Inverse skull conductivity estimation problems from EEG data

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A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [9] and serves for clinical (location of epilepsy foci) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly by the conductivity of the skull. The human skull is a bony tissue consisting of compact and spongy bone compartments, whose shape and size vary over the age and the individual’s anatomy making difficult to accurately model the skull conductivity.

1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations implies:
\[
\nabla \times (\sigma \nabla \varphi) = J
\]
for the electric fields \(E\) and the current density \(J\). The first, deduces that \(E = -\nabla \varphi\), while dividing the current density \(J\) into the ohmic current \(\sigma \nabla \varphi\) and the source current (also called primary current) \(J^p\) as \(J = \sigma \nabla \varphi + J^p\) leads to our general model for the electric potential \(U\) in terms of conductivity Poisson equation with source term in divergence form:
\[
\nabla \cdot (\sigma \nabla U) = \nabla \cdot J^p \text{ in } \Omega \subset \mathbb{R}^3
\]
where \(\sigma|\in\mathbb{R}\) is the real valued (isotropic assumption) conductivity of the medium at location \(r\). Modeling the primary current \(J^p\) as the result of the superposition of \(Q\) pointwise dipolar sources, leads to:
\[
\nabla \cdot (\sigma \nabla U) = \sum_{q=1}^{Q} \beta_q \nabla \times (p_q \nabla \varphi) \text{ in } \mathbb{R}^3
\]
where \(p_q\) is the moment of the source and \(\varphi\) is the Dirac distribution with mass at \(\varphi\).

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:
\[
U_i = \text{EEG} |_{S_i} \text{ pointwise values on } S_i \text{ at electrode locations}
\]
\(U_i, \partial U_i/\partial n_i = 0 \forall n_i \text{ flux current out the head}\)

where the source term is assumed to be already estimated, with the solution \(U_i\) in \(\Omega_i\), being expressed as the convolution of the source term \(\varphi\) with the fundamental solution (Green formula):
\[
U_i = \int_{S_i} \left( \frac{1}{4\pi r} \varphi(r) \right) dS
\]
\(\varphi\) in \(\mathbb{R}^3\) with \(r = \sqrt{x^2 + y^2 + z^2}\), and \(-k \leq m \leq k\).

with the later being transmitted over the spheres \(S_i\), \(S_j\) with the boundary conditions:
\[
U_i = U_j \text{ on } S_i \cap S_j
\]
\(n_i/\partial U_i/\partial n_i = 0 \text{ on } S_i \cap S_j\)

Numerical conductivity estimation results are shown in Fig. 1, 2, 3, where the mean value \(\bar{\sigma}\) of the estimated \(\sigma_{est}\) is the one to be compared with the actual conductivity value \(\sigma_{true}\).

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \(\sigma_s\). The data transmission \(U_i, \partial U_i/\partial n_i \text{ from a neighbouring sphere interface } S_j\) to a sphere interface \(S_i\) can be expressed by the following general matrix equation:
\[
T_{ij} (r, \sigma) = \left[ \begin{array}{c} 1 \sigma \times (r \times (1 - \kappa)) \left( I_{3 	imes 3} - \frac{1}{\sigma} r r^{\top} \right) \left( 1 \sigma \times (r \times (1-k)) \right) \end{array} \right]_{0 \times 1} T_{ij}(r)
\]
where \(s = 2 \kappa + 1 - \kappa^2\). The data transmission over the several spherical interfaces the spherical harmonics coefficients of the EEG measurements \(g_{\text{meas}}\) can be linked to the spherical harmonic coefficients of the source term \(g_{\text{true}}\) as:
\[
\beta_{\text{true}} = 0 \quad \text{if } \sigma_{\text{true}} \text{ is spherical harmonic spherical harmonic basis function}
\]

Solving this equation in terms of \(\sigma_s\) leads to a polynomial equation \(P_{\sigma_s}(\beta_{\text{true}}) = 0\) of degree \(P_{\text{true}} = 2\) in \(\sigma_s\).

Let \(\sigma_{\text{true}}\) be the one of the two roots of the polynomial \(P_{\sigma_s}(\beta_{\text{true}})\) for the spherical harmonic basis function. The unique admissible solution \(\sigma_{\text{true}}\) is the solution which satisfies the constraint \(0 < \sigma_{\text{true}} < \min(\sigma_{\text{true}})\) and make \(P_{\text{true}}\) achieve its minimal value \(\min(P_{\text{true}})\), up to a tolerance value \(\varepsilon\).

As the reconstruction of the conductivity \(\sigma_s\) does not depend on the spherical harmonics index \(m\), in order to increase the robustness of our reconstruction algorithm, the following normalization is applied over the different spherical harmonics index \(k\):
\[
\beta_k = \sum_{a=1}^{K} \sum_{i=1}^{N} g_{i,a} \bar{\sigma}_{i,a} = \sum_{a=1}^{K} \sum_{i=1}^{N} g_{i,a} \beta_k
\]

6. Further work

- Stability properties and error estimates of the inverse problem.
- Robustness analysis of the recovery algorithm and dependence on the number of sources.
- Simultaneous recovery of source term and skull conductivity. First, stop with known quantity of sources \(Q\) and locations \(C_Q\). If the location of the source, \(\partial U_i/\partial n_i\) influence of the known parameters of the problem on the estimation.
- Modeling the spongiosa layer and estimating its conductivity.
- Comparison of results with more realistic head models and spongiosa layer: joint work in progress.
- Conductivity estimations using additional magnetoencephalography data.

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