Inverse skull conductivity estimation problems from EEG data
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Introduction

A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [9] and serves for clinical (localization of epilepsy focus) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly by the conductivity of the skull. The human skull is a bony tissue consisting of compact and spongy bone compartments, whose shape and size vary over the age and the individual’s anatomy making difficult to accurately model the skull conductivity.

1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:
\[ \nabla \times (\sigma \nabla \phi) = -\nabla \cdot \mathbf{J}, \]

for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, deduce that \( \mathbf{E} = -\nabla \phi \), while dividing the current density \( \mathbf{J} \) into the ohmic current \( \mathbf{J}_0 \) and the source current (also called primary current) \( \mathbf{J}^{\text{est}} \) as \( \mathbf{J} = \mathbf{J}_0 + \mathbf{J}^{\text{est}} \) leads to our general model for the electric potential \( U \) in terms of conductivity \( \sigma \) with source term in divergence form:
\[ \nabla \cdot (\sigma \nabla U) = \nabla \cdot \mathbf{J}^{\text{est}} \]

where \( \omega \subset \mathbb{R}^3 \) be the real valued (isotropic assumption) conductivity of the medium at location \( r \).

Modeling the current density \( \mathbf{J}^{\text{est}} \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:
\[ \nabla \cdot (\sigma \nabla U) = \sum_{q=1}^{Q} p_q \nabla \cdot U_{q,r} \in \mathbb{R}^3, \quad C_q \subset \omega \]

where \( p_q \) is the moment of the source and \( C_q \) is the Dirac distribution with mass at \( C_q \).

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:
\[ \{ U_i \mid \mathcal{E} = \{ \phi \} \} \]

with no current flux outside the head while the source term \( \mathcal{S} \) is assumed to be already estimated, with the solution \( \beta \mid \mathcal{D} \), being expressed as the convolution of the source term \( \mathcal{S} \) with the fundamental solution \( G \) (Green formula).
\[ \beta \mid \mathcal{D} = G \ast \mathcal{S} \]

The sources \( \mathcal{U}_i \) and the EEG data \( \mathcal{E} \) are expanded on spherical harmonics basis:
\[ \mathcal{U}_{\mathcal{S},i} = \sum_{q=1}^{Q} P_{q,r} \mathcal{U}_{q,r} \in \mathbb{C}, \quad r \in \mathcal{D} \cap \{ C_q \} \]

with the later being transmitted over the spheres \( S \) with the boundary conditions:
\[ \begin{cases} \mathcal{U}_{\mathcal{S},i} = \mathcal{U}_i & \text{on } S \nonumber \\
\sigma_i \partial_{\mathcal{N}} \mathcal{U}_{\mathcal{S},i} = -\mathcal{M}_{\mathcal{S},i} & \text{on } S \nonumber \end{cases} \]

This approximation enables us to study the uniqueness of the solution to the inverse problem by extending the source term to \( \omega \) and by a tolerance value \( \tau \). As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still well estimated with a small number of them.

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_1 \). The data transmission \( \mathcal{U}_{\mathcal{S},i} \mid \mathcal{E} \) from a spherical interface \( S_i \) to a neighboring spherical interface \( S_j \) can be expressed by the following general matrix equation:
\[ T_{ij}(\sigma, \mathcal{S}) = 0 \]

where \( T_{ij}(\sigma, \mathcal{S}) \) is the transmission matrix. The inverse transmission matrix \( T_{ij}^{-1}(\sigma) \) is also defined. Computing the data transmission over the several spherical interfaces the spherical harmonics coefficients of the EEG measurements \( \beta \mid \mathcal{D} \) can be linked to the spherical harmonic coefficients of the source term \( \beta \mid \mathcal{D} \) as:
\[ \beta \mid \mathcal{D} = \left( \prod_{i} T_{i+1}^{-1}(\sigma_i) \right) \beta \mid \mathcal{S} \]

Solving this equation in terms of \( \sigma_1 \), leads to a polynomial equation \( P_{\sigma_1}(\sigma_1) = 0 \) of degree \( P_{\sigma_1}(\sigma_i) = 2 \) in \( \sigma_1 \), with dependencies \( \mathcal{P}_{\sigma_1} \). Let \( \sigma_{best} \) be the one of the two roots of the polynomial \( P_{\sigma_1}(\sigma_1) \) for the \( \sigma_1 \) spherical harmonic basis.

The unique admissible solution \( \sigma_1 \) is the solution which satisfies the constraint \( 0 < \sigma_{non} < \min(\sigma_1, \sigma_2) \) and make \( \sigma_1 \) achieve its minimal value \( (\mathcal{F}(\sigma_1) = 0) \), up to a tolerance value \( \sigma_{tol} \).

As the reconstruction of the conductivity \( \sigma_1 \) does not depend on the spherical harmonics index \( m_i \), in order to increase the robustness of our reconstruction algorithm, the following normalization is applied over the different spherical harmonics indices \( k \):
\[ \sigma_i = \sum_{m=1}^{M} \rho_i \end{cases} \]

5. Computational algorithm and improvements

We performed numerical analysis of the inverse conductivity estimation problem, using measurements and source activities expanded on spherical harmonics basis \((\mathcal{M}, \mathcal{S})\), simulated by the FindSources3D (FS3D) [4] software, while our simulations were performed in MATLAB.

The EEG data are subject to ambient noise and measurements errors, while the estimation of the sources is not perfect. In our simulation, the inverse conductivity estimation problem is sensitive to such perturbations, forcing us to decrease the tolerance of our reconstruction algorithm to \( \tau = 0.5 \% \). As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still well estimated with a small number of them.

Conductivity estimations results are shown in Fig. 1. 2. 3, where the mean value \( \bar{\sigma}_{est} \) of the estimated conductivity \( \sigma_{est} \) is the one to be compared with the actual conductivity value \( \sigma_{true} \).

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