Inverse skull conductivity estimation problems from EEG data

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A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [9] and serves for clinical (location of epilepsy foci) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly by the conductivity of the skull. The human skull is a bony tissue consisting of compact and spongy bone compartments, whose shape and size vary over the age and the individual's anatomy making difficult to accurately model the skull conductivity.

1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:
\[ \nabla \times \mathbf{E} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{J} = 0 \]
for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, that \( \mathbf{E} = - \nabla \Phi \), while dividing the current density \( \mathbf{J} \) into the ohmic current \( \mathbf{J}_o \) and the source current (also called primary current) \( \mathbf{J}_s \) as \( \mathbf{J} = - \nabla \Phi + \mathbf{J}_s \), leads to our general model for the electric potential \( \Phi \) in terms of conductivity Poisson equation with source term in divergence form:
\[ \nabla \cdot \left( \sigma(\mathbf{r})(\nabla \Phi(\mathbf{r}) + \nabla \Phi(\mathbf{r})) \right) = \mathbf{S}(\mathbf{r}) \]
where \( \sigma(\mathbf{r}) \) is the real valued (isotopic assumption) conductivity of the medium at location \( \mathbf{r} \).
Modeling the current primary \( \mathbf{J}_s \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:
\[ \nabla \times \left( \sigma(\mathbf{r}) \nabla \mathbf{J}(\mathbf{r}) \right) = \mathbf{S}(\mathbf{r}) \]
where \( \mathbf{p}_r \) is the moment of the source and \( \mathbf{c}_i \) is the Dirac distribution with mass at \( \mathbf{c}_i \).

\[ \nabla \times \mathbf{E} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{J} = 0 \]

where \( \mathbf{p}_r \) is the moment of the source and \( \mathbf{c}_i \) is the Dirac distribution with mass at \( \mathbf{c}_i \).

2. Mathematical formulation: Simplified model

We consider the inverse skull conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:

- In a three spherical homogenous volume each.
- Made of three concentric isolated spheres, each of them modeling the scalp \( \Omega_2 \), skull \( \Omega_1 \) and brain \( \Omega_0 \) tissue.
- The head is assumed to be piecewise homogenous.
- The sources are known to have a constant conductivity \( \sigma_0 \) and \( \sigma_{\min} < \sigma_0 < \sigma_{\max} \).
- The sources \( \mathbf{P}_r \) are modeled as dipolar sources \( \mathbf{J}(\mathbf{r}) = \sum_{i=1}^{Q} \mathbf{p}_r \nu_x \mathbf{c}_i \) in \( \Omega_0 \).
- The electrical potential satisfies the following equations:
\[ \begin{align*}
\sigma_0 \Delta \Phi &= \nabla \cdot \mathbf{J} = 0 \quad &\text{in} \quad \Omega_0 \\
\Delta \Phi &= 0 \quad &\text{on} \quad \partial \Omega_0 \\
\end{align*} \]

In each domain \( \Omega_i \), the electric potential satisfies the following equations:
\[ \begin{align*}
\sigma_{\max} \Delta \Phi &= \nabla \cdot \mathbf{J} = 0 \quad &\text{in} \quad \Omega_i \\
\Delta \Phi &= 0 \quad &\text{on} \quad \partial \Omega_i \\
\end{align*} \]
with \( \mathbf{U}_0 \), \( \mathbf{U}_1 \) being the solution in \( \Omega_i \).
We also assume that the conductivities of the brain \( \sigma_0 \) and the scalp \( \sigma_{\max} \) are known (\( \sigma_0 = \sigma_{\max} \)), and while the conductivity is recovered in the one of the intermediate spherical layer, the skull conductivity \( \sigma_0 \).

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:
\[ \mathbf{U}_0 \text{ at electrode locations} \quad \mathbf{U}_1 \text{ on spherical interface boundary} \]
with dependences:
\[ \sum_{k=0}^{1} \sigma_{km} \mathbf{U}_0 - \frac{1}{\mathbf{P}_r} \mathbf{P}_r \mathbf{U}_1 = \mathbf{b} \quad \text{in} \quad \Omega_i \]
\[ \mathbf{U}_0 \text{ on spherical interface boundary} \]
Note that the moment \( \mathbf{p}_r \) of the source \( \mathbf{P}_r \) is the moment of the source \( \mathbf{P}_r \) over the sphere \( \partial \Omega_i \).

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_0 \).

The data transmission \( \mathbf{U}_0 \mathbf{U}_1 \) from a spherical interface \( S_1 \) to a neighbouring spherical interface \( S_2 \) can be expressed by the following general matrix equation:
\[ T_r(\sigma_0) \sigma_0 \]

As def \( T_r(\sigma_0) = - (2k + 1) \Omega_{\max}^{-2} \neq 0 \) the inverse transmission matrix \( T_r^{-1}(\sigma_0) \) is also defined. Computing the data transmission over the spherical interface the spherical harmonics coefficients of the EEG measurements \( \mathbf{g}_m \), can be linked to the spherical harmonic coefficients of the source term \( \mathbf{b} \) as:
\[ \mathbf{g}_m = [0 \ 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \end{bmatrix} T_r^{-1}(\sigma_0) \begin{bmatrix} \mathbf{b}_m \end{bmatrix} \]
with \( \mathbf{b}_m \) the one of the two roots of the polynomial \( P(\sigma_0) \) for the \( \sigma_0 \) spherical harmonic basis.

The unique admissible solution \( \mathbf{g}_m \) is the solution which satisfies the constraint \( 0 < \sigma_{\min} < \min(\mathbf{g}_m, \sigma_{\max}) \) and make \( P(\sigma_0) \) achieve a minimal value \( \mathcal{F}(\sigma_0) = 0 \) up to a tolerance value tol.

As the reconstruction of the conductivity \( \sigma_0 \) does not depend on the spherical harmonics indices \( m \), in order to increase the robustness of our reconstruction algorithm, the following normalization is applied over the different spherical harmonics indices \( k \):
\[ \begin{align*}
\mathbf{g}_m &= \mathbf{g}_{m} \frac{1}{\sqrt{\mathbf{g}_{m} \mathbf{g}_{m}}} \\
\mathbf{b}_m &= \mathbf{b}_{m} \frac{1}{\sqrt{\mathbf{b}_{m} \mathbf{b}_{m}}} \\
\end{align*} \]

6. Further work

- Stability properties and error estimates of the inverse problem.
- Robustness analysis of the recovery algorithm and dependence on the number of sources.
- Simultaneous recovery of source term and skull conductivity. First, stop with known quantity of sources \( \mathbf{Q} \) and locations \( \mathbf{C}_0 \).
- Influence of the known parameters of the problem on the estimation.
- Modeling the spongy layer and estimating its conductivity.
- Comparison of results with more realistic head models and spongiosa layer: joint work in progress.
- Conductivity estimations using additional magnetoencephalography data.

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