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Velocity Measurement from the Spectral Phase of a Match-Filtered LFM Chirp.

Samuel Pinson

Abstract—Chirp signals are very convenient to perform acoustic measurement with high signal-to-noise ratios. But measurement using those signals may present some drawbacks if a transducer or a target is moving. Indeed match-filtered chirp signals are not Doppler tolerant. The Doppler effect affects their amplitude, delay and resolution. To perform a correct match filtering that includes the Doppler shift requires a prior knowledge of the Doppler velocity. In this paper, it is demonstrated that the Doppler velocity can be extracted directly from the Doppler cross-power spectrum. More precisely, the quadratic coefficient of the Doppler cross-power-spectrum phase is proportional to the relative velocity. Consequently, the method has a low computational cost.

Index Terms—Doppler effect, ambiguity function, linear-frequency-modulation (LFM) chirp, velocity measurement, pulse compression.

I. INTRODUCTION

WHEN USING A CHIRP for acoustic measurement, the match-filter output can be strongly affected by the Doppler effect. Pulse compression without a proper compensation of the Doppler effect affects amplitude, delay and resolution of the signal [1]. To perform the correct correlation to compensate the relative motion between a source and a receiver or between an active system and a target, it is necessary to first measure the relative velocity or the target relative velocity. One way to measure the relative velocity is to calculate the maximum likelihood by cross-correlating the match-filter output with the ambiguity function calculated for a wide range of velocities. But that operation might be computationally expensive for embedded system or to deal with an important number of measurements.

For particular applications such as underwater acoustic communications [2] a rough velocity estimation can be made measuring the time interval between two pulses obtained after match filtering and use a phase locked loop [3] to optimize the Doppler effect compensation. Under some conditions, it is possible to extract directly the information from the match-filtered signal. In [4], it is shown that the relative velocity can be measured from the instant frequency variation in a match-filtered echo if the source signal is linear-period modulated (LPM). In this paper, it is shown that the relative velocity can be assessed directly from a selected echo in the signal when the chirp has a linear-frequency modulation (LFM).

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II. THEORY

Considering a moving source transmitting a LFM chirp and a fixed receiver, the analytical emitted signal is given by:

$$s(t) = w(t) \exp \left[i2\pi \left(f_0 t + \frac{\alpha}{2} t^2 \right) \right], \quad (1)$$

where $w(t)$ is an envelope centered at the origin (typically a Gaussian or a rectangle function can be used). $f_0 + \alpha t$ is the instant frequency. The Doppler dependent received signal is:

$$r(t) = A s(\eta(t - \Delta t)), \quad (2)$$

where A is an amplitude factor due to the wave propagation, $\eta = 1 + v/c$ is the time-stretching factor, c is the sound-speed and v is the radial relative velocity and is defined positive when the source is moving towards the receiver.

After match filtering, the received signal becomes:

$$\chi_r(t) = A s(\eta(t - \Delta t)) * s(-t), \quad (3)$$

where $*$ is the convolution operator and Δt is the travel time between source and receiver.

A. Ambiguity function

To deduce the relative velocity v , one can match the received signal with numerous Doppler stretched emitted signals $s(\eta t)$ and find the maximum of correlation. Nevertheless, the emitted chirp has a long duration and it might be a better strategy to work on the match-filtered signal. Thus the compressed pulse can be windowed and the processing can be performed on a shorter signal. Then the match-filtered signal can be compared to the ambiguity function:

$$\chi_{af}(t, \eta) = s(\eta(t)) * s(-t), \quad (4)$$

by searching for the maximum of correlation between $\chi_r(t)$ and $\chi_{af}(t, \eta)$:

$$\eta = \underset{\eta}{\operatorname{argmax}} [\chi_r(t) * \chi_{af}(-t, \eta)]. \quad (5)$$

This operation requires to compute convolutions for a large number of relative velocity and is consequently computationally expensive.

B. Doppler cross-power spectrum

Instead of correlating the match-filtered signal with the ambiguity function for various relative velocity, it is preferable to extract the velocity directly from the signal. More precisely, the velocity can be extracted from the Doppler cross-power spectrum:

$$\operatorname{FT}\{\chi_r(t)\} = \operatorname{FT}\{r(t)\} \times \operatorname{FT}\{s(t)\}^* = R(f) \times S^*(f), \quad (6)$$

where $FT\{\cdot\}$ means the Fourier transform.

To derive the Doppler cross-power spectrum, the Fourier transform of the emitted LFM signal is first calculated:

$$\begin{aligned} S(f) &= FT\{s(t)\} \\ &= \int_{-\infty}^{\infty} w(t) \exp \left[i2\pi \left(f_0 t + \frac{\alpha}{2} t^2 - ft \right) \right] dt. \end{aligned} \quad (7)$$

Inside the integral, the exponential term is varying much faster than the smooth function $w(t)$. This kind of integral can be evaluated by the asymptotic approximation of stationary phase [1]:

$$\int_{-\infty}^{\infty} w(t) \exp [i\phi(t)] dt = w(t_{st}) \sqrt{\frac{i2\pi}{\phi''(t_{st})}} \exp [i\phi(t_{st})]. \quad (8)$$

where t_{st} is the time of stationary phase where $\phi'(t) = 0$. The phase term in 7, its derivative and its second derivative are:

$$\begin{aligned} \phi(t) &= 2\pi \left(f_0 t + \frac{\alpha}{2} t^2 - ft \right), \\ \phi'(t) &= 2\pi (\alpha t + f_0 - f), \\ \phi''(t) &= 2\pi \alpha, \end{aligned} \quad (9)$$

One can see that the phase is stationary at $t = (f - f_0)/\alpha$. Using 8, the equation 7 results in:

$$S(f) = w \left(\frac{f - f_0}{\alpha} \right) \sqrt{\frac{i}{\alpha}} \exp \left[-i\pi \frac{(f - f_0)^2}{\alpha} \right]. \quad (10)$$

To calculate the Fourier transform of the received signal, one can make the same calculation with the stationary phase at $t = (f/\eta - f_0)/(\eta\alpha)$, or use the Fourier transform property $FT\{s(\eta t)\}(f) = 1/\eta S(f/\eta)$:

$$\begin{aligned} R(f) &= \frac{A}{\eta} w \left(\frac{f/\eta - f_0}{\alpha} \right) \sqrt{\frac{i}{\alpha}} \\ &\times \exp \left[-i\pi \frac{(f/\eta - f_0)^2}{\alpha} - i2\pi f \Delta t \right]. \end{aligned} \quad (11)$$

$S(f)$ has a wideband spectrum. So it is not possible to extract the velocity information by looking at the spectrum shift in frequency. But one can notice the Doppler phase dependence by looking at the argument of the Doppler cross-power spectrum:

$$\arg [R(f) \times S^*(f)] = \pi \frac{(f - f_0)^2}{\alpha} - 2\pi f \Delta t - \pi \frac{(f/\eta - f_0)^2}{\alpha}, \quad (12)$$

which is a quadratic polynomial:

$$\begin{aligned} \arg [R(f) \times S^*(f)] &= p_2 f^2 + p_1 f + p_0, \text{ with:} \\ p_2 &= \frac{\pi}{\alpha} \left(1 - \frac{1}{\eta^2} \right), \\ p_1 &= -\frac{2\pi}{\alpha} f_0 \left(1 - \frac{1}{\eta} \right) - 2\pi \Delta t, \\ p_0 &= 0. \end{aligned} \quad (13)$$

The coefficient p_2 depends only on fixed parameters and the relative velocity. Note that the coefficient p_1 depends on the time delay Δt plus an additional delay $f_0 \alpha^{-1} (1 - 1/\eta)$ due to the Doppler effect. Making the approximations $\eta^2 \approx 1 + 2v/c$

and $(1 + 2v/c)(1 - 2v/c) \approx 1$, the relative velocity can be written as a simple function of the quadratic term coefficient:

$$v \approx \frac{p_2 \alpha c}{2\pi}. \quad (14)$$

For the active case (a fixed monostatic emitter/receiver and a moving target), we have:

$$\eta = \frac{1 + v/c}{1 - v/c} \approx 1 + 2v/c. \quad (15)$$

In that case, it is straightforward that equation 14 simply has to be divided by 2 to obtain the target relative speed ($v \approx p_2 \alpha c / (4\pi)$).

C. Implementation

To extract the coefficient p_2 and deduce v , the following algorithm is proposed.

- 1) Perform the match filtering $\chi_r(t) = r(t) * s(-t)$.
- 2) Make an automatic detection of the compressed pulse of interest.
- 3) Select a small portion of the signal that contains the pulse.
- 4) Compensate the pulse delay to center it around $t = 0$.
- 5) Compute the fast Fourier transform.
- 6) Unwrap the spectral phase.
- 7) Fit the unwrapped phase with a quadratic polynomial.
- 8) Calculate v using equation 14.

Unwrapping the spectral phase is generally a delicate operation. This is why step 4 consists in delaying the selected pulse such that the spectral-phase variation is minimized.

III. EXAMPLE

For the example, a Gaussian function is chosen for the LFM chirp envelope $w(t)$:

$$w(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{t^2}{\sigma^2} \right], \quad (16)$$

where $\sigma = 125$ ms. The frequency is modulated around $f_0 = 1/2(f_{max} + f_{min})$ with $\alpha = (f_{max} - f_{min})/(4\sigma)$. Frequencies $f_{min} = 1$ kHz and $f_{max} = 10$ kHz are the instant frequencies at $t = \mp 2\sigma$. The wave speed is set to the water sound speed $c = 1500$ m/s. A single pulse in the received signal is considered with a time delay $\Delta t = 200$ ms. Match-filtered signals for $v = -2, 0$ and 2 m/s are plotted in Fig. 1. The effect of $v \neq 0$ on the simulated signals shows a lowered maximum amplitude, a time spreading with a varying instant frequency, and a shifted maximum peak time of the envelope. The additional delay corresponding to the Doppler effect $f_0 \alpha^{-1} (1 - 1/\eta) = \pm 0.2$ ms for $v = \mp 2$ m/s is visible in Fig. 1a and 1c.

In practice the match-filtered signal may be composed of multiple echoes. Taking the application example of an at sea seismic measurement with a hydrophone moored on the seafloor and a towed source, the pulse corresponding to the direct path between source and receiver must be distinct from

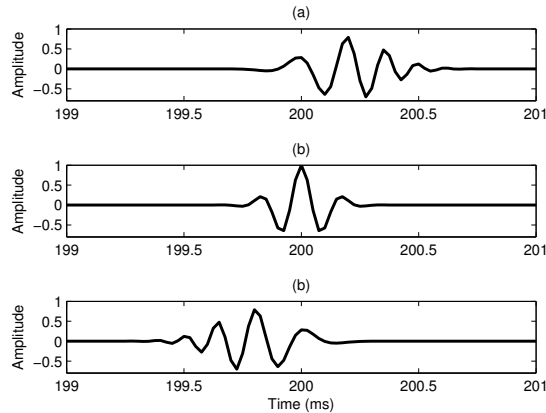


Fig. 1: Simulated match-filtered signal for $v = -2$ m/s (a), 0 m/s (b) and 2 m/s (c)

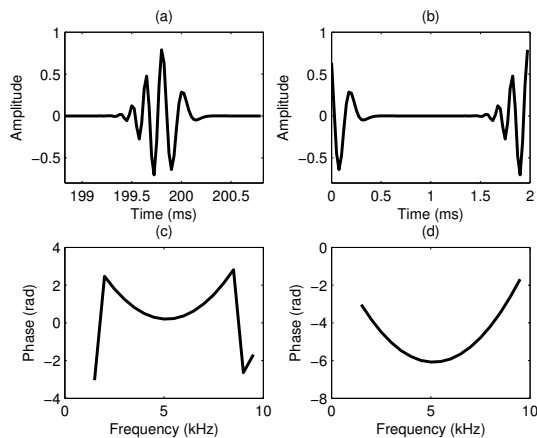


Fig. 2: (a) Pulse extracted from the simulated signal. (b) Pulse with delay compensation. (c) Phase of the pulse Fourier transform. (d) Unwrapped phase.

the seafloor or sea-surface reflection such that they do not overlap. Thus the pulse can be detected by a peak detection and the signal portion containing that pulse can be extracted. The selected portion of the simulated pulse with $v = 2$ m/s is displayed in Fig. 2a. The pulse appears in the middle of the selected portion that is considered as a delay by the fast Fourier transform algorithm (FFT). The consequence is a rapid phase change as a function of frequency and possible difficulties to unwrap the phase. This is overcome by swapping left and right signal portion halves (Fig. 2b). Then the phase can be taken from the FFT for frequencies between f_{min} and f_{max} (Fig. 2c) and be unwrapped (Fig. 2d). The final step consists in fitting the unwrapped phase with a quadratic polynomial to obtain its coefficient p_2 and calculate v .

IV. CONCLUSION

A method has been described to measure radial relative velocity between a source and a receiver or between an active system and a target when using a LFM chirp source signal. It has a low computational cost and operates on match-filtered signal if the echo of interest does not overlap another one. It has been demonstrated that the Doppler velocity is

proportional to the quadratic coefficient of the Doppler cross-power-spectrum unwrapped phase. Thus there is no need to calculate various correlations to find the best match between the match-filtered signal and the ambiguity function.

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