Inverse skull conductivity estimation problems from EEG data
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Introduction

A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [9] and serves for clinical (location of epilepsy foci) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly by the conductivity of the skull. The human skull is a bony tissue consisting of compact and spongy bone compartments, whose shape and size vary over the age and the individual’s anatomy making difficult to accurately model the skull conductivity.

1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:

\[ \nabla \cdot \mathbf{E} = 0 \quad \text{and} \quad \nabla \times \mathbf{E} = 0 \]

for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, that dictates \( \mathbf{E} = -\nabla \phi \), while dividing the current density \( \mathbf{J} \) into the ionic current \( \mathbf{j}_i \) and the source current (also called primary current) \( \mathbf{j}_s \), leads to our general model for the electric potential \( U \) in terms of conductivity Poisson equation with source term in divergence form:

\[ \nabla \cdot (\sigma \nabla U) = \nabla \cdot \mathbf{J}_s = \nabla \cdot \mathbf{j}_s \]

where \( \sigma \) is the real valued (isotropic assumption) conductivity of the medium at location \( r \).

Modeling the primary current \( \mathbf{j}_s \) as the result of the superposition of \( Q \) point source dipolar sources, leads to:

\[ \nabla \cdot (\sigma \nabla U) = \sum_{i=1}^{Q} \mathbf{p}_i \cdot \nabla \delta(r-r_i) \quad \text{in } \mathbb{R}^3 \]

where \( \mathbf{p}_i \) is the moment of the source and \( \delta(r) \) is the Dirac distribution with mass at \( r \).

\[ \nabla \sim \text{grad}, \quad \nabla \times \sim \text{div}, \quad \nabla \sim \text{curl} \]

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:

\[ \mathbf{U}_t = \mathbf{E} |_{\partial D_t} \quad \text{and} \quad \mathbf{U}_b = \mathbf{E} |_{\partial D_b} \quad \text{where} \quad \partial D_t \cup \partial D_b \quad \text{is the whole boundary} \]

while the source term is assumed to be already estimated, with the solution \( \mathbf{U}_t \) in \( D_t \), being expressed as the convolution of the source term \( \mathbf{J}_s(r) \) with the fundamental solution (Green formula):

\[ \mathbf{U}_t = \frac{1}{4\pi} \int_{\Omega} \mathbf{J}_s(r) \delta(r-r') \ dV \quad \text{in } D_t \]

The source \( \mathbf{U}_t \) and the EEG data \( \mathbf{E} \) are expressed on spherical harmonics basis:

\[ \mathbf{E} = \sum_{k,m} E_{km} \mathbf{Y}_{km}(\theta, \phi) \quad \text{on } \Omega \]

3. Data, boundary conditions and expansions

where \( \mathbf{Y}_{km}(\theta, \phi) \) are the spherical harmonics of degree \( k \) and order \( m \).

The source \( \mathbf{U}_t \) and the EEG data \( \mathbf{E} \) are expanded on spherical harmonics basis:

\[ \mathbf{U}_t = \sum_{k,m} U_{km} \mathbf{Y}_{km}(\theta, \phi) \quad \text{on } \Omega \]

with the later being transmitted over the spheres \( S_j \) with the boundary conditions:

\[ \mathbf{U}_t |_{S_j} = \mathbf{U}_b |_{S_j} \quad \text{on } S_j \]

\[ \mathbf{n}_i \cdot \mathbf{U}_t - \mathbf{n}_i \cdot \mathbf{U}_b = \mathbf{n}_i \cdot \mathbf{J}_s \quad \text{on } S_i \]

5. Computational algorithm and improvements

We performed numerical analysis of the inverse conductivity estimation problem, using measurements and sources activities expanded on spherical harmonics basis (\( \mathbf{E}_{km}, \mathbf{U}_{km} \)) simulated by the FindSources3D (FSD3) [4] software, while our simulations were performed in MATLAB.

The EEG data are subject to ambient noise and measurements errors, while the estimate of the sources is not perfect. In our simulation, the inverse conductivity estimation problem is sensitive to such perturbations, forcing us to decrease the tolerance of our reconstruction algorithm to \( tol = 10^{-2} \). As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still quite well estimated with a small number of them.

Numerical conductivity estimation results are shown in Fig. 1, 2, 3, where the mean value \( \overline{\sigma}_{est} \) of the estimated \( \sigma_{est} \) is the one to be compared with the actual conductivity value \( \sigma_{ACT} \).

2. Mathematical formulation: Simplified model

We consider the inverse skull conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:

- In a three layered spherical head model
- Made of three concentric nested spheres, each of them modeling the skull \( D_S \), skull \( D_B \) and brain \( D_B \) tissues
- The head is assumed to be piecewise homogenous: each of the three layers is supposed to have a constant conductivity \( \sigma_i \), \( i \in \{0, 1, 2\} \) and the scalp \( \sigma_0 \) is known (currently \( \sigma_0 = \sigma_S \)), while the conductivity to be recovered is the one of the intermediate spherical layer, the skull \( \sigma_2 \).

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_2 \). The data transmission \( \mathbf{U}_t \) from a spherical interface \( S_j \) to a neighbouring spherical interface \( S_i \) can be expressed by the following general matrix equation:

\[ \mathbf{T}_{ij}(\sigma_2) = \mathbf{G}_{ij}(\sigma_2) - \sigma_0 \mathbf{G}_{ij}(\sigma_0) \]

Equation (1) is also defined. Computing the data transmission over the several spherical interfaces the spherical harmonics coefficients of the EEG measurements \( \mathbf{E}_{km} \) can be linked to the spherical harmonic coefficients of the source term \( \mathbf{J}_s \) as:

\[ \mathbf{E}_{km} = \sum_{i=1}^{Q} \mathbf{G}_{ki}(\sigma_2) \mathbf{J}_{ki} \]

Solving this equation in terms of \( \mathbf{J}_{ki} \), leads to a polynomial equation \( P(\sigma_2) = 0 \) of degree \( P \) in \( \sigma_2 \) with dependencies \( \mathbf{P} = \mathbf{P}(\mathbf{E}_{km}) \). Let \( \sigma_{min} \) be the one of the two roots of the polynomial \( P(\sigma_2) \) for the \( k \)-th spherical harmonic basis. The unique admissible solution \( \sigma_{ik} \) is the solution which satisfies the constraint \( 0 < \sigma_{ik} < \min(\sigma_{min}, \sigma_S) \) and make \( P(\sigma_{ik}) \) achieve its minimal value \( \left( F(\sigma_{ik}) \right) \), up to a tolerance value tol. As the reconstruction of the conductivity \( \sigma_2 \) does not depend on the spherical harmonic index \( m \) in order to increase the robustness of our reconstruction algorithm, the following normalization is applied over the different spherical harmonics index \( k \): \( \left( \sigma_{ik} = \sum_{m} \sigma_{ik,m} \right) \)

6. Further work

- Stability properties and error estimates of the inverse problem
- Robustness analysis of the recovery algorithm and dependence on the number of sources
- Simultaneous recovery of source term and skull conductivity. First, stop with known quantity of sources \( Q \) and locations \( C_{Q} \)
- Influence of the known parameters of the problem on the estimation
- Modeling the aperiodic layer and estimating its conductivity
- Comparison of results with more realistic head models and aperiodic layer: joint work in progress
- Conductivity estimations using additional magnetoencephalography data.

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