Inverse skull conductivity estimation problems from EEG data
Juliette Leblond, Jean-Paul Marmorat, Christos Papageorgakis

To cite this version:
Juliette Leblond, Jean-Paul Marmorat, Christos Papageorgakis. Inverse skull conductivity estimation problems from EEG data. 1st International Conference on Mathematical Neuroscience (ICMNS), Jun 2015, Juan-les-Pins, France. <http://icmns2015.inria.fr/>. <hal-01243059>

HAL Id: hal-01243059
https://hal.archives-ouvertes.fr/hal-01243059
Submitted on 23 Dec 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed under a Creative Commons Attribution| 4.0 International License
A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [9] and serves for clinical (location of epilepsy foci) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly by the conductivity of the skull. The human skull is a bony tissue consisting of compact and spongy bone compartments, whose shape and size vary over the age and the individual’s anatomy making difficult to accurately model the skull conductivity.

1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:

\[ \nabla \times \mathbf{E} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{J} = 0 \]

for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, deduce that \( \mathbf{E} = -\nabla V \), while dividing the current density \( \mathbf{J} \) into the ohmic current \( \mathbf{J}_o \) and the source current (also called primary current) \( \mathbf{J}_s \) as \( \mathbf{J} = \mathbf{J}_o + \mathbf{J}_s \) leads to our general model for the electric potential \( U \) in terms of conductivity Poisson equation with source term in divergence form:

\[ \nabla \cdot \left( \sigma(\mathbf{r}) \nabla U(\mathbf{r}) \right) = \mathbf{J}_s(\mathbf{r}) \quad \text{in} \quad \mathbb{R}^3 \]

where \( \sigma(\mathbf{r}) \in \mathbb{R} \) be the real valued (isotropic assumption) conductivity of the medium at location \( \mathbf{r} \).

Modeling the primary current \( \mathbf{J}_s \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:

\[ \nabla \cdot \left( \rho_s \nabla U(\mathbf{r}) \right) = \delta(\mathbf{r}) - \rho_s \nabla U(\mathbf{r}) \quad \text{in} \quad \mathbb{R}^3 \]

where \( \rho_s \) is the moment of the source and \( \nabla \cdot U \) is the Dirac distribution with mass at \( \rho_s \).

\[ \nabla \times \nabla \cdot \delta(\mathbf{r}) = \delta(\mathbf{r}) \quad \text{curl} \quad \nabla \times \nabla \cdot \delta(\mathbf{r}) \neq \delta(\mathbf{r}) \]

2. Mathematical formulation: Simplified model

We consider the inverse skull conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:

- A three layered spherical model
- Made of three concentric nested spheres, each of them modeling the scalp \( \Omega_1 \), skull \( \Omega_2 \), and brain \( \Omega_3 \) tissues
- The head is assumed to be piecewise homogeneous: each of the three layers has a constant conductivity \( \sigma_0 \leq \sigma_i < \min(\sigma_1, \sigma_2) \)
- The sources \( \mathbf{S} \) are modeled as dipolar sources \( \mathbf{J}_s = \sum_{i=1}^{Q} \mathbf{J}_{si}(\mathbf{r}) + \mathbf{J}_o(\mathbf{r}) \)

In each domain \( \Omega_i \), the electric potential satisfies the following equations:

- \( \mathbf{J}_o(\mathbf{r}) = 0 \)
- \( \mathbf{J}_s(\mathbf{r}) = 0 \)

with \( \mathbf{U}_0, \mathbf{U}_1, \mathbf{U}_2 \) being the solution in \( \Omega_0 \), \( \Omega_1 \), and \( \Omega_2 \), respectively.

We also assume that the conductivities of the brain \( \sigma_1 \) and the scalp \( \sigma_2 \) are known (currently \( \sigma_0 = \sigma_2 \)), while the conductivity to be recovered is in the one of the intermediate spherical layer, the skull \( \sigma_2 \).

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:

\[ \{ \mathbf{U}_0 \quad \text{and} \quad \mathbf{U}_2 \} | \nabla \mathbf{U}_0 = 0 \quad \text{on} \quad \partial \Omega_0 \]

while the source term is assumed to be already estimated, with the solution \( \mathbf{U}_0 \) in \( \Omega_0 \), being expressed as the convolution of the source term \( \mathbf{J}_s(\mathbf{r}) \) with the fundamental solution (Green formula):

\[ \mathbf{U}_0(\mathbf{r}) = \int_{\Omega} \mathbf{H}(\mathbf{r}, \mathbf{r}') \mathbf{J}_s(\mathbf{r}') \, d\mathbf{r}' \]

The source activity \( \mathbf{U}_0 \) and the EEG data \( \mathbf{E}(\mathbf{r}) \) are expanded on spherical harmonics basis:

\[ \mathbf{E}(\mathbf{r}) = \sum_{k=0}^{\infty} \sum_{m=-k}^{k} \mathbf{E}_k^m(\theta, \phi) \quad (\theta, \phi) \in \Omega \]

where \( \mathbf{E}_k^m(\theta, \phi) \) is supposed to have a constant conductivity \( \sigma_0 \) and each component \( \mathbf{E}_k^m \) is the solution in \( \Omega \) of a homogeneous linear problem using spherical harmonics basis. The electric potential satisfies the following equations:

\[ \nabla \cdot \nabla U(\mathbf{r}) = \mathbf{J}_s(\mathbf{r}) + \mathbf{J}_o(\mathbf{r}) \quad \text{in} \quad \Omega \]

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_2 \). The data transmission from a spherical interface \( S_i \) to a neighbouring spherical interface \( S_{i+1} \) can be expressed by the following general matrix equation:

\[ \mathbf{T}_{i} \mathbf{r}(\sigma_i) = 0 \]

Let \( \mathbf{T}_i \mathbf{r}(\sigma_i) \) be the inverse transmission matrix \( \mathbf{T}_i \mathbf{r}(\sigma_i) \) is also defined. Computing the data transmission over the several spherical interfaces the spherical harmonics coefficients of the EEG measurements \( \mathbf{g}_m \) can be linked to the spherical harmonics coefficients of the source term \( \mathbf{h}_m \) as:

\[ \mathbf{h}_m \approx [0 \ 1 \ 0] \mathbf{g}_m \]