Performances of Weighted Cyclic Prefix OFDM with Low-Complexity Equalization

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Abstract—In this paper, we justify low-complexity equalization techniques for weighted cyclic prefix (WCP)-OFDM. This modulation technique refers to filter bank based multicarrier (FBMC) transmission system provided with short filters. It allows the use of non-rectangular waveforms in order to mitigate interference caused by time-frequency selective channels while preserving an efficient implementation.

Index Terms—time-varying multipath channels, filter bank based multicarrier modulations, equalization, efficient realization.

I. INTRODUCTION

Mobile radio applications in terrestrial environment usually imply multipath propagation and motion-induced Doppler spread. Such a channel may be modeled as a time-frequency spreading operator. Under narrowband assumptions, it simplifies to a linear time-varying (LTV) system. The approximate eigenstructure of LTV systems has been discussed in [1]. This study reveals the need of pulse shaped multicarrier systems in order to match the time-frequency characteristics of doubly selective channels.

A practical implementation of the resulting transceiver relies on filter bank based multicarrier (FBMC) systems. Efficient realization schemes have been proposed in [2], providing orthogonal sub-channels and making use of fast Fourier transform (FFT) algorithm. Apart from the implementation, the design of pulse-shaping filters is an active area of research [3]–[5].

Since the pulse shapes are usually longer than each data block, the underlying transceiver requires a polyphase decomposition and advanced equalization techniques. Consequently, despite attractive performances results [6], FBMC systems are not widely used because of their relative complexity compared to traditional block transmission frameworks.

Therefore, the most common multicarrier scheme remains cyclic prefix orthogonal frequency-division multiplexing (CP-OFDM) scheme. It is based on a rectangular pulse-shaping and presents the advantage of diagonalizing time-invariant channels if a guard interval longer than the channel impulse response is used. Assuming invertibility of this eigensystem, perfect reconstruction of the transmitted symbols is performed using a single-tap per sub-channel equalizer [7]. However, the Doppler spread introduced by time-variant channels breaks orthogonality between sub-channels, resulting in inter-carrier interference (ICI).

In this work, we focus on short pulse shape filters, whose impulse response is shorter than data blocks [8]. This particular class of FBMC system is referred to as weighted cyclic prefix orthogonal frequency-division multiplexing (WCP-OFDM). Such a generalization of CP-OFDM through the use of non-rectangular filters offers an interesting trade-off between channel-induced interference mitigation and complexity [9].

Previous work has shown that WCP-OFDM outperforms CP-OFDM for several mobile radio environment with a single-tap per sub-channel equalizer [10]. We propose here more-than-one coefficient per sub-channel equalization scenarios, keeping a linear inversion complexity and show that WCP-OFDM still outperforms CP-OFDM using these scenarios.

II. GENERAL FRAMEWORK

In this section, we describe the input-output relation of a WCP-OFDM transmultiplexer (fig. 1) in presence of a doubly selective channel.

A. WCP-OFDM transmultiplexer structure

Let \( c_n = [c_{0,n}, \ldots, c_{M-1,n}]^T \) be the \( n \)th block of \( M \) complex symbols, with \( .^T \) being the transpose operator. The corresponding block at the output of the transmitter consists in \( N \) samples \( s_n = [s_n[0], \ldots, s_n[N-1]]^T \), with \( \Delta = N - M \geq 1 \). Given \( \gamma[k] = 0 \) if \( k < 0 \) or \( k > N - 1 \), the transmitted signal fulfills the relation

\[
s_n[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} c_{m,n} \gamma[k] e^{\frac{2 \pi m k}{M}}. \tag{1}
\]

If we let \( F_M \) be the \( M \)-size discrete Fourier transform (DFT) matrix with entries \( F_M[k,l] = 1/\sqrt{M} \exp(-j2\pi kl/M) \) for \( 0 \leq k, l \leq M - 1 \) and \( P_\Delta \) the \( N \times M \) cyclic extension matrix with entries \( P_\Delta[k,m] = \delta_{k,m} + \delta_{k-M,m} \) for \( 0 \leq k \leq N - 1 \) and \( 0 \leq m \leq M - 1 \) we may write in a similar way

\[
s_n = D_\gamma P_\Delta F_M^H c_n \tag{2}
\]

where \( .^H \) denotes the transpose conjugate operation and \( D_\gamma = \text{diag}(\gamma[0], \ldots, \gamma[N-1]) \).

Let \( r_n = [r_n[0], \ldots, r_n[N-1]]^T \) be the \( n \)th block of \( N \) received symbols. The corresponding \( M \) estimated symbols are given by

\[
\hat{c}_{m,n} = \frac{1}{\sqrt{M}} \sum_{k=0}^{N-1} r[k] \gamma[k] e^{-j\frac{2\pi m k}{M}}. \tag{3}
\]
If we denote \( D_\gamma = \text{diag}(\gamma[0], \ldots, \gamma[N-1]) \), we can write equivalently

\[
\hat{c}_n = F_M P_\Delta^T D_\gamma F_\Delta c_n.
\]

In the case of a perfect channel without noise, namely if \( r_n = s_n \), the combination of (2) and (4) gives

\[
\hat{c}_n = F_M P_\Delta^T D_\gamma F_\Delta c_n.
\]

Since \( F_M^H = F_M^{-1} \), the perfect reconstruction (PR) of the transmitted symbols is achieved when

\[
P_\Delta^T D_\gamma F_\Delta = I_M
\]

with \( I_M \) the \( M \)-size identity matrix. A PR system is said biorthogonal if \( \gamma[k] \neq \tilde{\gamma}[k] \) and orthogonal if \( \gamma[k] = \tilde{\gamma}[k] \).

As an example, CP-OFDM is a biorthogonal system. The transmitter pulse shape is given by \( \gamma_{\text{CP}}[k] = 1 \) for \( 0 \leq k \leq N-1 \) whereas the receiver pulse shape is written as \( \gamma_{\text{CP}}[k] = 0 \) for \( 0 \leq k \leq \Delta - 1 \) and \( \gamma_{\text{CP}}[k] = 1 \) for \( \Delta \leq k \leq N-1 \).

Two orthogonal systems have been proposed in [8], assuming \( N/\Delta = M/\Delta + 1 \) and odd values of \( M/\Delta \). One of them is optimized with respect to time-frequency localization (TFL) criterion and the underlying pulse shapes are denoted \( \gamma_{\text{TFL}}[k] = \tilde{\gamma}_{\text{TFL}}[k] \) (fig. 2). Compared to rectangular pulses used in CP-OFDM, a previous study confirms the interest of the TFL criterion for mobile channels [10]. The orthogonal setup also offers an optimal approach over additive white Gaussian noise channels [11, p. 160].

![Diagram of WCP-OFDM transmultiplexer](image)

**Fig. 1:** Efficient implementation of a WCP-OFDM transmultiplexer.

**B. Multipath time-variant channel model**

In a mobile radio environment, we consider a time-variant channel \( \mathcal{H} \), with \( I \) resolvable paths. Let \( s(t) \) be the baseband equivalent transmitted signal, limited to a band \( B \). In a narrowband context, the received signal is given by

\[
r(t) = (\mathcal{H} s(t)) + z(t) = \sum_{i=1}^I \alpha_i(t) s(t - \tau_i) + z(t)
\]

where \( \alpha_i(t) \) is a complex gain associated to the \( i \)th path, at delay \( \tau_i \). The signal \( z(t) \) is a complex bandlimited Gaussian noise characterised by its power spectral density \( 2 N_0 = \sigma_z^2 / B \) for \( |f| \leq B/2 \) and 0 otherwise.

In order to relate the channel model to a filter bank transceiver, we express a discrete-time version of \( \mathcal{H} \), denoted \( \tilde{\mathcal{H}} \). Using the bandlimited hypothesis and if we let \( s_n[k] = s[k + nN] = s((k + nN)/B) \), \( z_n[k] = z[k + nN] = z((k + nN)/B) \) and \( r_n[k] = r[k + nN] = r((k + nN)/B) \), for \( 0 \leq k \leq N-1 \), the input-output relation becomes

\[
r[q] = (\tilde{\mathcal{H}} s)[q] + z[q] = \sum_{l \in \mathbb{Z}} \tilde{\alpha}_l[q] s[q - l] + z[q]
\]

with

\[
\tilde{\alpha}_l[q] = \sum_{i=1}^I \alpha_i \left( \frac{q}{B} \right) \text{sinc}(B \tau_i - l).
\]

Bandlimiting operation leads to an infinite number of coefficients \( \tilde{\alpha}_l(t) \) \( (l \in \mathbb{Z}) \). In practice, the sequence is truncated to \( L \) taps whenever \( \{ |\tilde{\alpha}_l(t)|^2 \}_{t > L} \approx 0 \).

In the context of short filters used by WCP-OFDM, the input-output relation given in (8) may be simplified if \( L \leq N \). In other words, if the channel delay is less than the block length, interblock interference (IBI) is restricted to two consecutive blocks and we have

\[
r_n = H_n s_n + G_n s_{n-1} + z_n, \quad n \in \mathbb{N}
\]

where \( z_n = [z_n[0] \ldots z_n[N-1]]^T \). The transfer function of the discrete-time equivalent channel over the \( n \)th block is...
described by two $N \times N$ matrices defined as

$$H_n = \begin{bmatrix}
\tilde{\alpha}_1[nN] & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\tilde{\alpha}_L[nN] & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \alpha_L[nN+2] & \cdots & \tilde{\alpha}_1[nN+N-1]
\end{bmatrix}$$

(11)

$$G_n = \begin{bmatrix}
0 & \cdots & \cdots & \alpha_2[nN-1] & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\alpha_2[nN-1] & \cdots & \cdots & \cdots & \cdots & \vdots \\
\vdots & \cdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \vdots \\
\alpha_2[nN-1] & \cdots & \cdots & \cdots & \cdots & 0
\end{bmatrix}$$

(12)

Combining (2), (4) and (10), the overall transmultiplexer input-output relation writes

$$\tilde{c}_n = \underbrace{F_MP_T^TD_yH_yD_yP_HM}_{A_n}c_n + \underbrace{F_MP_T^TD_yG_yD_yP_HM}_{B_n}c_{n-1} + \underbrace{F_MP_T^TD_yz_n}_{\zeta_n},$$

(13)

where $\zeta_n$ corresponds to the noise term projection on the receiver and $A_n$ and $B_n$ represent the WCP-OFDM transfer matrices associated to the $n$th block.

III. WCP-OFDM EQUALIZATION SCHEMES FOR A GIVEN SIMULATION FRAMEWORK

As stated above, IBI and ICI may occur in the general case of WCP-OFDM transmultiplexer in presence of a time-frequency selective channel. However, IBI involves at most two consecutive blocks. As a consequence, the analysis of $E\{|A[m,p]|^2\}$ and $E\{|B[m,p]|^2\}$ is sufficient to determine the most appropriate equalization scheme for a given application.

In this simulation framework, we consider a QPSK modulated transmission system, using a band $B = 8$ MHz, centered around a frequency $f_c = 5$ GHz. We use a 6-path WSSUS channel model where the last path occurs at $5$ µs (COST 207 TUx6 [12]). It implies a highly frequency selective behavior over the band $B$. Two mobility scenarios are developed with regard to the fast fading assumption: pedestrian ($v_{max} = 3$ km/h) and vehicular ($v_{max} = 350$ km/h).

Using TFL optimized pulses, figures 3 and 4 confirm that IBI and ICI decrease as $N/M$ increases. As expected, ICI increases with the Doppler spread. Furthermore, IBI is negligible compared to ICI in both mobility scenarios. As a consequence, a single-tap per sub-carrier equalizer may be sufficient in the low mobility scenario whereas ICI from adjacent sub-channels should be mitigated in the high mobility case.

In the following, we consider two low-complexity equalizers and we denote $\tilde{c}_n$ the equalized signal. Channel impulse response estimation is beyond the scope of this paper.

1) Equalizer 1: We first propose a single tap per sub-channel equalizer. This approach considers that IBI and ICI terms are negligible. It relies on the approximation of $A_n$ as a diagonal matrix written

$$\tilde{A}_n[m,p] = A_n[m,p]\delta_{m,p}.$$  

(14)

Assuming that $\tilde{A}_n$ is invertible, the equalized symbols are given by

$$\tilde{c}_n = \tilde{A}_n^{-1}\tilde{c}_n = \tilde{A}_n^{-1}A_n c_n + \tilde{A}_n^{-1}B_n c_{n-1} + \tilde{A}_n^{-1}\zeta_n.$$  

(15)

This equalizer may lead to perfect reconstruction in the case of a CP-OFDM facing a frequency selective channel, provided a prefix greater than the last echo is used. However, if $\tilde{A}_n$ has close-to-zero diagonal terms, noise and interference terms may be amplified. In the general case of WCP-OFDM with a severe time-frequency selective channel, IBI and ICI terms may remain preponderant over the noise term [10].

2) Equalizer 2: In the case of mainly time-selective channels, the major performances degradation is caused by ICI and it becomes interesting to cancel interferences induced by adjacent sub-channels. To this end, we approximate $A_n$ as a tridiagonal matrix defined by

$$\tilde{A}_n[m,p] = A_n[m,p](\delta_{m,p} + \delta_{m,p-1} + \delta_{m,p+1}).$$  

(16)

If $\tilde{A}_n$ is invertible, we retrieve the expression given in (15). Thanks to Thomas algorithm [13], the resolution of a tridiagonal system results in an $O(N)$ operation which is much more affordable than a general $N \times N$ matrix inversion whose complexity is usually $O(N^3)$. 

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**Fig. 3:** WCP-OFDM system mean gains $E\{|A[m,p]|^2\}$ and $E\{|B[m,p]|^2\}$ with $M = 512$, $v_{max} = 3$ km/h.

**Fig. 4:** WCP-OFDM system mean gains $E\{|A[m,p]|^2\}$ and $E\{|B[m,p]|^2\}$ with $M = 512$, $v_{max} = 350$ km/h.
IV. BER SIMULATION RESULTS

We compare the performances of CP-OFDM and WCP-OFDM with TFL pulses using $M = 512$ and $N/M = 5/4$. We focus our analysis on the two equalization scenarios described above. Bit error rate (BER) is plotted as a function of $E_b/N_0$, where $E_b = \sigma^2 \|c\|^2 / 2MB$ and $N_0 = \sigma^2 / 2B$ (fig. 5).

When the Doppler spread tends to zero (fig. 5a), ICI is negligible and equalizers 1 and 2 yield the same results. WCP-OFDM with TFL pulses shows better results than CP-OFDM in presence of noise, thanks to prototypes functions orthogonality.

In a high mobility scenario (fig. 5b), an interference floor appears at high $E_b/N_0$ values. It demonstrates the interest of three taps per sub-carrier equalization for both WCP-OFDM and CP-OFDM. The clear advantage of WCP-OFDM is justified by the good frequency containment of the TFL pulses.

V. CONCLUSION

The focus of our work on short filters leads to an efficient FBMC modulator-demodulator referred to as WCP-OFDM and whose complexity is similar to traditional cyclic prefix OFDM.

Through the analysis of the transfer matrices of a realistic system, we proposed two block equalization scenarios, preserving a linear inversion complexity.

Finally, BER simulation results confirm the interest of WCP-OFDM with TFL filters with regard to CP-OFDM. This system, provided with one to three coefficients per sub-channel, is particularly resilient to ICI in the case of highly time-selective channels.

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