Inverse skull conductivity estimation problems from EEG data
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1. Physical formulation

Under the quasi-static approximation for the EEG case, Maxwell’s equations imply:
\[ \nabla \times \mathbf{E} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{J} = 0 \]
for the electric fields \( \mathbf{E} \) and the current density \( \mathbf{J} \). The first, deduces that \( \mathbf{E} = -\nabla \psi \); while dividing the current density \( \mathbf{J} \) into the ohmic current \( \mathbf{J}^o \) and the source current (also called primary current) \( \mathbf{J}^s \) as \( \mathbf{J} = \mathbf{J}^o + \mathbf{J}^s \) leads to our general model for the electric potential \( U \) in terms of conductivity Poisson equation with source term in divergence form:
\[ \nabla \cdot (\sigma \nabla U) = \nabla \cdot \mathbf{J}^s \]
\[ \nabla \cdot \mathbf{J}^o = 0 \]
where \( \sigma \) is the real valued (isotropic assumption) conductivity of the medium at location \( r \).

Modeling the primary current \( \mathbf{J}^s \) as the result of the superposition of \( Q \) pointwise dipolar sources, leads to:
\[ \nabla \cdot (\mathbf{J}^o) = \sum_{p=1}^{Q} \mathbf{P}_p \cdot \nabla \mathbf{c}_p \text{ in } \mathbb{R}\]
\[ \nabla \cdot \mathbf{J}^o = 0 \text{ in } \mathbb{R} \]
where \( \mathbf{P}_p \) is the moment of the source and \( \mathbf{c}_p \) is the Dirac distribution with mass at \( \mathbf{c}_p \).

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG partial boundary data:
\[ \mathbf{U}_I = \mathbf{E}_{GEC}(\text{pointwise}) \text{ values on } S_I \text{ at electrode locations} \]
\[ \mathbf{U}_I \cdot \mathbf{n} = 0 \text{ no current flux outside the head} \]
while the source term is assumed to be already estimated, with the solution \( \mathbf{U}_I \) on \( \Omega \), being expressed as
\[ \mathbf{U}_I = \mathbf{U}_I^o + \mathbf{U}_I^s \]
\[ \mathbf{U}_I^o = \mathbf{U}_I^o(\mathbf{c}_p, r, \mathbf{P}_p) \text{ for } \mathbf{c}_p \text{ in } S \]
\[ \mathbf{U}_I^s = \mathbf{U}_I^s(\mathbf{c}_p, r) \text{ for } \mathbf{c}_p \text{ in } S \]
with the later being transmitted on the spherical set \( S \) and \( S_I \) with the boundary conditions:
\[ \mathbf{U}_I|_{\Sigma_I} = \mathbf{U}_I^o|_{\Sigma_I} \text{ on } \Sigma_I \]
\[ \mathbf{U}_I|_{\Sigma_I} = \mathbf{U}_I^s|_{\Sigma_I} \text{ on } \Sigma_I \]
Numerical conductivity estimation results are shown in Fig. 1, 2, 3, where the mean value \( \bar{\sigma}_{\text{ns}} \) of the estimated \( \sigma_{\text{ns}} \) is the one to be compared with the actual conductivity value \( \sigma_{\text{ns}} \).

2. Mathematical formulation: Simplified model

We consider the inverse skull conductivity estimation problem using partial boundary EEG data, in the preliminary case of an homogeneous skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:

\[ \text{In a three layer spherical head model} \]
\[ \text{Made of three concentric nested spheres, each of them modeling the} \]
\[ \text{scalp \( \Omega_2 \), skull \( \Omega_1 \), and brain \( \Omega_0 \) tissues} \]
\[ \text{The head is assumed to be piecewise homogenous: each of the three layers has a constant} \]
\[ \text{conductivity} \]
\[ \beta_{\Omega_i} = \{ \beta_{\text{skull}}, \beta_{\text{scalp}}, \beta_{\text{brain}} \} \text{ with } \beta_{\text{skull}} < \beta_{\text{scalp}} < \beta_{\text{brain}} \]
\[ \text{The sources } \mathbf{S}_{\text{ns}} \text{ are modelled as dipolar sources } \mathbf{S}_{\text{ns}} = \mathbf{Q}\mathbf{P} \text{ with } \mathbf{Q} \in \mathbb{C} \]

In each domain \( \Omega_i \), the electric potential satisfies the following equations:
\[ \Delta U^i(\mathbf{r}) = 0 \text{ in } \Omega_i \]
\[ \mathbf{n} \cdot \mathbf{n} = 0 \text{ on } \partial \Omega_i \]
with \( \mathbf{U}_I, \mathbf{U}_I^o, \mathbf{U}_I^s \) being the solution in \( \Omega_i \).

We also assume that the conductivities of the brain \( \sigma_{\text{brain}} \) and the scalp \( \sigma_{\text{scalp}} \) are known (currently \( \sigma_{\text{scalp}} = \sigma_{\text{brain}} \)), while the conductivity to be recovered is in the one intermediate spherical layer, the skull \( \sigma_{\text{skull}} \).

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish uniqueness properties and a reconstruction algorithm for the skull conductivity \( \sigma_{\text{skull}} \). The data transmission through the spherical interfaces the spherical harmonics coefficients of the EEG measurements \( \mathbf{P} \) can be linked to the spherical harmonic coefficients of the source term \( \mathbf{P} \) as:
\[ \mathbf{P} = \mathbf{Q}\mathbf{P} \]
Solving this equation in terms of \( \sigma_{\text{skull}} \), leads to a polynomial equation \( \mathbf{P} = \mathbf{Q}\mathbf{P} \) of degree \( \leq 2 \) in \( \sigma_{\text{skull}} \), with dependencies \( \mathbf{P} = \mathbf{P}_{\text{skull}} \) and \( \mathbf{P} = \mathbf{P}_{\text{scalp}} \).

Let \( \sigma_{\text{skull}} \) be one of the two roots of the polynomial \( \mathbf{P} = \mathbf{Q}\mathbf{P} \) for the \( \mathbf{P}_{\text{skull}} \) spherical harmonic basis.

The unique admissible solution \( \sigma_{\text{skull}} \) is the solution which satisfies the constraint \( 0 < \sigma_{\text{skull}} < \min(\sigma_{\text{scalp}}, \sigma_{\text{brain}}) \) and makes \( \mathbf{P} \) achieve its minimal value (\( \mathbf{F}(\mathbf{P}) = 0 \)), up to a tolerance value tol. As the reconstruction of the conductivity \( \sigma_{\text{skull}} \) does not depend on the spherical harmonics index \( k \), in order to increase the robustness of our reconstruction algorithm, the following normalization is applied over the different spherical harmonics index \( k \):
\[ \sigma_{\text{skull}} = \left( \sum_{k=1}^{K} \sigma_{\text{skull}}^{2k} \right)^{-1} \]

6. Further work

- Stability properties and error estimates of the inverse problem.
- Robustness analysis of the recovery algorithm and dependency on the number of sources.
- Simultaneous recovery of source term and skull conductivity. First, stop with known quantity of sources \( Q \) and locations \( \mathbf{C}_Q \).
- Influence of the known parameters of the problem on the estimation.
- Modeling the sponging layer and estimating its conductivity.
- Comparison of results with more realistic head models and sponging layer: joint work in progress.
- Conductivity estimations using additional magnetoencephalography data.

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References