Performances of Weighted Cyclic Prefix OFDM with Low-Complexity Equalization

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***Abstract*—In this paper, we justify low-complexity equalization techniques for weighted cyclic prefix (WCP)-OFDM. This modulation technique refers to filter bank based multicarrier (FBMC) transmission system provided with short filters. It allows the use of non-rectangular waveforms in order to mitigate interference caused by time-frequency selective channels while preserving an efficient implementation.**

***Index Terms*—time-varying multipath channels, filter bank based multicarrier modulations, equalization, efficient realization.**

I. Introduction

Mobile radio applications in terrestrial environment usually imply multipath propagation and motion-induced Doppler spread. Such a channel may be modeled as a time-frequency spreading operator. Under narrowband assumptions, it simplifies to a linear time-varying (LTV) system. The approximate eigenstructure of LTV systems has been discussed in [1]. This study reveals the need of pulse shaped multicarrier systems in order to match the time-frequency characteristics of doubly selective channels.

A practical implementation of the resulting transceiver relies on filter bank based multicarrier (FBMC) systems. Efficient realization schemes have been proposed in [2], providing orthogonal sub-channels and making use of fast Fourier transform (FFT) algorithm. Apart from the implementation, the design of pulse-shaping filters is an active area of research [3]–[5].

Since the pulse shapes are usually longer than each data block, the underlying transceiver requires a polyphase decomposition and advanced equalization techniques. Consequently, despite attractive performances results [6], FBMC systems are not widely used because of their relative complexity compared to traditional block transmission frameworks.

Therefore, the most common multicarrier scheme remains cyclic prefix orthogonal frequency-division multiplexing (CP-OFDM) scheme. It is based on a rectangular pulse-shaping and presents the advantage of diagonalizing time-invariant channels if a guard interval longer than the channel impulse response is used. Assuming invertibility of this eigensystem, perfect reconstruction of the transmitted symbols is performed using a single-tap per sub-channel equalizer [7]. However, the Doppler spread introduced by time-variant channels breaks orthogonality between sub-channels, resulting in inter-carrier interference (ICI).

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In this work, we focus on short pulse shape filters, whose impulse response is shorter than data blocks [8]. This particular class of FBMC system is referred to as weighted cyclic prefix orthogonal frequency-division multiplexing (WCP-OFDM). Such a generalization of CP-OFDM through the use of non-rectangular filters offers an interesting trade-off between channel-induced interference mitigation and complexity [9].

Previous work has shown that WCP-OFDM outperforms CP-OFDM for several mobile radio environment with a single-tap per sub-channel equalizer [10]. We propose here more-than-one coefficient per sub-channel equalization scenarios, keeping a linear inversion complexity and show that WCP-OFDM still outperforms CP-OFDM using these scenarios.

II. General framework

In this section, we describe the input-output relation of a WCP-OFDM transmultiplexer (fig. 1) in presence of a doubly selective channel.

A. WCP-OFDM transmultiplexer structure

Let $\boldsymbol{c}_n = [c_{0,n} \dots c_{M-1,n}]^T$ be the nth block of M complex symbols, with $\cdot^T$ being the transpose operator. The corresponding block at the output of the transmitter consists in N samples $\boldsymbol{s}_n = [s_n[0] \dots s_n[N-1]]^T$, with $\Delta = N - M \geq 1$. Given $\gamma[k] = 0$ if $k < 0$ or $k > N-1$, the transmitted signal fullfils the relation

$$s_n[k] = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} c_{m,n} \gamma[k] e^{j\frac{2\pi mk}{M}}. \tag{1}$$

If we let $\boldsymbol{F}_M$ be the M-size discrete Fourier transform (DFT) matrix with entries $F_M[k,l] = 1/\sqrt{M}\exp(-j2\pi kl/M)$ for $0 \leq k, l \leq M-1$ and $\boldsymbol{P}_\Delta$ the $N \times M$ cyclic extension matrix with entries $P_\Delta[k,m] = \delta_{k,m} + \delta_{k-M,m}$ for $0 \leq k \leq N-1$ and $0 \leq m \leq M-1$ we may write in a similar way

$$\boldsymbol{s}_n = \boldsymbol{D}_\gamma \boldsymbol{P}_\Delta \boldsymbol{F}_M^H \boldsymbol{c}_n \tag{2}$$

where $\cdot^H$ denotes the transpose conjugate operation and $\boldsymbol{D}_\gamma = \operatorname{diag}(\gamma[0], \dots, \gamma[N-1])$.

Let $\boldsymbol{r}_n = [r_n[0] \dots r_n[N-1]]^T$ be the nth block of N received symbols. The corresponding M estimated symbols are given by

$$\tilde{c}_{m,n} = \frac{1}{\sqrt{M}} \sum_{k=0}^{N-1} r[k] \check{\gamma}[k] e^{-j\frac{2\pi mk}{M}}. \tag{3}$$

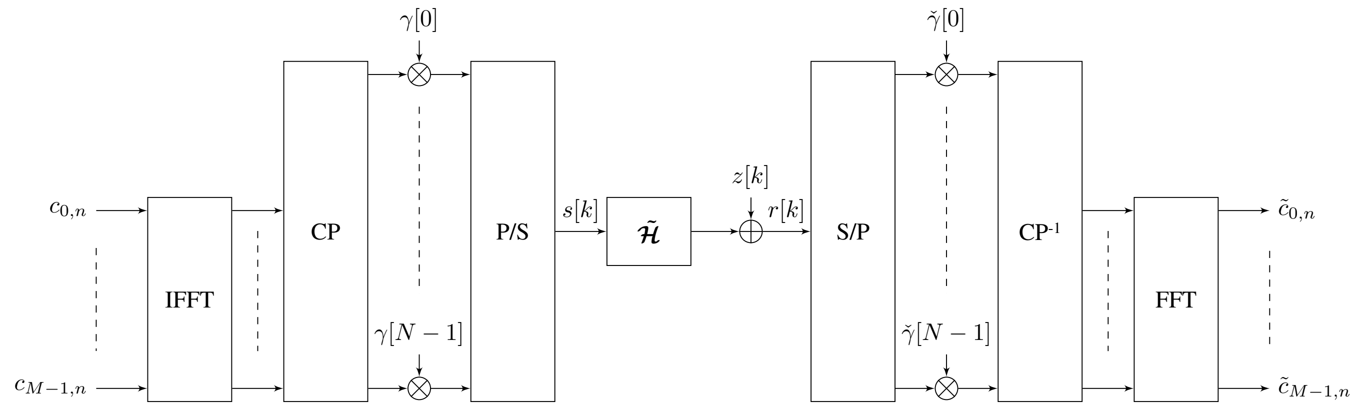


Fig. 1: Efficient implementation of a WCP-OFDM transmultiplexer.

If we denote $\boldsymbol{D}_\gamma = \operatorname{diag}(\gamma[0], \ldots, \gamma[N-1])$, we can write equivalently

$$\tilde{\boldsymbol{c}}_n = \boldsymbol{F}_M \boldsymbol{P}_\Delta^T \boldsymbol{D}_{\check{\gamma}} \boldsymbol{r}_n. \tag{4}$$

In the case of a perfect channel without noise, namely if $\boldsymbol{r}_n = \boldsymbol{s}_n$, the combination of (2) and (4) gives

$$\tilde{\boldsymbol{c}}_n = \boldsymbol{F}_M \boldsymbol{P}_\Delta^T \boldsymbol{D}_{\check{\gamma}} \boldsymbol{D}_\gamma \boldsymbol{P}_\Delta \boldsymbol{F}_M^H \boldsymbol{c}_n. \tag{5}$$

Since $\boldsymbol{F}_M^H = \boldsymbol{F}_M^{-1}$, the perfect reconstruction (PR) of the transmitted symbols is achieved when

$$\boldsymbol{P}_\Delta^T \boldsymbol{D}_{\check{\gamma}} \boldsymbol{D}_\gamma \boldsymbol{P}_\Delta = \boldsymbol{I}_M \tag{6}$$

with $\boldsymbol{I}_M$ the M-size identity matrix. A PR system is said biorthogonal if $\gamma[k] \neq \check{\gamma}[k]$ and orthogonal if $\gamma[k] = \check{\gamma}[k]$.

As an example, CP-OFDM is a biorthogonal system. The transmitter pulse shape is given by $\gamma^{\text{CP}}[k] = 1$ for $0 \leq k \leq N-1$ whereas the receiver pulse shape is written as $\check{\gamma}^{\text{CP}}[k] = 0$ for $0 \leq k \leq \Delta - 1$ and $\check{\gamma}^{\text{CP}}[k] = 1$ for $\Delta \leq k \leq N-1$.

Two orthogonal systems have been proposed in [8], assuming $N/\Delta = M/\Delta + 1$ and odd values of M/Δ. One of them is optimized with respect to time-frequency localization (TFL) criterion and the underlying pulse shapes are denoted $\gamma^{\text{TFL}}[k] = \check{\gamma}^{\text{TFL}}[k]$ (fig. 2). Compared to rectangular pulses used in CP-OFDM, a previous study confirms the interest of the TFL criterion for mobile channels [10]. The orthogonal setup also offers an optimal approach over additive white Gaussian noise channels [11, p. 160].

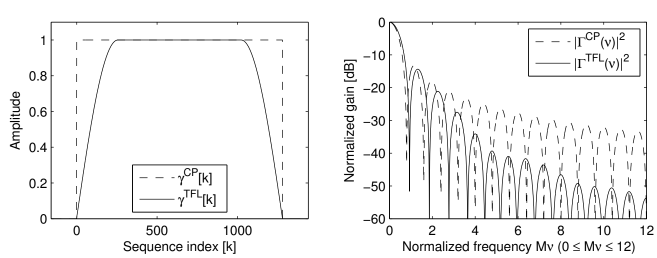


Fig. 2: Time and frequency responses of the prototype filters with $\Gamma(\nu) = 1/\|\gamma\|_\infty \sum_{k=0}^{N-1} \gamma[k] \exp(-j2\pi\nu k/N)$ and $\|\gamma\|_\infty = \max\{|\gamma[k]|\}_{k\in\mathbf{Z}}$, using $M = 1024$ and $N = 1280$.

B. Multipath time-variant channel model

In a mobile radio environment, we consider a time-variant channel $\boldsymbol{\mathcal{H}}$, with I resolvable paths. Let $s(t)$ be the baseband equivalent transmitted signal, limited to a band B. In a narrowband context, the received signal is given by

$$r(t) = (\boldsymbol{\mathcal{H}}s)(t) + z(t) = \sum_{i=1}^{I} \alpha_i(t) s(t - \tau_i) + z(t) \tag{7}$$

where $\alpha_i(t)$ is a complex gain associated to the ith path, at delay τ_i. The signal $z(t)$ is a complex bandlimited Gaussian noise characterised by its power spectral density $2N_0 = \sigma_z^2/B$ for $|f| \leq B/2$ and 0 otherwise.

In order to relate the channel model to a filter bank transceiver, we express a discrete-time version of $\boldsymbol{\mathcal{H}}$, denoted $\tilde{\boldsymbol{\mathcal{H}}}$. Using the bandlimited hypothesis and if we let $s_n[k] = s[k+nN] = s((k+nN)/B)$, $z_n[k] = z[k+nN] = z((k+nN)/B)$ and $r_n[k] = r[k+nN] = r((k+nN)/B)$, for $0 \leq k \leq N-1$, the input-output relation becomes

$$r[q] = \left(\tilde{\boldsymbol{\mathcal{H}}} s\right)[q] + z[q] = \sum_{l \in \mathbf{Z}} \tilde{\alpha}_l[q] s[q-l] + z[q] \tag{8}$$

with

$$\tilde{\alpha}_l[q] = \sum_{i=1}^{I} \alpha_i \left(\frac{q}{B}\right) \operatorname{sinc}(B\tau_i - l). \tag{9}$$

Bandlimiting operation leads to an infinite number of coefficients $\tilde{\alpha}_l(t)$ ($l \in \mathbf{Z}$). In practice, the sequence is truncated to L taps whenever $\{|\tilde{\alpha}_l(t)|^2\}_{l>L} \approx 0$.

In the context of short filters used by WCP-OFDM, the input-output relation given in (8) may be simplified if $L \leq N$. In other words, if the channel delay is less than the block length, interblock interference (IBI) is restricted to two consecutive blocks and we have

$$\boldsymbol{r}_n = \boldsymbol{H}_n \boldsymbol{s}_n + \boldsymbol{G}_n \boldsymbol{s}_{n-1} + \boldsymbol{z}_n, \qquad n \in \mathbf{N} \tag{10}$$

where $\boldsymbol{z}_n = [z_n[0] \ldots z_n[N-1]]^T$. The transfer function of the discrete-time equivalent channel over the nth block is described by two $N \times N$ matrices defined as

$$\boldsymbol{H}_n = \begin{bmatrix} \tilde{\alpha}_1[nN] & 0 & 0 & \cdots & 0 \\ \vdots & \tilde{\alpha}_1[nN+1] & 0 & \cdots & 0 \\ \tilde{\alpha}_L[nN] & \cdots & \ddots & \cdots & 0 \\ \vdots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & \tilde{\alpha}_L[nN+2] & \cdots & \tilde{\alpha}_1[nN+N-1] \end{bmatrix} \quad (11)$$

$$\boldsymbol{G}_n = \begin{bmatrix} 0 \cdots & \tilde{\alpha}_L[nN-L+1] & \cdots & \tilde{\alpha}_2[nN-1] \\ \vdots \ddots & 0 & \ddots & \vdots \\ 0 \cdots & \ddots & \cdots & \tilde{\alpha}_L[nN-1] \\ \vdots \; \vdots & \vdots & \ddots & \vdots \\ 0 \cdots & 0 & \cdots & 0 \end{bmatrix}. \quad (12)$$

Combining (2), (4) and (10), the overall transmultiplexer input-output relation writes

$$\tilde{\boldsymbol{c}}_n = \underbrace{\boldsymbol{F}_M \boldsymbol{P}_\Delta^T \boldsymbol{D}_{\tilde{\gamma}} \boldsymbol{H}_n \boldsymbol{D}_\gamma \boldsymbol{P}_\Delta \boldsymbol{F}_M^H}_{\boldsymbol{A}_n} \boldsymbol{c}_n + \underbrace{\boldsymbol{F}_M \boldsymbol{P}_\Delta^T \boldsymbol{D}_{\tilde{\gamma}} \boldsymbol{G}_n \boldsymbol{D}_\gamma \boldsymbol{P}_\Delta \boldsymbol{F}_M^H}_{\boldsymbol{B}_n} \boldsymbol{c}_{n-1} + \underbrace{\boldsymbol{F}_M \boldsymbol{P}_\Delta^T \boldsymbol{D}_{\tilde{\gamma}} \boldsymbol{z}_n}_{\boldsymbol{\zeta}_n}. \quad (13)$$

where $\boldsymbol{\zeta}_n$ corresponds to the noise term projection on the receiver and $\boldsymbol{A}_n$ and $\boldsymbol{B}_n$ represent the WCP-OFDM transfer matrices associated to the nth block.

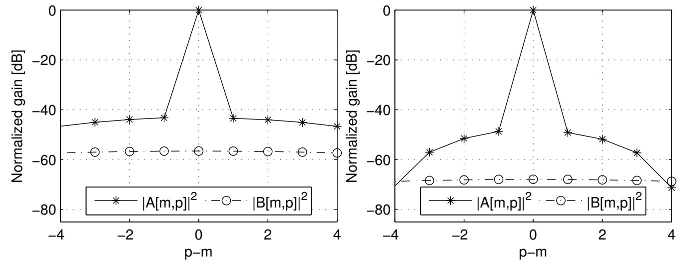
III. WCP-OFDM EQUALIZATION SCHEMES FOR A GIVEN SIMULATION FRAMEWORK

As stated above, IBI and ICI may occur in the general case of WCP-OFDM transmultiplexer in presence of a time-frequency selective channel. However, IBI involves at most two consecutive blocks. As a consequence, the analysis of $\mathrm{E}\{|A[m,p]|^2\}$ and $\mathrm{E}\{|B[m,p]|^2\}$ is sufficient to determine the most appropriate equalization scheme for a given application.

In this simulation framework, we consider a QPSK modulated transmission system, using a band $B = 8$ MHz, centered around a frequency $f_c = 5$ GHz. We use a 6-path WSSUS channel model where the last path occurs at 5 μs (COST 207 TUx6 [12]). It implies a highly frequency selective behavior over the band B. Two mobility scenarios are developed with regard to the fast fading assumption: pedestrian ($v_{\max} = 3$ km/h) and vehicular ($v_{\max} = 350$ km/h).

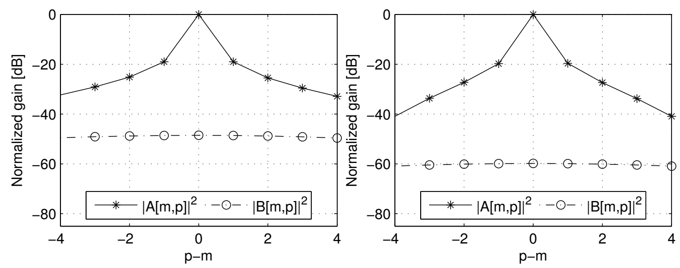
Using TFL optimized pulses, figures 3 and 4 confirm that IBI and ICI decrease as N/M increases. As expected, ICI increases with the Doppler spread. Furthermore, IBI is negligible compared to ICI in both mobility scenarios. As a consequence, a single-tap per sub-carrier equalizer may be sufficient in the low mobility scenario whereas ICI from adjacent sub-channels should be mitigated in the high mobility case.

In the following, we consider two low-complexity equalizers and we denote $\bar{\boldsymbol{c}}_n$ the equalized signal. Channel impulse response estimation is beyond the scope of this paper.



(a) $N/M = 9/8$. (b) $N/M = 5/4$.

Fig. 3: WCP-OFDM system mean gains $\mathrm{E}\{|A[m,p]|^2\}$ and $\mathrm{E}\{|B[m,p]|^2\}$ with $M = 512$, $v_{\max} = 3$ km/h.



(a) $N/M = 9/8$. (b) $N/M = 5/4$.

Fig. 4: WCP-OFDM system mean gains $\mathrm{E}\{|A[m,p]|^2\}$ and $\mathrm{E}\{|B[m,p]|^2\}$ with $M = 512$, $v_{\max} = 350$ km/h.

1) Equalizer 1: We first propose a single tap per sub-channel equalizer. This approach considers that IBI and ICI terms are negligible. It relies on the approximation of $\boldsymbol{A}_n$ as a diagonal matrix written

$$\tilde{A}_n[m,p] = A_n[m,p]\delta_{m,p}. \quad (14)$$

Assuming that $\tilde{\boldsymbol{A}}_n$ is invertible, the equalized symbols are given by

$$\bar{\boldsymbol{c}}_n = \tilde{\boldsymbol{A}}_n^{-1} \tilde{\boldsymbol{c}}_n = \tilde{\boldsymbol{A}}_n^{-1} \boldsymbol{A}_n \boldsymbol{c}_n + \tilde{\boldsymbol{A}}_n^{-1} \boldsymbol{B}_n \boldsymbol{c}_{n-1} + \tilde{\boldsymbol{A}}_n^{-1} \boldsymbol{\zeta}_n. \quad (15)$$

This equalizer may lead to perfect reconstruction in the case of a CP-OFDM facing a frequency selective channel, provided a prefix greater than the last echo is used. However, if $\tilde{\boldsymbol{A}}_n$ has close-to-zero diagonal terms, noise and interference terms may be amplified. In the general case of WCP-OFDM with a severe time-frequency selective channel, IBI and ICI terms may remain preponderant over the noise term [10].

2) Equalizer 2: In the case of mainly time-selective channels, the major performances degradation is caused by ICI and it becomes interesting to cancel interferences induced by adjacent sub-channels. To this end, we approximate $\boldsymbol{A}_n$ as a tridiagonal matrix defined by

$$\tilde{A}_n[m,p] = A_n[m,p](\delta_{m,p} + \delta_{m,p-1} + \delta_{m,p+1}). \quad (16)$$

If $\tilde{\boldsymbol{A}}_n$ is invertible, we retrieve the expression given in (15). Thanks to Thomas algorithm [13], the resolution of a tridiagonal system results in a $\mathcal{O}(N)$ operation wich is much more affordable than a general $N \times N$ matrix inversion whose complexity is usually $\mathcal{O}(N^3)$.

IV. BER simulation results

We compare the performances of CP-OFDM and WCP-OFDM with TFL pulses using $M = 512$ and $N/M = 5/4$. We focus our analysis on the two equalization scenarios described above. Bit error rate (BER) is plotted as a function of E_b/N_0, where $E_b = \sigma_c^2 \|\gamma\|^2 / 2MB$ and $N_0 = \sigma_z^2/2B$ (fig. 5).

When the Doppler spread tends to zero (fig. 5a), ICI is negligible and equalizers 1 and 2 yield the same results. WCP-OFDM with TFL pulses shows better results than CP-OFDM in presence of noise, thanks to prototypes functions orthogonality.

In a high mobility scenario (fig. 5b), an interference floor appears at high E_b/N_0 values. It demonstrates the interest of three taps per sub-carrier equalization for both WCP-OFDM and CP-OFDM. The clear advantage of WCP-OFDM is justified by the good frequency containment of the TFL pulses.

V. Conclusion

The focus of our work on short filters leads to an efficient FBMC modulator-demodulator referred to as WCP-OFDM and whose complexity is similar to traditional cyclic prefix OFDM.

Through the analysis of the transfer matrices of a realistic system, we proposed two block equalization scenarios, preserving a linear inversion complexity.

Finally, BER simulation results confirm the interest of WCP-OFDM with TFL filters with regard to CP-OFDM. This system, provided with one to three coefficients per sub-channel, is particularly resilient to ICI in the case of highly time-selective channels.

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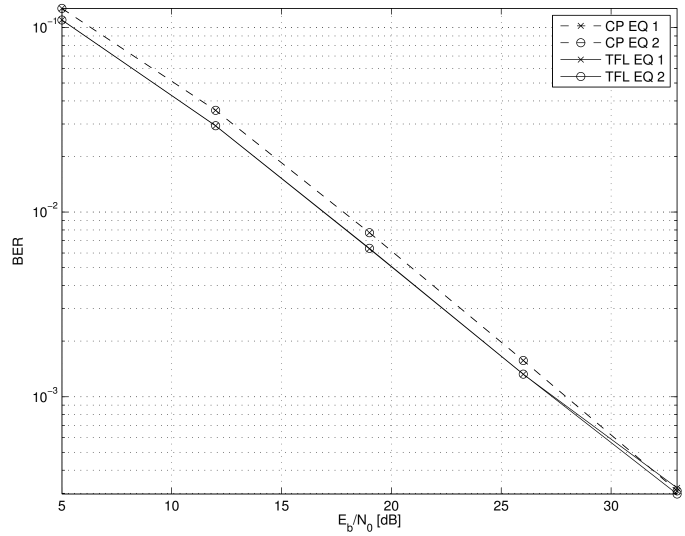
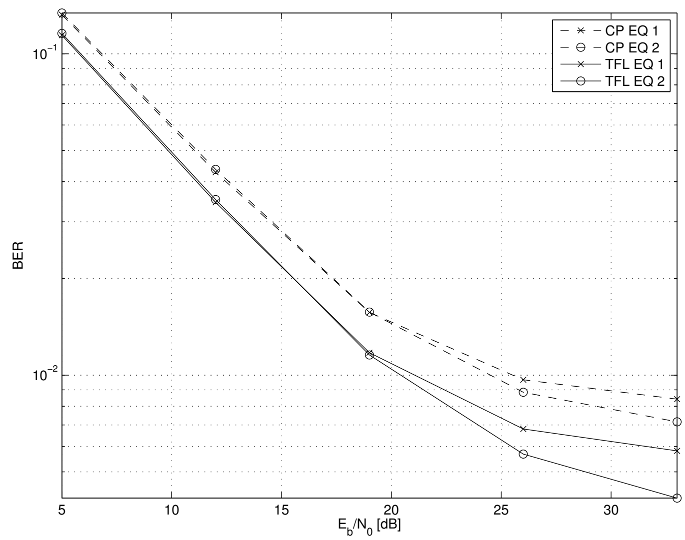


(a) BER performances for $v_{\max} = 3$ km/h.



(b) BER performances for $v_{\max} = 350$ km/h.

Fig. 5: BER performances for pedestrian and vehicular scenarios in COST 207 TUx6 channel. Comparison between CP-OFDM and WCP-OFDM (with TFL pulses) for $M = 512$ and $N/M = 5/4$.

References

[1] W. Kozek and A. F. Molisch, “On the eigenstructure of underspread WSSUS channels,” in *Proc. First IEEE Signal Processing Workshop Signal Processing Advances in Wireless Communications*, 1997, pp. 325–328.

[2] C. Siclet, P. Siohan, and D. Pinchon, “Analysis and design of OFDM/QAM and BFDM/QAM oversampled orthogonal and biorthogonal multicarrier modulations,” in *IEEE ICASSP'02*, Orlando, USA, May 2002, student forum.

[3] T. Strohmer and S. Beaver, “Optimal OFDM design for time-frequency dispersive channels,” *IEEE Trans. Commun.*, vol. 51, no. 7, pp. 1111–1122, 2003.

[4] C. Siclet, P. Siohan, and D. Pinchon, “Perfect reconstruction conditions and design of oversampled DFT-modulated transmultiplexers,” *EURASIP journal on applied signal processing*, vol. 2006, pp. 1–14, 2006.

[5] P. Jung and G. Wunder, “WSSUS pulse design problem in multicarrier transmission,” *IEEE Trans. Commun.*, vol. 55, no. 9, p. 1822, 2007.

[6] A. M. Tonello and F. Pecile, “Analytical results about the robustness of FMT modulation with several prototype pulses in time-frequency selective fading channels,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1634–1645, 2008.

[7] Z. Wang and G. B. Giannakis, “Wireless multicarrier communications,” *IEEE Signal Process. Mag.*, vol. 17, no. 3, pp. 29–48, 2000.

[8] D. Pinchon and P. Siohan, “Closed-form expressions of optimal short PR FMT prototype filters,” in *Proc. IEEE Global Telecommunications Conf. GLOBECOM '11*, 2011.

[9] N. Moret and A. M. Tonello, “Design of orthogonal filtered multitone modulation systems and comparison among efficient realizations,” *EURASIP J. Adv. Signal Process*, vol. 2010, pp. 10:1–10:11, January 2010.

[10] D. Roque, C. Siclet, and P. Siohan, “A performance comparison of FBMC modulation schemes with short perfect reconstruction filters,” in *Proc. IEEE International Conference on Telecommunications, ICT '12*, 2012.

[11] J. Proakis and M. Salehi, *Digital communications*, 5th ed. McGraw-Hill, 2006.

[12] M. Failli, “COST 207 digital land mobile radio communications,” *Commission of the European Communities*, p. 137, 1988.

[13] S. Conte and C. Boor, *Elementary numerical analysis: an algorithmic approach*. McGraw-Hill Higher Education, 1980.