Inverse skull conductivity estimation problems from EEG data

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1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:

\[ \nabla \cdot \mathbf{E} = 0 \quad \text{and} \quad \nabla \times \mathbf{H} = \mathbf{J} \]

for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, deducing that \( \mathbf{E} = -\nabla \varphi \), while dividing the current density \( \mathbf{J} \) into the ohmic current \( \mathbf{J}_o \) and the source current (also called primary current) \( \mathbf{J}_s \) as \( \mathbf{J} = \mathbf{J}_o + \mathbf{J}_s \), leads to our general model for the electric potential \( U \) in terms of conductivity \( \sigma \) equation with source term in divergence form:

\[ \nabla \cdot (\sigma \nabla U) = -\nabla \cdot \mathbf{J}_s \]

where \( \sigma \) is the real valued (isotropic assumption) conductivity of the medium at location \( r \).

Modeling the primary current \( \mathbf{J}_s \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:

\[ \nabla \cdot (\sigma \nabla U) = -\sum_{\mathbf{J}_s} \mathbf{p}_s \cdot \nabla \mathbf{C}_s, \quad \mathbf{C}_s \in \mathbb{R}^3 \]

where \( \mathbf{p}_s \) is the moment of the source and \( \mathbf{C}_s \) is the Dirac distribution mass with mass at \( \mathbf{C}_s \).

\[ \nabla \sim \text{grad,} \quad \nabla \cdot \sim \text{div,} \quad \nabla \times \sim \text{curl} \]

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:

\[ U_t = \text{EEG,} \quad \text{pointwise values on} \quad \mathcal{S}_t \quad \text{at electrode locations} \]

\[ U_t, U_o \in \mathcal{O}_t, \quad \text{no current flux outside the head} \]

while the source term is assumed to be already estimated, with the solution \( U_t \) in \( \mathcal{O}_t \), being expressed as the convolution of the source term \( \mathbf{J}_s \) with the fundamental solution (Green formula):

\[ U_t(r) = \frac{1}{4\pi} \int_{\mathcal{S}_t} \frac{1}{|r - \mathbf{C}_s|} \mathbf{J}_s(\mathbf{C}_s) \, d\mathbf{S}_s \]

where \( \mathbf{C}_s \) are dipolar sources inside \( \mathcal{O}_t \) with source values on \( \mathcal{S}_t \).

The solution \( U_t \) and the EEG data \( U_{EEG} \) are expanded on spherical harmonics basis:

\[ U_t(r) = \sum_{k,m} U_t^{km} Y_{km}(\hat{r}), \quad r \in \mathcal{O}_t \subset \mathcal{O}_1 \]

\[ U_{EEG}(r) = \sum_{k,m} U_{EEG}^{km} Y_{km}(\hat{r}), \quad r \in \mathcal{S}_t \subset \mathcal{S}_1 \]

with the later being transmitted over the spheres \( \mathcal{S}_t, \mathcal{S}_1 \) with the boundary conditions:

\[ \begin{align*}
U_t^{km} &= U_{EEG}^{km} & \text{on} \quad \mathcal{S}_t \\
\sigma_m U_t^{km} &= \rho_m U_{EEG}^{km} & \text{on} \quad \mathcal{S}_t
\end{align*} \]

5. Computational algorithm and improvements

We performed numerical analysis of the inverse conductivity estimation problem, using measurements and sources activities expanded on spherical harmonics basis \((g_{km}, h_{km})\) simulated by the Finess3D (FS3D) [4] software, while our simulations were performed in MATLAB.

The EEG data are subject to ambient noise and measurements errors, while the estimation of the sources is not perfect. In our simulation, the inverse conductivity estimation problem is sensitive to such perturbations, forcing us to decrease the tolerance of our reconstruction algorithm to \( 10^{-4} - 10^{-6} \). As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still quite well estimated with a small number of them.

Numerical conductivity estimation results are shown in Fig. 1-3, 2, where the mean value \( \bar{\sigma}_{RMSE} \) of the estimated \( \sigma_{RMSE} \) is the one to be compared with the actual conductivity value \( \sigma_{RMSE} \).

2. Mathematical formulation: Simplified model

We consider the inverse conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:

\[ \begin{align*} &\text{in a three layer spherical head model} \\
&\text{Made of three concentric nested spheres, each of them modeling the} \\
&\text{skull \( \Omega_3 \), skull \( \Omega_2 \) and brain \( \Omega_1 \) tissue.} \\
&\text{The head is assumed to be piecewise homogenous: each of the three layers has a constant} \\
&\text{conductivity \( \sigma_0 \), \( 0 < \sigma_0 < \min(\sigma_{\Omega_1}, \sigma_{\Omega_2}) \).} \\
&\text{The sources \( C \) are modeled as dipolar} \]

\[ \sigma_1(r) = \left\{ \begin{array}{ll}
\sigma_0 & \text{in} \quad \Omega_3 \cup \Omega_2 \\
\sigma_1 & \text{in} \quad \Omega_1
\end{array} \right. \]

In each domain \( \Omega_i \), the potential satisfies the following equations:

\[ \sigma_i \nabla^2 U_i(r) - \nabla U_i(r) = 0 \quad \text{in} \quad \Omega_i \]

with \( U_t, U_0, U_3 \) being the solution in \( \Omega_t, \Omega_0, \Omega_3 \), respectively.

We also assume that the conductivities of the brain \( \sigma_1 \) and the scalp \( \sigma_2 \) are known (currently \( \sigma_0 = \sigma_{\Omega_1} \)), while the conductivity to be recovered is in the one of the intermediate spherical layer, the skull \( \sigma_2 \).

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_2 \). The data transmission \( \{ U_t(r) \} \) from a spherical interface \( \mathcal{S}_t \) to a neighbouring spherical interface \( \mathcal{S}_1 \) can be expressed by the following general matrix equation:

\[ T_{\mathcal{S}_1}[r, \sigma] = 0 \quad \text{with} \quad \mathcal{S}_1 \subset \mathcal{S}_t \]

where \( T_{\mathcal{S}_1} \) is a \( n \times m \) matrix where \( n \) is the number of equations and \( m \) the number of unknowns.

As \( \sigma_i(r) \neq \sigma_0 \), the inverse transmission matrix \( T_{\mathcal{S}_1}[r, \sigma] \) is also defined. Computing the data transmission over the spherical interfaces the spherical harmonics coefficients of the EEG measurements \( g_{km} \), can be linked to the spherical harmonics coefficients of the source term \( h_{km} \) as:

\[ \begin{align*}
\beta_{km} &= \left( \begin{array}{c}
\phi_{km}(1) \\
\phi_{km}(2)
\end{array} \right) \quad \text{on} \quad \mathcal{S}_t \\
\phi_{km}(4) &= \left( \begin{array}{c}
\phi_{km}(1) \\
\phi_{km}(2)
\end{array} \right) \quad \text{on} \quad \mathcal{S}_1
\end{align*} \]

Solving this equation in terms of \( \beta_{km} \), leads to a polynomial equation \( P_{\beta_{km}}(\alpha) = 0 \). Let \( \alpha_{km} \) be the one of the two roots of the polynomial \( P_{\beta_{km}} \) for the \( k \)-th spherical harmonic basis.

The unique admissible solution \( \sigma_2 \) is the solution which satisfies the constraint \( 0 < \alpha_{km} < \min(\sigma_{\Omega_1}, \sigma_{\Omega_2}) \) and \( \mathcal{S}_1 \) achieving a minimal Frobenius norm \( \|F\| = \sqrt{\sum_{km} \|\alpha_{km}\|^2} \) up to a tolerance value tol.

As the reconstruction of the conductivity \( \sigma_2 \) does not depend on the spherical harmonics index \( k \), in order to increase the robustness of our reconstruction algorithm, the following normalization is applied over different spherical harmonics indices \( k \):

\[ \begin{align*}
 \beta_k &= \sum_{m=1}^{M} \beta_{km} h_{km} \\
 \alpha_k &= \sum_{m=1}^{M} \alpha_{km} h_{km}
\end{align*} \]

6. Further work

- Stability properties and error estimates of the inverse problem.
- Robustness analysis of the recovery algorithm and dependence on the number of sources.
- Simultaneous recovery of source term and skull conductivity. First, stop with known quantity of sources \( Q \) and locations \( C_Q \).
- Influence of the known parameters of the problem on the estimation.
- Modeling the spongius layer and estimating its conductivity.
- Comparison of results with more realistic head models and spongius layer: joint work in progress.
- Conductivity estimations using additional magnetoencephalography data.

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References