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Inverse skull conductivity estimation problems from EEG data

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Introduction

A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [2] and serves for clinical (location of epilepsy foci) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly by the conductivity of the skull. The human skull is a bony tissue consisting of compact and spongy bone compartments, whose shape and size vary over the age and the individual’s anatomy making difficult to accurately model the skull conductivity.

1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:
\[ \nabla \times \mathbf{E} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{J} = 0 \]

for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, deduce that \( \mathbf{E} = -\nabla U \), while dividing the current density \( \mathbf{J} \) into the ohmic current \( \mathbf{J}^T \) and the source current \( \mathbf{J}^S \). The second equation \( \nabla \cdot \mathbf{J} = 0 \) leads to our general model for the electric potential \( U \) in terms of conductivity Poisson equation with source term in divergence form:
\[ \nabla \cdot \sigma \nabla U = \mathbf{J}^S, \quad \text{in} \quad \Omega \]

where \( \sigma \) is the real valued (isotropic assumption) conductivity of the medium at location \( r \). Modeling the primary current \( \mathbf{J}^S \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:
\[ \nabla \cdot \sigma \nabla U = \sum_{i=1}^{Q} \mathbf{p}_i \cdot \nabla U_i, \quad \text{in} \quad \Omega \]

where \( \mathbf{p}_i \) is the moment of the source and \( U_i \) is the Dirac distribution with mass at \( \mathbf{p}_i \).

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:
\[ U_i = \text{EEG} \text{ pointwise values on } S_i \text{ at electrode locations } \] 

\( \partial U_i = 0 \), no current flux outside the head

while the source term is assumed to be already estimated, with the solution \( U_i \) in \( \Omega_i \), being expressed as the convolution of the source term \( \mathbf{J}^S(r) \) with the fundamental solution (Green formula):
\[ U_i(r) = \frac{1}{4\pi} \int_{S_i} \nabla U_i(r') \cdot (r-r') \, ds_i, \quad r \in \Omega_i \]

The source activity \( U_i \) and the EEG data \( \text{EEG} \) are expanded on spherical harmonics basis:
\[ \text{EEG} = \sum_{k=0}^{\infty} \sum_{m=-k}^{k} \xi_{km} \mathbf{Y}_{km}(\theta, \phi), \quad r \in \partial \Omega_i \]

\[ U_i = \sum_{k=0}^{\infty} \sum_{m=-k}^{k} U_{km} \mathbf{Y}_{km}(\theta, \phi), \quad r \in \partial \Omega_i \]

with the later being transmitted over the spheres \( S_i \) with the boundary conditions:
\[ U_i = U_i \text{ on } S_i \]

\[ \sigma U_i = \sigma U_i \text{ on } S_i \]

Numerical conductivity estimation results are shown in Fig. 1, 2, 3, where the mean value \( \sigma_{est} \) of the estimated conductivity is the one to be compared with the actual conductivity value \( \sigma_{act} \).

2. Mathematical formulation: Simplified model

We consider the inverse skull conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:

- In a three layer spherical head model
- Made of three concentric nested spheres, each of them modeling the scalp \( \Omega_3 \), skull \( \Omega_2 \) and brain \( \Omega_1 \) tissues
- The head is assumed to be piecewise homogeneous, each of the three layers having a constant conductivity.
- The sources \( \mathbf{J}^S \) are modelled as dipolar sources:
\[ \mathbf{J}^S = \sum_{i=1}^{Q} \mathbf{p}_i \cdot \nabla U_i, \quad C_i \subset \Omega_i \]

in each domain \( \Omega_i \), the electric potential satisfies the following equations:
\[ \nabla \cdot (\sigma_i \nabla U_i) = \mathbf{J}^S_i \]

\[ \nabla \cdot (\sigma_i \nabla U_i) = 0 \text{ in } \Omega_i \]

with \( U_1, U_2, U_3 \) being the solution in \( \Omega_i \).

Such we also assume that the conductivities of the brain \( \sigma_1 \) and the scalp \( \sigma_2 \) are known (currently \( \sigma_2 = \sigma_3 \)), while the conductivity to be recovered is the one of the intermediate spherical layer, the skull \( \sigma_3 \).

4. Uniqueness properties and reconstruction algorithm

Linear algebraic computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_3 \). The data transmission \( U_i \) from a spherical interface \( S_i \) to a neighbouring spherical interface \( S_{i+1} \) can be expressed by the following general matrix equation:
\[ T_2(r, \sigma_1, \sigma_2) \cdot \mathbf{Y}_{km} = 0 \]

as \( \sigma_3 \) is a unique solution of \( \nabla \cdot (\sigma_3 \nabla U_3) = 0 \) if the inverse transmission matrix \( T_2^{-1}(r, \sigma_1) \) is also defined. Computing the data transmission over the several spherical interfaces the spherical harmonics coefficients of the EEG measurements \( \mathbf{g}_{km} \) can be linked to the spherical harmonic coefficients of the source term \( \mathbf{J}^S \) as:
\[ \mathbf{J}^S = \sum_{k=0}^{\infty} \sum_{m=-k}^{k} J_{km} \mathbf{Y}_{km}(\theta, \phi) \]

where \( J_{km} \) are the admissible unique \( \sigma_3 \) which satisfy the constraint \( 0 < \sigma_3 < \min(\sigma_1, \sigma_2) \) and \( \mathbf{g}_{km} \) achieving its minimal value.

As a reconstruction of the conductivity \( \sigma_3 \) does not depend on the spherical harmonics index \( m \), in order to increase the robustness of our reconstruction algorithm, the following normalization is applied over the different spherical harmonics index \( k \):
\[ \sigma_3 = \sum_{m=0}^{\infty} \sigma_{km} \mathbf{Y}_{km}(\theta, \phi) \]

5. Computational algorithm and improvements

We performed numerical analysis of the inverse conductivity estimation problem, using measurements and sources activities expanded on spherical harmonics basis \( (\xi_{km}, \mathbf{g}_{km}) \) simulated by the FieldTrip3D (F3D) [4] software, while our simulations were performed in MATLAB.

The EEG data are subject to ambient noise and measurements errors, while the estimation of the sources is not perfect. In our simulation, the inverse conductivity estimation problem is sensitive to such perturbations, forcing us to decrease the tolerance of our reconstruction algorithm to \( 10^{-2} \) to \( \sigma_3 \). As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still quite well estimated with a small number of them.

6. Further work

- Stability properties and error estimates of the inverse problem.
- Robustness analysis of the recovery algorithm and dependence on the number of sources.
- Simultaneous recovery of source term and skull conductivity. First, stop with known quantity of sources \( Q \) and locations \( C_i \).
- Influence of the known parameters of the problem on the estimation.
- Modelling the spongiosa layer and estimating its conductivity.
- Comparison of results with more realistic head models and spongiosa layer: joint work in progress.
- Conductivity estimations using additional magnetoencephalography data.

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