Inverse skull conductivity estimation problems from EEG data

Juliette Leblond, Jean-Paul Marmorat, Christos Papageorgakis

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Introduction

A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [9] and serves for clinical (location of epilepsy foci) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly accurate to the model of conductivity.

1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:

\[ \nabla \times \mathbf{E} = \mathbf{0} \quad \text{and} \quad \nabla \cdot \mathbf{J} = 0 \]

for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, deduces that \( \mathbf{E} = -\nabla V \), while dividing the current density \( \mathbf{J} \) into the ohmic current \( \mathbf{J}_o \) and the source current \( \mathbf{J}_s \) as \( \mathbf{J} = \mathbf{J}_o + \mathbf{J}_s \) leads to our general model for the electric potential \( U \) in terms of conductivity Poisson equation with source term in divergence form:

\[ \nabla \cdot (\sigma \nabla U) = -\nabla \cdot \mathbf{J}_s \]

where \( \sigma \) is the real valued (isotropic assumption) conductivity of the medium at location \( r \).

Modeling the primary current \( \mathbf{J}_s \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:

\[ \nabla \cdot (\sigma \nabla U) = \sum_{q=1}^{Q} \mathbf{p}_q \cdot \nabla \mathbf{C}_q \]

where \( \mathbf{p}_q \) is the moment of the source and \( \mathbf{C}_q \) is the Dirac distribution with mass at \( \mathbf{C}_q \).

\[ \nabla \cdot \mathbf{J} = \nabla \cdot \mathbf{J}_o + \nabla \cdot \mathbf{J}_s = \nabla \cdot \mathbf{J}_o \]

\[ \mathbf{E} = \nabla \times \mathbf{B} \]

with \( \mathbf{B} \) the magnetic induction.

2. Mathematical formulation: Simplified model

We consider the inverse skull conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:

\[ \hat{\sigma} = 0 \quad \text{in} \quad \Omega \quad \text{with} \quad \Delta \hat{\sigma} = 0 \quad \text{in} \quad \Omega \]

\[ \text{boundary conditions} \]

\[ \sigma^\text{est} = \text{the solution which satisfies the constraint} \]

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:

\[ \mathbf{U}_q \in \mathcal{E} \quad \text{pointwise} \quad \text{values on} \quad S \quad \text{at} \quad \text{electrode locations} \]

\[ \mathbf{b}_0 \text{, no current flux outward the head} \]

while the source term is assumed to be already estimated, with the solution \( \mathbf{U}_0 \) in \( \Omega_0 \), being expressed as the convolution of the source term \( \mathbf{J}_s \) with the fundamental solution (Green formula):

\[ \mathbf{b}_0 \quad \text{for} \quad \mathbf{U}_0 \in \mathcal{E} \quad \text{on} \quad S \quad \text{at} \quad \text{electrode locations} \]

\[ \mathbf{b}_0 \text{, no current flux outward the head} \]

The source activity \( \mathbf{U}_q \) and the EEG data \( \mathcal{E} \) are expanded on spherical harmonics basis:

\[ \mathbf{U}_q \quad \text{expansion} \quad \text{on} \quad \text{spherical harmonics basis} \]

\[ \mathbf{b}_0 \quad \text{for} \quad \mathbf{U}_0 \in \mathcal{E} \quad \text{on} \quad S \quad \text{at} \quad \text{electrode locations} \]

\[ \text{boundary conditions} \]

\[ \sigma^\text{est} = 0 \quad \text{on} \quad \Omega \quad \text{and} \quad \Delta \sigma^\text{est} = 0 \quad \text{on} \quad \Omega \]

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_1 \). The data transmission \( \mathbf{U}_0 \) from the frontal interface \( S \) to a neighbouring spherical interface \( S_1 \) can be expressed by the following general matrix equation:

\[ \mathbf{T}_{21}(r, \sigma_1) = \int_0^r \frac{r'}{4 \pi} \left( \frac{r^2}{(r')^2} + \frac{4}{3} \right) \, d\sigma_1 \]

As \( \mathbf{T}_{21}(r, \sigma_1) = \mathbf{0} \) if \( \sigma_1 = 0 \), the inverse transmission matrix \( \mathbf{T}_{12}(r, \sigma) \) is also defined. Computing the data transmission over the several spherical interfaces the spherical harmonics coefficients of the EEG measurements \( \mathbf{b}_0 \), can be linked to the spherical harmonic coefficients of the source term \( \mathbf{b}_0 \) as:

\[ \beta_{0m} = \sum_{i=0}^{\infty} \int_0^r \frac{r'}{4 \pi} \left( \frac{r^2}{(r')^2} + \frac{4}{3} \right) \, d\sigma_1 \]

Solving this equation in terms of \( \beta_{0m} \), leads to a polynomial equation \( P_2(\sigma_1) = 0 \) of degree \( 2 \) in \( \sigma_1 \), with dependencies \( \sigma_1 = P_{2,\sigma_1}(\sigma_1) \).

5. Computational algorithm and improvements

We performed numerical analysis of the inverse conductivity estimation problem, using measurements and sources activities expanded on spherical harmonics basis \( \mathbf{b}_0 \) simulated by the FindSources3D (FS3D) [4] software, while our simulations were performed in MATLAB.

The EEG data are subject to ambient noise and measurements errors, while the estimation of the sources is not perfect. In our simulation, the inverse conductivity estimation problem is sensitive to such perturbations, forcing us to decrease the tolerance of our reconstruction algorithm to \( \epsilon_{\text{tol}} = 10^{-3} \). As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still quite well estimated with a small number of them.

6. Further work

\[ \hat{\sigma} = 0 \quad \text{on} \quad \Omega \quad \text{and} \quad \Delta \hat{\sigma} = 0 \quad \text{on} \quad \Omega \]

\[ \text{boundary conditions} \]

\[ \sigma^\text{est} = 0 \quad \text{on} \quad \Omega \quad \text{and} \quad \Delta \sigma^\text{est} = 0 \quad \text{on} \quad \Omega \]

\[ \text{boundary conditions} \]

\[ \mathbf{T}_{21}(r, \sigma_1) = \int_0^r \frac{r'}{4 \pi} \left( \frac{r^2}{(r')^2} + \frac{4}{3} \right) \, d\sigma_1 \]

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