Inverse skull conductivity estimation problems from EEG data
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Introduction

A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [1] and serves for clinical (location of epilepsy foci) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly by the conductivity of the skull. The human skull is a bony tissue consisting of compact and spongy bone compartments, whose shape and size vary over the age and the individual’s anatomy making difficult to accurately model the skull conductivity.

1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:

\[ \nabla \times (\sigma \nabla \times \mathbf{E}) = \nabla \times \mathbf{J} \]

\[ \nabla \cdot \mathbf{E} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{J} = 0 \]

for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, deduces that \( \mathbf{E} = -\nabla \mathbf{U} \), while dividing the current density \( \mathbf{J} \) into the ohmic current \( \mathbf{J}^o \) and the source current (also called primary current) \( \mathbf{J}^p \) as \( \mathbf{J} = \mathbf{J}^o + \mathbf{J}^p \) leads to our general model for the electric potential \( \mathbf{U} \) in terms of conductivity Poisson equation with source term in divergence form:

\[ \nabla \cdot (\sigma \nabla \mathbf{U}) = \nabla \cdot \mathbf{J} = 0 \]

where \( \sigma \) is the real valued (isotropic assumption) conductivity of the medium at location \( r \). The source activity \( \mathbf{J}^p \) is the convolution of the source term \( \beta \) with the fundamental solution (Green formula):

\[ \mathbf{J}^p(r) = \int_{\Sigma} \beta(r') G(r,r') \, dS(r') \]

where \( \beta \) is the moment of the source and \( C_i \) is the Dirac distribution with mass at \( C_i \).

\[ \nabla \cdot \mathbf{U} = 0 \quad \text{and} \quad \nabla \times \mathbf{U} = \mathbf{0} \]

\[ \nabla \cdot \mathbf{J} = 0 \]

\[ \nabla \sim \text{grad} \quad \nabla \sim \text{div} \quad \nabla \sim \text{curl} \]

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:

\[ \mathbf{U}_L = \mathbf{E}_{\Sigma}, \quad \mathbf{U}_R = 0 \quad \text{and} \quad \nabla \cdot \mathbf{J}_L = 0 \]

where the source term is assumed to be already estimated, with the solution \( \mathbf{U}_R \) in \( \Omega_o \), being expressed as the convolution of the source term \( \beta(r) \) with the fundamental solution (Green formula):

\[ \mathbf{U}_R(r) = \int_{\Sigma} \beta(r') G(r,r') \, dS(r') \]

The source activity \( \mathbf{U}_L \) and the EEG data \( \mathbf{E}_{\Sigma} \) are expanded on spherical harmonics basis:

\[ \mathbf{E}_{\Sigma} = \sum_{k} \sum_{s} \mathbf{U}_{L,R,k,s} \mathbf{Y}_{k,s}(\Omega_o) \quad \beta = \sum_{k} \sum_{s} \beta_{k,s} \mathbf{Y}_{k,s}(\Omega_o) \]

The source activity \( \mathbf{U}_L \) and the EEG data \( \mathbf{E}_{\Sigma} \) are transmitted through the sources \( S_j \) and the boundary conditions:

\[ \mathbf{U}_L = \mathbf{U}_R = 0 \quad \text{on} \quad S_i \quad \text{and} \quad \mathbf{U}_L = \mathbf{U}_R = 0 \quad \text{on} \quad S_j \]

5. Computational algorithm and improvements

We performed numerical analysis of the inverse conductivity estimation problem, using measurements and sources activities expanded on spherical harmonics basis \( \mathbf{u}_{l,k,s} \) simulated by the FindSources3D (F3SD) [4] software, while our simulations were performed in MATLAB.

The EEG data are subject to ambient noise and measurements errors, while the estimation of the sources is not perfect. In our simulation, the inverse conductivity estimation problem is sensitive to such perturbations, forcing us to decrease the tolerance of our reconstruction algorithm to \( tol = 5 \times 10^{-5} \). As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still quite well estimated with a small number of them.

Numerical conductivity estimation results are shown in Fig. 1, 2, 3, where the mean value \( \bar{\sigma}_{well} \) of the estimated conductivity is the one to be compared with the actual conductivity value \( \bar{\sigma}_{true} \). The conductivity estimation errors for the three used source terms are shown in Fig. 1, 2, 3, where the mean value \( \bar{\sigma}_{well} \) is the one to be compared with the actual conductivity value \( \bar{\sigma}_{true} \).

2. Mathematical formulation: Simplified model

We consider the inverse conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:

- In a three layer spherical head model
- Made of three concentric nooted spheres, each of them modeling the scalp \( \Omega_S \), skull \( \Omega_k \) and brain \( \Omega_r \) tissues
- The head is assumed to be piecewise homogenous: each of the three layers is supposed to have a constant conductivity

\[ \bar{\sigma}_r \leq \bar{\sigma}_k \leq \bar{\sigma}_S \]

The sources \( C_i \) are modeled as dipolar sources \( \mathbf{J}^p = \sum_{i} \mathbf{p}_i \nabla \mathbf{c}_i \), \( \mathbf{c}_i \in \Omega_k \)

In each domain \( \Omega_d \), the electric potential satisfies the following equations:

\[ \mathbf{A} \mathbf{U} = \mathbf{V} \quad \text{in} \quad \Omega_d \]

\[ \mathbf{A} = \nabla \mathbf{A} = 0 \quad \text{in} \quad \Omega_{k,r} \]

\[ \mathbf{A} = \nabla \mathbf{A} = 0 \quad \text{in} \quad \Omega_{k,r} \]

with \( \mathbf{U}_L, \mathbf{U}_R, \mathbf{U}_L \) being the solution in \( \Omega_d \).

We also assume that the conductivities of the brain \( \bar{\sigma}_r \), and the scalp \( \sigma_s \) are known (current \( \bar{\sigma}_r \approx \bar{\sigma}_s \), while the conductivity to be recovered is the one of the intermediate spherical layer, the skull \( \bar{\sigma}_r \).

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \bar{\sigma}_r \). The data transmission \( \mathbf{U}_L \) from a spherical interface \( S_i \) to a neighbouring spherical interface \( S_j \) can be expressed by the following general matrix equation:

\[ \mathbf{T}_{d}(r,s) = \int_{S_i} \mathbf{U}_L(r') \mathbf{G}(r,s) \, dS(r') \]

\[ r \neq s \quad \mathbf{T}_{d}(r,s) = \mathbf{0} \]

Solving this equation in terms of \( \bar{\sigma}_r \) leads to a polynomial equation \( P_{\bar{\sigma}_r} = 0 \) of degree \( P_{\bar{\sigma}_r} = 2 \) in \( \bar{\sigma}_r \), with dependences \( r = \mathbf{P}_{\bar{\sigma}_r}(r) \).

Let \( \bar{\sigma}_r \) be the one of the two roots of the polynomial \( P_{\bar{\sigma}_r} \) for the \( \bar{\sigma}_r \) spherical harmonic basis. The unique admissible solution \( \bar{\sigma}_r \) is the solution which satisfies the constraint \( 0 < \bar{\sigma}_r \leq \bar{\sigma}_{well} \), being expressed as a minimal polynomial \( F(\bar{\sigma}_r) = 0 \), up to a tolerance value tol.

As the reconstruction of the conductivity \( \bar{\sigma}_r \) does not depend on the spherical harmonics index \( m \), in order to increase the robustness of our reconstruction algorithm, the following normalization is applied over the different spherical harmonics indices \( k \):

\[ \bar{\sigma}_l = \frac{1}{\bar{\sigma}_l} \sum_{k} \bar{\sigma}_{l,k,s} = \sum_{k} \frac{\bar{\sigma}_{l,k,s}}{\bar{\sigma}_{l,k,s}} \]

6. Further work

- Stability properties and error estimates of the inverse problem
- Robustness analysis of the recovery algorithm and dependence on the number of sources
- Simultaneous recovery of source term and skull conductivity
- Comparison of results with more realistic head models and spongya layer: joint work in progress
- Conductivity estimation using additional magnetoencephalography data

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References


