Inverse skull conductivity estimation problems from EEG data
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1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:

\[ \nabla \cdot \mathbf{E} = 0 \quad \text{and} \quad \nabla \times \mathbf{H} = \mathbf{J}_e \]

for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J}_e \). The first, deduced that \( \mathbf{E} = - \nabla \psi \), while dividing the current density \( \mathbf{J}_e \) into the ohmic current \( \sigma \vec{E} \) and the source current (also called primary current) \( \mathbf{J}^p \) as \( \mathbf{J}_e = \sigma \vec{E} + \mathbf{J}^p \), leads to our general model for the electric potential \( U \) in terms of conductivity

\[ \nabla \cdot (\sigma \nabla U) = \nabla \cdot \mathbf{J}^p \quad \text{in} \quad \Omega \]

where \( \sigma \in \mathbb{R} \) be the real valued (isotropic assumption) conductivity of the medium at location \( \mathbf{r} \).

Modeling the primary current \( \mathbf{J}^p \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:

\[ \nabla \cdot (\sigma \nabla U) = \sum_{i=0}^{Q} \mathbf{p}_i \cdot \nabla \psi_i \quad \text{in} \quad \Omega, \quad \mathbf{C}_i \in \mathbb{R}^3 \]

where \( \mathbf{p}_i \) is the moment of the source and \( \psi_i \) is the Dirac distribution with mass at \( \mathbf{C}_i \):

\[ \nabla \psi_i \rightarrow \text{grad} \quad \nabla \psi_i \rightarrow \text{div} \quad \nabla \psi_i \rightarrow \text{curl} \]

2. Mathematical formulation: Simplified model

We consider the inverse conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays[1]. The following problem is thus examined:

- In a three layered spherical head model
- Made of three concentric neotod spheres, each of them modeling the scalp \( \Omega_1 \), skull \( \Omega_2 \), and brain \( \Omega_3 \) tissues
- The head is assumed to be piecewise homogenous: each of the three layers is supposed to have a constant conductivity
- The sources \( \mathbf{C}_i \) are modeled as dipolar sources \( \mathbf{J}_i^p = \mathbf{p}_i \times \mathbf{r} \in \mathbb{R}^3 \)

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:

\[ U_l \rightarrow \mathbf{E}_l \quad \text{pointwise values on} \quad S_l \quad \text{at electrode locations} \]

\( \partial U_l \bigg|_{\partial \Omega} = 0 \), no current flux outside the head

while the source term is assumed to be already estimated, with the solution \( U_l \in \Omega_2 \), being expressed as the convolution of the source term \( \mathbf{J}^p \) with the fundamental solution (Green formula):

\[ U_l(\mathbf{r}) = \frac{1}{4\pi} \int_{S} \mathbf{p}_i \cdot \nabla \psi_i \bigg|_{\mathbf{r}} \quad \mathbf{r} \in \Omega_2 \]

The source \( U_l \) and the EEG data \( \mathbf{E}_l \) are expressed on spherical harmonics basis:

\[ U_l(\mathbf{r}, \lambda) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} U_{l,m} Y_{l,m}(\lambda, \varphi) \quad \mathbf{r} \in \Omega_2 \]

where the later being transmitted over the spheres \( S_l \) with the boundary conditions:

\[ U_l = U_l \quad \text{on} \quad S_l, \quad \mathbf{n}_l \cdot \mathbf{U}_l = \sigma_l \mathbf{n}_l \cdot \mathbf{U}_l \quad \text{on} \quad S_l \]

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_2 \). The data transmission \( (U_l, \mathbf{E}_l) \) from a spherical interface \( S_1 \) to a neighboring spherical interface \( S_2 \) can be expressed by the following general matrix equation

\[ \mathbf{T}_{21}(\sigma_{1}) \begin{bmatrix} T_{21} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \quad \left( \begin{bmatrix} \sigma_1 \end{bmatrix} \right) \begin{bmatrix} 1 \end{bmatrix} \}

Solving this equation in terms of \( \sigma_1 \), leads to a polynomial equation \( P_n(\sigma_{1}) = 0 \) of degree \( n \) in \( \sigma_{1} \), with dependences \( \sigma_{1} = \sigma_{1}(\xi_1, \xi_2) \). Let \( \sigma_{1,m} \) be the one of the two roots of the polynomial \( P_n(\sigma_{1}) \) for the \( i \)-th spherical harmonic. The unique admissible solution \( \sigma_{1,m} \), is the solution which satisfies the constraint \( 0 < \sigma_{1,m} < min(\sigma_{1}, \sigma_{2}) \) and make \( \sigma_{1} \) achieving its minimal value \( (\sigma_{1} = \sigma_{1,m}) \), up to a tolerance value tol.

As the reconstruction of the conductivity \( \sigma_2 \) does not depend on the spherical harmonics index \( m \), in order to increase the robustness of our reconstruction algorithm, the following normalization is applied over the different spherical harmonics index \( k \):

\[ \sigma_{2,m} = \sum_{l} \sum_{m} \sigma_{1,m} Y_{l,m}(\lambda, \varphi) \]

5. Computational algorithm and improvements

We performed numerical analysis of the inverse conductivity estimation problem, using measurements and sources activities expanded on spherical harmonics basis \( (g_{mn}, h_{mn}) \) simulated by the FindSources3D (FS3D) [4] software, while our simulations were performed in MATLAB.

The EEG data are subject to ambient noise and measurements errors, while the estimation of the sources is not perfect. In our simulation, the inverse conductivity estimation problem is sensitive to such perturbations, forcing us to decrease the tolerance of our reconstruction algorithm to tol = 10^{-5}. As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still quite well estimated with a small number of terms.

6. Further work

- Stability properties and error estimates of the inverse problem.
- Robustness analysis of the recovery algorithm and dependence on the number of sources.
- Simultaneous recovery of source term and skull conductivity. First, stop with known quantity of sources \( Q \) and locations \( C_F \).
- Influence of the known parameters of the problem on the estimation.
- Comparison of results with more realistic head models and spongiosa layer: joint work in progress.
- Conductivity estimations using additional magnetoencephalography data.

References


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