not chattering. Indeed, the same level of noise affect the system with PID controller.

The only difference between the delay-free case control (Figures 2 and 3) and the control with the new predictive scheme is the transient regime: the overshoot is larger for the input delay system. Indeed, the prediction is not exact during a few time after the perturbation kicked the system but then the same rejection as in the delay-free case is achieved.

Remark 18. Note that the new predictive scheme achieves better disturbance attenuation than the standard scheme for constant perturbation but for time-varying disturbances as well (see Léchappé et al. (2015a)). However, it has not been illustrated through experiments because it was not easy to generate a reproducible time-varying perturbation.
Fig. 7. Experimental results with $\tau = 1 \text{ s}$ – Top: PID with new prediction – Bottom: PID with standard prediction

Fig. 8. Experimental results with $\tau = 1 \text{ s}$ – Top: STA with new prediction – Bottom: STA with standard prediction

Fig. 9. Experimental results $\tau = 1 \text{ s}$ – Top: convergence errors for PID – Bottom: convergence errors for STA

5. CONCLUSION

In this paper, the new predictive scheme presented in Léchappé et al. (2015a) is applied to a DC motor subjected to a constant perturbation. A PID controller and a Super Twisting algorithm are then designed in order to reject the disturbance in spite of the large input delay (1 s). Simulation and experimental results confirm that the new scheme is more efficient than the standard one to reject constant disturbances. Consequently, the practical feasibility of this new method is demonstrated.

The extension to unknown and/or time-varying delays is currently under investigation.

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