Inverse skull conductivity estimation problems from EEG data

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Introduction

A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [1] and serves for clinical (location of epilepsy foci) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly to accurately model the skull conductivity.

1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:

\[ \nabla \times E = 0 \quad \text{and} \quad \nabla \cdot J = \sigma \nabla \times H = 0 \]

for the electric fields \( E \) and the current density \( J \). The first, deduces that \( E = -\nabla V \), while dividing the current density \( J \) into the electric current \( \sigma \) and the source current (also called primary current) \( J^p \), as \( J = -\nabla \Phi + J^p \). This leads to our general model for the electric potential \( U \) in terms of conductivity Poisson equation with source term in divergence form:

\[ \nabla \cdot (\sigma \nabla U) = -\nabla \cdot J^p \text{ in } \Omega \]

where \( \sigma \) is the electric conductivity of the medium at location \( r \).

Modelling the primary current \( J^p \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:

\[ \nabla \cdot (\sigma \nabla U) = -\sum_{i=1}^{Q} \mathbf{p}_i \cdot \nabla \Phi_i \text{ in } \Omega \]

where \( \mathbf{p}_i \) is the moment of the source and \( \Phi_i \) is the Dirac distribution with mass at \( \mathbf{p}_i \).

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:

\[ U_\partial = \mathbf{G}(\tilde{E}) \text{ pointwise values on } S_1 \text{ at electrode locations} \]

\( \partial U_\partial = 0 \), no current flux outside the head

while the source term is assumed to be already estimated, with the solution \( U_\partial \) in \( \Omega \), being expressed as the convolution of the source term \( \nabla \cdot J^p \) with the fundamental solution (Green formula):

\[ U_\partial(r) = \frac{1}{4\pi} \int_{S_1} \mathbf{G}(\tilde{E})(r) \cdot \mathbf{p}_i \cdot \nabla \Phi_i \text{ dS} \]

The source \( U_\partial \) and the EEG data \( \mathbf{G}(\tilde{E}) \) are expanded on spherical harmonics basis:

\[ \mathbf{G}(\tilde{E}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\tilde{E}), \quad \ell \in \mathcal{D} \]

The source term \( U_\partial \), with matrix elements

\[ U_{\partial}\ell_1\ell_2 = \frac{1}{4\pi} \int_{S_1} \mathbf{G}(\tilde{E})(r) Y_{\ell_1 m_1}(\tilde{E}) \cdot \mathbf{p}_i \cdot \nabla \Phi_{\ell_2 m_2}(\tilde{E}) \text{ dS} \]

with the latter being transmitted on the spheres \( S_1 \) and \( S_2 \) with the boundary conditions:

\[ U_{\partial i} = U_{\partial i} \text{ on } S_1 \]

\[ 0 = \partial U_{\partial i} / \partial n_{\partial i} \text{ on } S_2 \]

with \( \partial U_{\partial i} / \partial n_{\partial i} \) the normal derivative.

Numerical conductivity estimations are shown in Fig. 1. 2. 3, where the mean value \( \bar{\sigma}_{\text{act}} \) of the estimated \( \sigma_{\text{act}} \) is the one to be compared with the actual conductivity value \( \sigma_{\text{true}} \).

5. Computational algorithm and improvements

We performed numerical analysis of the inverse conductivity estimation problem, using measurements and sources activities expanded on spherical harmonics basis \( (\mathbf{G}(\tilde{E}), U_\partial) \) simulated by the FindSources3D (FS3D [4]) software, while our simulations were performed in MATLAB.

The EEG data are subject to ambient noise and measurements errors, while the estimation of the sources is not perfect. In our simulation, the inverse conductivity estimation problem is sensitive to such perturbations, forcing us to decrease the tolerance of our reconstruction algorithm to \( tol \rightarrow 0.5 \). As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still quite well estimated with a small number of them.

3.2. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_1 \). The data transmission \( U_{\partial i}(\ell_1,\ell_2) \) from a spherical interface \( S_1 \) to a neighbouring spherical interface \( S_2 \) can be expressed by the following general matrix equation.

\[ T_{\sigma_1}(\ell_2) \cdot U_{\partial i}(\ell_1) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\tilde{E}), \quad \ell \in \mathcal{D} \]

with \( \ell \) the degree of the spherical harmonic coefficient of the source term \( \mathbf{G}(\tilde{E}) \), \( \ell_2 \) the one of the two roots of the polynomial \( \mathcal{P}(\sigma_1) \) for the spherical harmonic basis.

4. References