Inverse skull conductivity estimation problems from EEG data

Juliette Leblond, Jean-Paul Marmorat, Christos Papageorgakis

Inverse skull conductivity estimation problems from EEG data

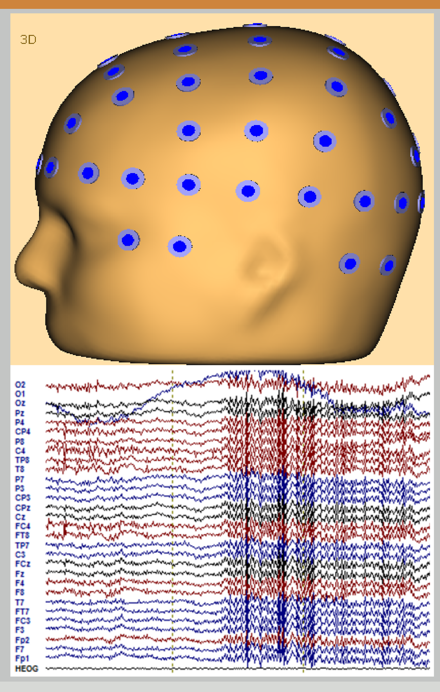
1[st] ICMNS

Juliette Leblond[1], Jean-Paul Marmorat[2], Christos Papageorgakis[1]

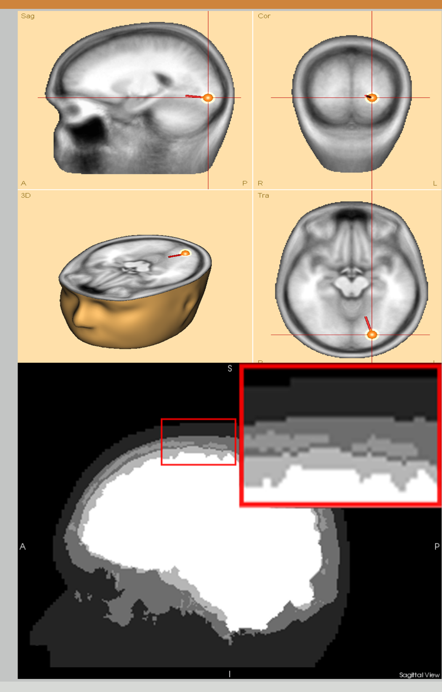
[1]APICS Research team, Inria Sophia Antipolis Méditerranée, France
[2]Ecole des Mines ParisTech, France

June 8-10, 2015

Introduction



A fundamental problem in theoretical neurosciences is the inverse problem of source localization, which aims at locating the sources of the electric activity of the functioning human brain using measurements usually acquired by non-invasive imaging techniques, such as the electroencephalography (EEG). EEG measures the effect of the electric activity of active brain regions through values of the electric potential furnished by a set of electrodes placed at the surface of the scalp [3] and serves for clinical (location of epilepsy foci) and functional brain investigation. The inverse source localization problem in EEG is influenced by the electric conductivities of the several head tissues and mostly by the conductivity of the skull. The human skull is a bony tissue consisting of compact and spongy bone compartments, whose shape and size vary over the age and the individual's anatomy making difficult to accurately model the skull conductivity.



1. Physical formulation

Under the **quasi-static approximation** for the EEG case, **Maxwell's equations** implies:

$$\nabla \times \mathbf{E} = 0 \text{ and } \nabla \cdot \mathbf{J} = 0$$

for the electric fields $\mathbf{E}$ and the current density $\mathbf{J}$. The first, deduce that $\mathbf{E} = -\nabla U$, while dividing the current density $\mathbf{J}$ into the ohmic current $\sigma\mathbf{E}$ and the source current (also called primary current) $\mathbf{J}^P$ as: $\mathbf{J} = \sigma\mathbf{E} + \mathbf{J}^P$ leads to our general model for the electric potential U in terms of conductivity **Poisson equation** with source term in divergence form:

$$\nabla \cdot (\sigma(\mathbf{r})\nabla U) = \nabla \cdot \mathbf{J}^P(\mathbf{r}) := \mathscr{S}(\mathbf{r}) \text{ in } \mathbb{R}^3$$

where $\sigma(\mathbf{r}) \in \mathbb{R}$ be the real valued (isotropic assumption) conductivity of the medium at location $\mathbf{r}$.

Modeling the primary current $\mathbf{J}^P$ as the result of the superposition of Q pointwise dipolar sources, leads to:

$$\nabla \cdot (\sigma \nabla U) = \sum_{q=1}^{Q} \mathbf{p}_q \cdot \nabla \delta_{\mathbf{C}_q} \text{ in } \mathbb{R}^3, \ \mathbf{C}_q \in \mathbb{R}^3$$

where $\mathbf{p}_q$ is the moment of the source and $\delta_{\mathbf{C}_q}$ is the Dirac distribution with mass at $\mathbf{C}_q$.

$$\nabla \sim grad, \ \nabla\cdot \sim div, \ \nabla\times \sim curl$$

3. Data, boundary conditions and expansions

We solve the conductivity estimation problem from the available EEG **partial** boundary data:

$$\begin{cases} U_2 = g_{EEG}, \ \textbf{pointwise} \text{ values on } S_2 \text{ at electrode locations} \\ \partial_n U_2 = 0, \text{ no current flux outside the head} \end{cases}$$

while the source term is assumed to be already **estimated**, with the solution U_0 in Ω_0, being expressed as the convolution of the source term $\mathscr{S}(\mathbf{r})$ with the **fundamental solution** (Green formula):

$$U_0(\mathbf{r}) = \sum_{q=1}^{Q} \frac{< \mathbf{p}_q, \mathbf{r} - \mathbf{C}_q >}{4\pi\sigma_0 |\mathbf{r} - \mathbf{C}_q|^3}$$

The source activity U_0 and the **EEG data** g_{EEG} are expanded on **spherical harmonics basis**:

$$U_0(\mathbf{r}) = \sum_{k,m} \beta_{km} r^{-(k+1)} Y_{km}(\theta, \phi), \ \mathbf{r} \in \Omega_0 \setminus \{\mathbf{C}_q\}$$

$$g_{EEG} = \sum_{k,m} g_{km} Y_{km}(\theta, \phi), \text{ where } k \in \mathbb{Z}_+, \ m \in \mathbb{Z}, \text{ and } -k \leq m \leq k$$

with the later being **transmitted** over the spheres S_1, S_0 with the **boundary conditions**:

$$\begin{cases} U_{i-1} = U_i & \text{on } S_i \\ \sigma_{i-1}\partial_n U_{i-1} = \sigma_i \partial_n U_i & \text{on } S_i \end{cases}$$

5. Computational algorithm and improvements

We performed **numerical analysis** of the inverse conductivity estimation problem, using **measurements** and **sources activities** expanded on spherical harmonics basis (g_{km}, b_{km}) simulated by the FindSources3D (FS3D [4]) software, while our simulations were performed in MATLAB.

The EEG data are subject to **ambient noise** and **measurements errors**, while the estimation of the sources is **not perfect**. In our simulation, the inverse conductivity estimation problem is **sensitive** to such **perturbations**, forcing us to decrease the tolerance of our reconstruction algorithm to $tol = 5e^{-2}$. As a result a significant amount of spherical harmonic coefficients is rejected, but the conductivity is still quite **well estimated** with a small number of them.

Numerical conductivity estimation results are shown in Fig. 1, 2, 3, where the mean value $\bar{\sigma}_{est}$ of the estimated $\sigma_{est,k}$ is the one to be compared with the actual conductivity value σ_{act}.

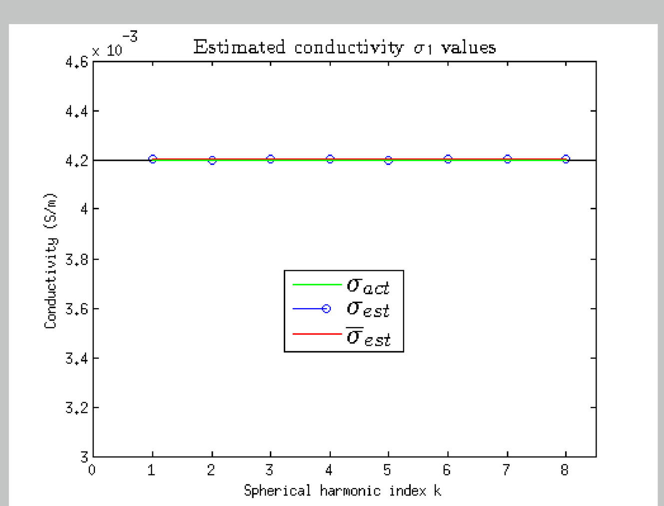


Fig.1: β_{km} from transmitted g_{km}.

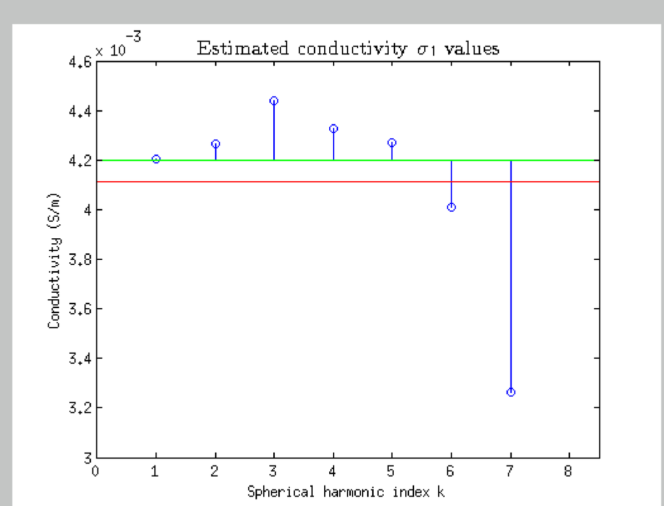


Fig.2: β_{km} from recovered sources by FS3D.

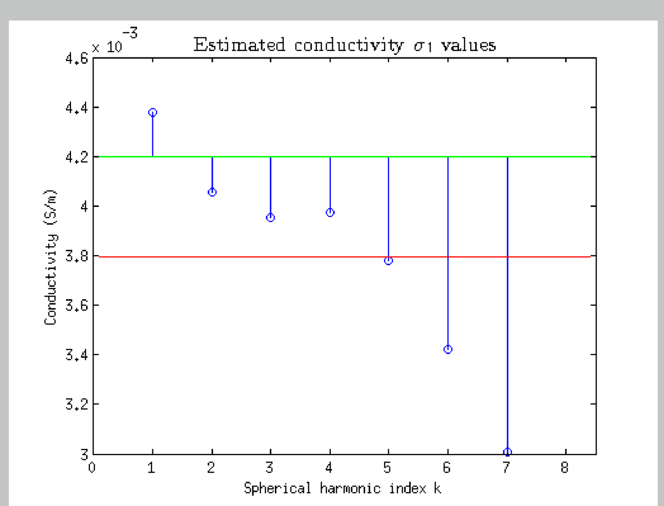


Fig.3: β_{km} from transmitted g_{km}, with noise.

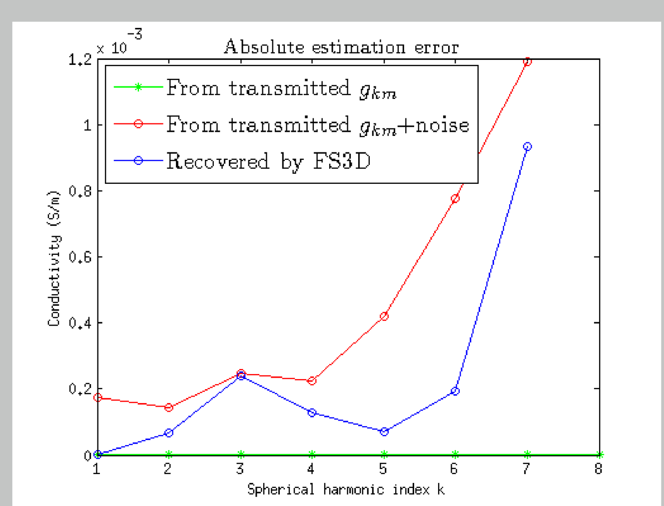
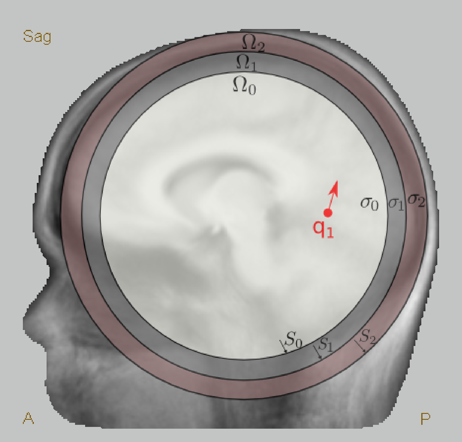


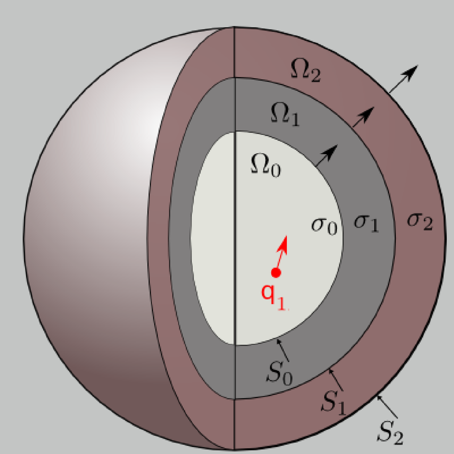
Fig.4: Conductivity estimation errors for the three used source terms.

2. Mathematical formulation: Simplified model

We consider the inverse **skull conductivity estimation** problem using **partial boundary EEG data**, in the preliminary case of an **homogeneous** skull conductivity. This is a version of the many inverse conductivity issues still under study nowadays [1]. The following problem is thus examined:



- In a three layer **spherical head** model
- Made of **three** concentric **nested spheres**, each of them modelling the scalp Ω_2, skull Ω_1 and brain Ω_0 tissues
- The head is assumed to be **piecewise homogeneous**: each of the three layers is supposed to have a constant conductivity
  $\sigma|_{\Omega_i} = \sigma_i, \ \ 0 < \sigma_1 < \min(\sigma_0, \sigma_2)$
- The sources C_q are modelled as **dipolar** sources $\mathbf{J}^P = \sum_{q=1}^{Q} \mathbf{p}_q \delta_{\mathbf{C}_q}, \ \mathbf{C}_q \in \Omega_0$



In each domain Ω_i, the electric potential satisfies the following equations:

$$\begin{cases} \sigma_0 \Delta U = \nabla \cdot \mathbf{J}^P & \text{in } \Omega_0 \\ \Delta U = \ \ 0 & in\ \Omega_1 \text{ and } \Omega_2 \end{cases}$$

with U_0, U_1, U_2 being the solution in Ω_i.

We also assume that the conductivities of the brain σ_0 and the scalp σ_2 are **known** (currently $\sigma_0 = \sigma_2$), while the conductivity to be recovered is the one of the **intermediate spherical layer**, the **skull** σ_1.

4. Uniqueness properties and reconstruction algorithm

Linear algebra computations allow us to establish **uniqueness properties** and a **reconstruction algorithm** for the skull conductivity σ_1. The data transmission $\begin{bmatrix} U_i \\ \partial_n U_i \end{bmatrix}$ from a spherical interface S_i to $\begin{bmatrix} U_{i+1} \\ \partial_n U_{i+1} \end{bmatrix}$ of a neighbouring spherical interface S_{i+1} can be expressed by the following general matrix equation.

$$T_k(r,\sigma) = \begin{bmatrix} 1 & 0 \\ 0 & \sigma \end{bmatrix} \begin{bmatrix} r^k & r^{-(k+1)} \\ kr^{k-1} & -(k+1)r^{-(k+2)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \sigma \end{bmatrix} T_k(r)$$

As $det(T_k) = -(2k+1)\sigma r^{-2} \neq 0$ the inverse transmission matrix $T_k^{-1}(r,\sigma)$ is also defined. Computing the data transmission over the several spherical interfaces the spherical harmonics coefficients of the EEG measurements g_{km} can be linked to the spherical harmonic coefficients of the source term β_{km} as:

$$\beta_{km} - [0,1]\, T_k^{-1}(r_0,\sigma_0) T_k(r_0,\sigma_1) T_k^{-1}(r_1,\sigma_1) T_k(r_1,\sigma_2) T_k^{-1}(r_2,\sigma_2) \begin{bmatrix} g_{km} \\ 0 \end{bmatrix} = 0$$

Solving this equation in terms of σ_1, leads to a **polynomial equation** $P(\sigma_1) = 0$ of $deg(P_{\sigma_1}) = 2$ **in** σ_1, with dependences: $P = P_{k,r_0,r_1,r_2,\sigma_0,\sigma_2}$.

Let σ_{est_k} be the one of the two roots of the polynomial $P(\sigma_1)$ for the k^{th} spherical harmonic basis. The **unique** admissible **solution** σ_{1_k}, is the solution which satisfies the constraint $0 < \sigma_{est_k} < \min(\sigma_0, \sigma_2)$ and make $|P|$ achieving its minimal value ($|P| = 0$), up to a tolerance value tol.

As the **reconstruction** of the conductivity σ_1 does not depend on the spherical harmonics index m, in order to increase the **robustness** of our **reconstruction algorithm**, the following **normalization** is applied over the different spherical harmonics index k:

$$\begin{cases} g_k = \sum_m g_{km}\bar{\beta}_{km} \\ \beta_k = \sum_m \beta_{km}\bar{\beta}_{km} = \sum_m |\beta_{km}|^2 \end{cases}$$

6. Further work

- Stability properties and error estimates of the inverse problem.
- Robustness analysis of the recovery algorithm and dependence on the number of sources.
- Simultaneous recovery of source term and skull conductivity. First, step with known quantity of sources Q and locations $\mathbf{C}_q$.
- Influence of the known parameters of the problem on the estimation.
- Modeling the spongiosa layer and estimating its conductivity.
- Comparison of results with more realistic head models and spongiosa layer: joint work in progress.
- Conductivity estimations using additional magnetoencephalography data.

Acknowledgements

This work was supported by the Région Provence-Alpes-Côte d'Azur and the BESA company.

References

[1] G. Alessandrini. Generic uniqueness and size estimates in the inverse conductivity problem with one measurement, *Le Matematiche* 54 pp. 5-14, 1999.

[2] G. Chavent, K. Kunisch. The Output Least Squares Identifiability of the Diffusion Coefficient from an H1–Observation in a 2–D Elliptic Equation. ESAIM: Control, Optimisation and Calculus of Variations, 8, 423-440, 2002.

[3] M. Clerc, J. Leblond, J.-P. Marmorat, T. Papadopoulo. Source localization in EEG using rational approximation on plane sections, *Inverse Problems* 28, 055018 (24p.), 2012.

[4] FindSources3D, `http://www-sop.inria.fr/apics/FindSources3D/`