Inverse skull conductivity estimation problems from EEG data
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1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell's equations imply:

\[ \nabla \times \mathbf{H} = \mathbf{J} \quad \text{with} \quad \mathbf{J} = \sigma \mathbf{E} \]

for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, deduces that \( \mathbf{E} = -\nabla \psi \), while dividing the current density \( \mathbf{J} \) into the ohmic current \( \sigma \mathbf{E} \) and the source current (also known as primary current) \( \mathbf{J}^s \) as \( \mathbf{J} = \sigma \mathbf{E} + \mathbf{J}^s \). This leads to our general model for the electric potential \( U \) in terms of conductivity \( \sigma \) and source term in divergence form:

\[ \nabla \cdot (\sigma \nabla U) = -\nabla \cdot \mathbf{J}^s \]

where \( \sigma \) is the real valued (isotropic assumption) conductivity of the medium at location \( r \). Modeling the primary current \( \mathbf{J}^s \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:

\[ \nabla \cdot (\sigma \nabla J^s) = \nabla \cdot \mathbf{J}^s = \sum_{q=1}^{Q} \mathbf{p}_q \times \nabla \mathbf{C}_k^r \quad \text{in} \quad \Omega \]

where \( \mathbf{p}_q \) is the moment of the source, and \( C_k^r \) is the Dirac distribution with mass at \( C_k^r \).

2. Mathematical formulation: Simplified model

We consider the inverse conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:

- In a three layer spherical model
- Made of three concentric neutral spheres, each of them modeling the scalp \( \Omega_0 \), skull \( \Omega_1 \), and brain \( \Omega_2 \) tissues.
- The head is assumed to be piecewise homogeneous: each of the three layers is supposed to have a constant conductivity \( \sigma = \sigma_1 \) in \( \Omega_1 \) and \( \sigma > \sigma_1 \) in \( \Omega_0 \) and \( \Omega_2 \).
- The sources \( S \) are modelled as dipolar sources \( \mathbf{J}^s = \sum_{q=1}^{Q} \mathbf{p}_q \times \nabla \mathbf{C}_k^r \) \( \in \Omega_0 \) in each domain \( \Omega_i \), the electric potential satisfies the following equations:

\[ \sigma_r \Delta U^r = -\mathbf{J}^s \quad \text{in} \quad \Omega_i \]

\[ \Delta U^r = 0 \quad \text{on} \quad \partial \Omega_i \]

with \( U_0, U_1, U_2 \) being the solution in \( \Omega_0, \Omega_1, \Omega_2 \), respectively.

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data: \( \{ U_i \} \), implying:

\[ \nabla \cdot (\sigma \nabla U_i) = \nabla \cdot \mathbf{J}^s \quad \text{on} \quad \partial \Omega_i \]

The source activity \( U_i \) and the EEG data \( \mathbf{J}^s \) are expanded on spherical harmonics basis:

\[ U_i(r) = \sum_{q=1}^{Q} \mathbf{p}_q \times \nabla \mathbf{C}_k^r \quad \text{on} \quad \partial \Omega_i \]

\[ \mathbf{J}^s(r) = \sum_{k=1}^{n} \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \mathbf{A}^{k \ell m}_{\text{dip}} Y_{\ell m}(\theta, \phi) \quad \text{on} \quad \partial \Omega_i \]

with the 

- \( \mathbf{A}^{k \ell m}_{\text{dip}} \) being the coefficients of the \( \mathbf{p}_q \times \nabla \mathbf{C}_k^r \) on \( \partial \Omega_i \), being expressed in terms of conductivity with dependences:

\[ \sigma_r \Delta U^r = -\mathbf{J}^s \quad \text{in} \quad \Omega_i \]

\[ \Delta U^r = 0 \quad \text{on} \quad \partial \Omega_i \]

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_1 \). The data transmission \( \{ U_i \} \) from a spherical interface \( S_i \) to a neighbouring spherical interface \( S_{i+1} \) can be expressed by the following general linear matrix equation:

\[ \mathbf{T}_{i+1} \mathbf{U}_{i+1} = \mathbf{T}_i \mathbf{U}_i \]

where \( \mathbf{T}_i \) is the transmission matrix. The data transmission over the whole sphere satisfies the spherical harmonics coefficients of the EEG measurements \( \mathbf{g}_{\text{EEG}} \) can be linked to the spherical harmonic coefficients of the source term \( \mathbf{A}^{\text{dip}} \) as:

\[ \mathbf{g}_{\text{EEG}} = \sum_{k=1}^{n} \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \mathbf{B}^{k \ell m}_{\text{dip}} Y_{\ell m}(\theta, \phi) \]

\[ \mathbf{B}^{k \ell m}_{\text{dip}} = \int_{S} \mathbf{C}_k^r \times \mathbf{A}^{k \ell m}_{\text{dip}} Y_{\ell m}(\theta, \phi) dS \]

Solving this equation in terms of \( \mathbf{A}_{\text{dip}} \) leads to a polynomial equation \( P_{\text{dip}}(\sigma_1) = 0 \) of degree \( P \), \( \leq 2n \) in \( \sigma_1 \), \( \sigma_2 \), \( \sigma_3 \). Let \( \sigma_{\text{est}} \) be the one of the two roots of the polynomial \( P_{\text{dip}}(\sigma_1) \) for the spherical harmonic basis.

The unique admissible solution \( \sigma_{\text{est}} \), is the solution which satisfies the constraint \( 0 < \sigma_{\text{est}} < \min(\sigma_0, \sigma_2) \).

At the reconstruction of the conductivity \( \sigma_1 \) does not depend on the spherical harmonics index \( m \), in order to increase the robustness of our reconstruction algorithm, the following normalisation is applied over the different spherical harmonics index \( k, \ell, m \):

\[ \sigma_{\text{est}} = \frac{\sum_{k=1}^{n} \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} |\mathbf{B}^{k \ell m}_{\text{dip}}|^2}{\sum_{k=1}^{n} \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} |\mathbf{A}^{k \ell m}_{\text{dip}}|^2} \]

5. Computational algorithm and improvements

We performed numerical analysis of the inverse conductivity estimation problem, using measurements and sources activities expanded on spherical harmonics basis \( \{ \mathbf{g}_{\text{EEG}}, \mathbf{g}_{\text{Dip}} \} \) simulated by the FindSources3D (FS3D) [4] software, while all simulations were performed in MATLAB.

The EEG data are subject to ambient noise and measurements errors, while the estimation of the sources is not perfect. In our simulation, the inverse conductivity estimation problem is sensitive to such perturbations, forcing us to decrease the tolerance of our reconstruction algorithm to \( \text{tol} = 10^{-4} \). As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still quite well estimated with a small number of them.

Numerical conductivity estimation results are shown in Fig. 1. 2. 3, where the mean value \( \sigma_{\text{est}} \) of the estimated \( \sigma_{\text{est}} \) is the one to be compared with the actual conductivity value \( \sigma_{\text{true}} \).

6. Further work

- Stability properties and error estimates of the inverse problem.
- Robustness analysis of the recovery algorithm and dependence on the number of sources.
- Simultaneous recovery of source term and skull conductivity. First, stop with known quantity of sources \( Q \) and locations \( C_j \).
- Influence of the known parameters of the problem on the estimation.
- Modeling the spongiosa layer and estimating its conductivity.
- Comparison of results with more realistic head models and spongiosa layer: joint work in progress.
- Conductivity estimations using additional magnetoencephalography data.

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References


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