Simple Discrete-Time Switched H ∞ Optimal Control: Application for Lateral Vehicle Control

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Abstract: This paper presents a switched H_∞ optimal control for a class of discrete-time switched linear systems. All sufficient conditions of the existence of the control law are proved and given in terms of LMI for any switching. Moreover, the proofs are established using an H_∞ norm and switched Lyapunov functions. Its performances are shown through a steering vehicle control application. In fact, the vehicle models are affected by several parameter variations like longitudinal speed, cornering stiffnesses coefficients. The validation step is conducted using real data acquired by a laboratory car under high lateral loads.

Keywords: Switched optimal control, discrete-time switched linear systems, H_∞ norm, LMI, steering vehicle control.

1. INTRODUCTION

This work proposes a switched H_∞ optimal control. This approach can be used to control switched discrete-time linear systems. Indeed, the knowledge of linear models of actual systems is local and affected by several modeling errors. Moreover, it is a very difficult task to quantify the bounds and the nature of these uncertainties to obtain a global model. To overcome this problem, several solutions like fuzzy, LPV, adaptive and switched systems are proposed. In fact, the switched systems are widely addressed in the literature Branicky [1998], Liberzon and Morse [1999], Daafouz et al. [2002], Prajna and Papachristodoulou [2003], Sun and Ge [2005], Lin and Antsaklis [2007], Koenig et al. [2008], Lin and Antsaklis [2009], Koenig and Marx [2009], D and Daafouz [2011]. These systems are defined by a set of sub-models and switching rules. The switching rule is used as a supervisor to determine the appropriate local model.

The major problem of switched systems is related to the stability analysis. Several developments on the stability analysis problem under arbitrary switching signals for some kinds of switched systems are presented for instance in Liberzon and Morse [1999], Sun and Ge [2005], Lin and Antsaklis [2009] and the references cited therein. Among the stability results on the switched systems we can find those on the stability of switched continuous-time systems Liberzon and Morse [1999], Sun and Ge [2005], Lin and Antsaklis [2007, 2009] using common Lyapunov functions,

and those on the stability of switched discrete-time systems Branicky [1998], Daafouz et al. [2002], Koenig et al. [2008], Du et al. [2007] using switched Lyapunov functions. Other switched systems are also treated like switched linear and nonlinear discrete-time descriptor systems Koenig et al. [2008], Koenig and Marx [2009], linear discrete-time switched systems with state delays Du et al. [2007].

Our objective is to develop a steering vehicle control by considering some parametric variations that affect the vehicle model. In fact, the proposed strategy takes advantages of switched Lyapunov functions Branicky [1998], Daafouz et al. [2002], Koenig et al. [2008], Du et al. [2007] and H_∞ criterion Du et al. [2007], Xie [1996], Xu et al. [2005], Zhou et al. [1994], Muradore and Picci [2003]. In fact, using switched Lyapunov functions, less restrictive sufficient conditions for the solution of switched H_∞ optimal control are expressed in terms of LMI.

In the literature, several kinds of lateral vehicle control are developed, see for example Ackermann et al. [1995], Cerone et al. [2009], Marino and Cinili [2009], Plochl and Edelmann [2007] and the references cited therein. Some of them are designed using the classical control techniques and taking into account some driver behavior like anticipation time and prediction distance Plochl and Edelmann [2007], Sharp et al. [2000] to perform some driving tasks like obstacle or pedestrian avoidance, lane-change maneuvers and lane keeping. However, many of them assume that the vehicle models are well known. To overcome such a problem, a steering vehicle control based on switched H_∞ optimal control is proposed. This

control can easily be used under normal or high lateral accelerations.

The paper is organized as follows. Section 2 describes the Linear Two Wheels Vehicle Model (L2WVM) and the problem statement. The switched H_∞ optimal control design method is given in Section 3. The simulation results conducted with real data are presented in Section 4. Section 5 presents conclusions and perspectives.

Control notations: The notations used in this paper are standard. The superscript "T" stands for the transpose matrix; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $P > 0$ ($\geqslant 0$) means that P denotes a symmetric positive definite matrix (semi-definite). In symmetric block matrices or complex matrix expressions, we use an asterisk ($*$) to represent a term that is induced by symmetry and $diag\{\cdots\}$ stands for a block-diagonal matrix.

Vehicle notations: V_x: longitudinal speed $[km.h]$, $\dot{\psi}$: yaw rate $[rad/s]$, ψ: yaw angle $[rad]$, $\dot{Y}$: first derivative of lateral deviation $[m/s]$, $\ddot{Y}$: second derivative of lateral deviation $[m/s^2]$, δ: wheel steer angle $[deg]$, ϕ_r: and road bank angle $[rad]$, $F_{y(f,r)}$: front and rear lateral forces in the vehicle coordinates $[N]$, $L_{(f,r)}$: distances from the CoG to the front and rear axles $[m]$, I_z: yaw moment of inertia $[Kg.m^{-2}]$, $C_{(f,r)}$: front and rear cornering stiffnesses $[N/rad]$, g: acceleration due to gravity $[m/s^2]$, , m: vehicle mass $[kgm^2]$.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Application example: single-track vehicle model

A linear two wheels vehicle model is considered. This model is composed of lateral and yaw motions. The lateral tire forces have been supposed to proportional to sideslip angles of each axle as follows:

$$F_{yf} = C_f \left(\delta - \frac{V_y + L_f \dot{\psi}}{V_x} \right), \ F_{yr} = -C_r \left(\frac{V_y - L_r \dot{\psi}}{V_x} \right)$$

$$(1)$$

Then, the state space representation can be written:

$$\begin{cases} \dot{x} = Ax + Bu + Ff \\ y = Cx \end{cases} \quad (2)$$

where: $x = \begin{bmatrix} \dot{Y} & \dot{\psi} & Y & \psi \end{bmatrix}^T$, $u = \delta$,

$$A = \begin{bmatrix} -\dfrac{2C_f + 2C_r}{mV_x(t)} & -\dfrac{2C_f L_f - 2C_r L_r}{mV_x(t)} & 0 & \dfrac{2C_f + 2C_r}{m} \\ -\dfrac{2C_f L_f - 2C_r L_r}{I_z V_x(t)} & -\dfrac{2C_f L_f^2 + 2C_r L_r^2}{I_z V_x(t)} & 0 & \dfrac{2C_f L_f - 2C_r L_r}{I_z} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$B = \begin{bmatrix} \dfrac{2C_f}{m} & \dfrac{2L_f C_f}{I_z} & 0 & 0 \end{bmatrix}^T$, $F = \begin{bmatrix} -g & 0 & 0 & 0 \end{bmatrix}^T$ and $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Here, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the control input, $y(t) \in \mathbb{R}^m$ is the output, and $f(t) \in \mathbb{R}^{n_\omega}$ is the disturbance input that satisfies $f \in L_2[0, \infty)$. A, B, C and F are system matrices with appropriate size.

2.2 Problem formulation

Model (2) is affected by several parametric variations. In fact, when the vehicle is subjected to high lateral accelerations and braking actions, the cornering stiffnesses C_f and C_r become coupled and nonlinear functions of some dynamical parameters of the vehicle like sideslip angles, longitudinal slip ratio, vertical forces, camber angle (for more details see Fig. 3 of section 4). Moreover, model (2) is also affected by variations of the longitudinal speed $V_x(t)$. Unfortunately, if we take into account all variations, (2) becomes nonlinear. Then, design of control and estimation algorithms becomes a hard task. To consider some uncertainties and parametric variations, we propose a switched continuous-time system:

$$\begin{cases} \dot{x} = \displaystyle\sum_{i=1}^{M} \alpha_i(t) \left[A_i x + B_i u + F_i f \right] \\ y = \displaystyle\sum_{i=1}^{M} \alpha_i(t) C_i x \end{cases} \quad (3)$$

where the function $\alpha_i(t)$ is a known switching signal

$$\alpha_i : \mathbb{R}^+ \longrightarrow \{0, 1\} \quad \sum_{i=1}^{M} \alpha_i(t) = 1, \quad t \in \mathbb{R}^+ \quad (4)$$

Our aim is to design a switched H_∞ controller, such that the output $y(t)$ of the closed-loop system tracks a given reference signal to satisfy the desired tracking performance. For this, consider the reference output $y_r(t)$ computed by the following dynamical reference model:

$$\begin{cases} y_r = \displaystyle\sum_{i=1}^{M} \alpha_i(t) C_{r_i} x_r \\ \dot{x}_r = \displaystyle\sum_{i=1}^{M} \alpha_i(t) \left[A_{r_i} x_r + F_{r_i} r \right] \end{cases} \quad (5)$$

where $y_r(t)$ has the same dimension as $y(t)$. $x_r(t) \in \mathbb{R}^{n_r}$ and $r(t) \in \mathbb{R}^{p_r}$ are respectively the reference state and the bounded reference input. A_{r_i}, C_{r_i} and F_{r_i} are appropriate matrices with A_{r_i} Hurwitz. Notice that the control design procedure assumes that both $y(t)$ and $y_r(t)$ are measurable outputs. For our purpose we define the following tracking output error:

$$\tilde{y}(t) = y(t) - y_r(t) \quad (6)$$

Therefore, the following augmented system can be obtained:

$$\begin{cases} \dot{\xi}(t) = \displaystyle\sum_{i=1}^{M} \alpha_i(t) \left[A_{ai} \xi(t) + B_{ai} u(t) + F_{ai} \omega(t) \right] \\ \tilde{y}(t) = \displaystyle\sum_{i=1}^{M} \alpha_i(t) C_{ai} \xi(t) \end{cases} \quad (7)$$

where $A_{ai} = \begin{bmatrix} A_i & 0 \\ 0 & A_{r_i} \end{bmatrix}$, $B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$, $F_{ai} = \begin{bmatrix} F_i & 0 \\ 0 & F_{r_i} \end{bmatrix}$, $C_{ai} = \begin{bmatrix} C_i & -C_{r_i} \end{bmatrix}$, $\xi(t) = \begin{bmatrix} x(t) \\ x_r(t) \end{bmatrix}$, $\omega(t) = \begin{bmatrix} f(t) \\ r(t) \end{bmatrix}$.

Applying the first order Euler approximation on model (7), the following discrete-time model is obtained:

$$\begin{cases} \xi(k+1) = \sum_{i=1}^{M} \alpha_i(k) \left[\bar{A}_i \xi(k) + \bar{B}_i u(k) + \bar{F}_i \omega(k) \right] \\ \tilde{y}(k) = \sum_{i=1}^{M} \alpha_i(k) \bar{C}_i \xi(k) \end{cases} \quad (8)$$

where the functional switching signal $\alpha_i(k)$ is

$$\alpha_i : Z^+ \longrightarrow \{0, 1\}, \quad \sum_{i=1}^{M} \alpha_i(k) = 1, \quad k \in Z^+ = \{0, 1, \cdots\} \quad (9)$$

For model (8), the following switched H_∞ optimal control problem is addressed.
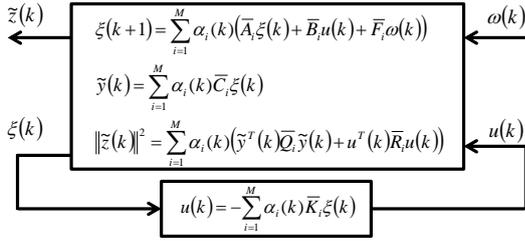

Fig. 1. Switched H_∞ State Feedback Optimal Control

Problem: Consider the following switched H_∞ state feedback optimal control for the switched model (8)

$$u(k) = -\sum_{i=1}^{M} \alpha_i(k) \bar{K}_i \xi(k) \quad (10)$$

where the gain $\bar{K}_i$ has to be computed such that:

C1. The closed-loop system $\xi(k+1) = \sum_{i=1}^{M} \alpha_i(k)(\bar{A}_i - \bar{B}_i \bar{K}_i)\xi(k)$ is globally asymptotically stable when $\omega(k) = 0$;

C2. the following optimization problem is feasible

$$\min \gamma$$

subject to $\qquad\qquad\qquad (11)$

$$\|\tilde{z}(k)\| < \gamma \|\omega(k)\|$$

where

$$\|\tilde{z}(k)\|^2 = \sum_{i=1}^{M} \alpha_i(k) \left[\tilde{y}^T(k) \bar{Q}_i \tilde{y}(k) + u^T(k) \bar{R}_i u(k) \right] \quad (12)$$

with $\bar{Q}_i = \bar{Q}_i^T \geq 0$ and $\bar{R}_i = \bar{R}_i^T > 0$ are chosen by the designer. Notice that the C2 combines H_2 and H_∞ performances Abu-Khalaf et al. [2010].

3. SWITCHED H_∞ OPTIMAL CONTROL

In this section, the switched H_∞ control (10) is designed using a switched Lyapunov function method. The main objective is to compute the switched control (10) in order that the following closed-loop system

$$\begin{cases} \xi(k+1) = \sum_{i=1}^{M} \alpha_i(k) \left[(\bar{A}_i - \bar{B}_i \bar{K}_i)\xi(k) + \bar{F}_i \omega(k) \right] \\ \tilde{y}(k) = \sum_{i=1}^{M} \alpha_i(k) \bar{C}_i \xi(k) \end{cases}$$
$$(13)$$

is stable (i.e. C1) and check the H_∞ norm (i.e. C2). For this, consider the following theorem.

Theorem 1. Suppose that for $(i,j) \in \{1, \cdots, M\}^2$, the pair $(\bar{A}_i, \bar{B}_i)$ is stabilizable. If there exist a positive constant $\gamma > 0$, symmetric positive definite matrices X_i, X_j and matrices Γ_i such that the following optimization problem is satisfied for $(i,j) \in \{1, \cdots, M\}^2$:

$$\min_{P_i, P_j, \Gamma_i} \gamma$$

subject to (14)

$$\begin{bmatrix} -X_i & 0 & X_i \bar{A}_i^T - \Gamma_i^T \bar{B}_i^T & X_i & \Gamma_i^T \\ * & -\gamma^2 I & \bar{F}_i^T & 0 & 0 \\ * & * & -X_j & 0 & 0 \\ * & * & * & -\bar{Q}_i^{-1} & 0 \\ * & * & * & * & -\bar{R}_i^{-1} \end{bmatrix} < 0$$

then, the gain of (10) are given by $\bar{K}_i = \Gamma_i X_i^{-1}$.

Proof 1. To establish sufficient conditions for the existence of switched H_∞ optimal control (10) such that the closed-loop system (13) satisfies the conditions C1 and C2, consider the following switched Lyapunov function:

$$V(\xi, k) = \sum_{i=1}^{M} \alpha_i(k) \xi^T(k) P_i \xi(k) \quad (15)$$

Sufficient conditions for the existence of (10), according to C1 and C2 are related to the existence of a switched Lyapunov function V_k such that the following inequality is satisfied:

$$V(k+1) - V(k) + \sum_{i=1}^{M} \alpha_i(k) \tilde{y}^T(k) \bar{Q}_i \tilde{y}(k)$$
$$+ \sum_{i=1}^{M} \alpha_i(k) u^T(k) \bar{R}_i u(k) - \gamma^2 \omega^T(k)\omega(k) < 0 \quad (16)$$

with $V(k+1) - V(k) = \sum_{j=1}^{M} \alpha_j(k+1)\xi^T(k+1)P_j\xi(k+1) - \sum_{i=1}^{M} \alpha_i(k)\xi^T(k)P_i\xi(k)$. For all switches, we consider the following particular case:

$$\begin{cases} \alpha_i(k) = 1 & \text{and } \alpha_{l \neq i}(k) = 0 \\ \alpha_j(k+1) = 1 & \text{and } \alpha_{l \neq j}(k+1) = 0 \end{cases} \quad (17)$$

According to (17), (13) and (16) become respectively

$$\begin{cases} \xi(k+1) = (\bar{A}_i - \bar{B}_i \bar{K}_i)\xi(k) + \bar{F}_i \omega(k) \\ \tilde{y}(k) = \bar{C}_i \xi(k) \end{cases} \quad (18)$$

and

$$\xi^T(k+1)P_j\xi(k+1) - \xi^T(k)P_i\xi(k) + \tilde{y}^T(k)\bar{Q}_i\tilde{y}(k)$$
$$+ \xi^T(k)\bar{K}_i^T \bar{R}_i \bar{K}_i \xi(k) - \gamma^2 \omega^T(k)\omega(k) < 0 \quad (19)$$

Computing (19) along the solution of the closed-loop system (18), gives:

$$[\, \xi(k) \ \omega(k) \,] \times$$
$$\begin{bmatrix} \bar{\Phi}_i - P_i & (\bar{A}_i - \bar{B}_i \bar{K}_i)^T P_j \bar{F}_i \\ \bar{F}_i^T P_j (\bar{A}_i - \bar{B}_i \bar{K}_i) & -\gamma^2 I + \bar{F}_i^T P_j \bar{F}_i \end{bmatrix} \begin{bmatrix} \xi(k) \\ \omega(k) \end{bmatrix} \quad (20)$$

where $\bar{\Phi}_i = (\bar{A}_i - \bar{B}_i \bar{K}_i)^T P_j (\bar{A}_i - \bar{B}_i \bar{K}_i) + \bar{C}_i^T \bar{Q}_i \bar{C}_i + \bar{K}_i^T \bar{R}_i \bar{K}_i$. Then, $\Delta V(x, k)$ is negative definite for any nonzero vector $[\, \xi(k) \ \omega(k) \,]$ if

$$\begin{bmatrix} \bar{\Phi}_i - P_i & (\bar{A}_i - \bar{B}_i \bar{K}_i)^T P_j \bar{F}_i \\ \bar{F}_i^T P_j (\bar{A}_i - \bar{B}_i \bar{K}_i) & -\gamma^2 I + \bar{F}_i^T P_j \bar{F}_i \end{bmatrix} < 0 \quad (21)$$

Using Schur complement and pre- and post-multiplying by $Z_i = diag(P_i^{-1}, I, I, I, I)$, (21) becomes

$$\begin{bmatrix} -P_i^{-1} & 0 & P_i^{-1}(\bar{A}_i^T - \bar{K}_i^T \bar{B}_i^T) & P_i^{-1} & P_i^{-1}\bar{K}_i^T \\ * & -\gamma^2 I & \bar{F}_i^T & 0 & 0 \\ * & * & -P_j^{-1} & 0 & 0 \\ * & * & * & -\bar{Q}_i^{-1} & 0 \\ * & * & * & * & -\bar{R}_i^{-1} \end{bmatrix} < 0 \tag{22}$$

Substituting $\Gamma_i = \bar{K}_i X_i$, $X_i = P_i^{-1}$ and $X_j = P_j^{-1}$ into (22), (14) is obtained.

4. SIMULATION RESULTS USING EXPERIMENTAL DATA

In this section, simulation tests of the proposed switched H_∞ optimal control using experimental data are presented. The experimental data are acquired by a laboratory vehicle at a frequency of $\frac{1}{h} = 200\,Hz$. These data are used to compute the reference trajectories. The aim important measurements provided by this vehicle are: longitudinal, lateral and vertical speeds, yaw, roll and pitch rates, driver steering angle, driver steering torque and some other measurements. The measurements used here are illustrated by the grey curves on Figs. 3, 4, 5, 6 and 7.
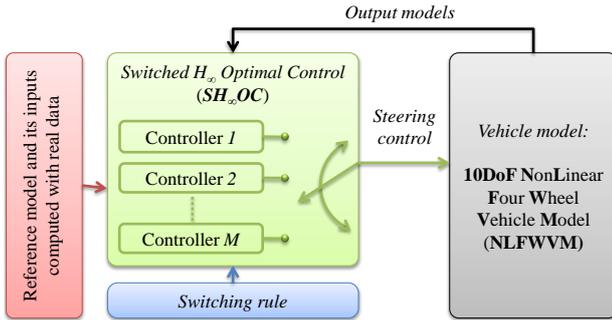

Fig. 2. A diagram block of switched H_∞ optimal control coupled reference model and nonlinear vehicle model

The reference model (5) used for our simulations describes the the first and second derivatives of lateral deviation is:

$$\begin{bmatrix} \dot{x}_{r1} \\ \dot{x}_{r2} \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{h} & 0 \\ 0 & \dfrac{-1}{h} \end{bmatrix} \begin{bmatrix} x_{r1} \\ x_{r2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \tag{23}$$

where h is the sample time, $r_1(t) = a_{y_r}(t) + (V_{y_r}(t) + V_{x_r}(t)\psi_r(t))/h$ and $r_2(t) = V_{y_r}(t) + V_{x_r}(t)\psi_r(t) + Y_r(t)/h$ are the bounded reference inputs. Fig. 2 shows the closed-loop system composed from: the reference model, the switched H_∞ optimal steering vehicle control and the vehicle model (NLFWVM). As pointed previously, the real data are used to compute the reference inputs r_1 and r_2 of model (23), then, the closed-loop obtained behavior is compared to the actual one.

To perform some simulation tests under high lateral accelerations, a more realistic Non Linear Four Wheels Vehicle Model [1] is used as a complete vehicle simulator.

[1] The 10DoF Non Linear Four Wheels Vehicle Model (NLFWVM) is composed of: three translational motions (longitudinal V_x, lateral V_y
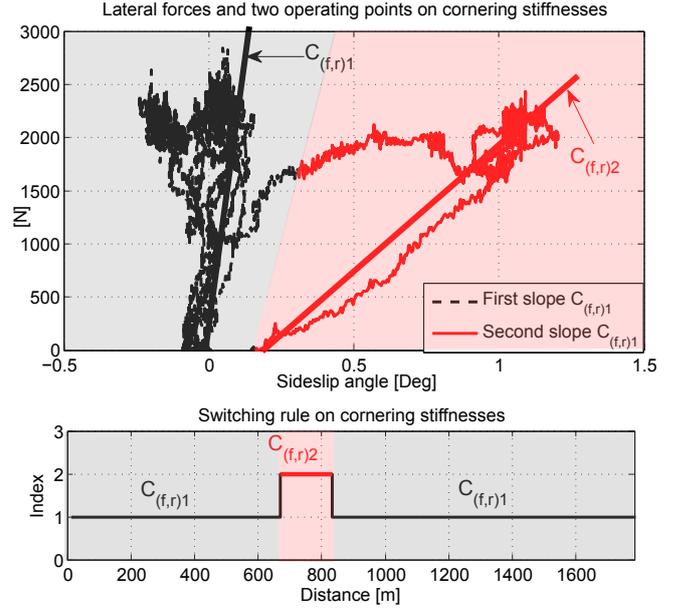

Fig. 3. Experimental lateral tire force characteristic, switching rule on cornering stiffnesses and two operating points $C_{(f,r)_1}$ and $C_{(f,r)_2}$
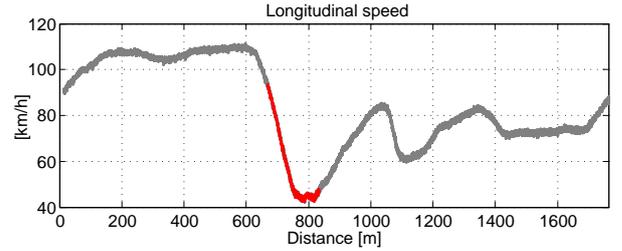

Fig. 4. Measured longitudinal speed and braking action between the positions $650\,m$ and $850\,m$

In this simulation results, the switching rule is defined on cornering stiffnesses C_f and C_r. This in order to overcome the assumption on the linearity of lateral tire forces. For this, two operating points are chosen as shown on Fig. 3. Moreover, C_f and C_r are the slopes of tire characteristic, and their variations can be observed on Fig. 3 during the braking action depicted on Fig. 4 of longitudinal speed between abscissa $630\,m$ and $830\,m$. In fact, as depicted on Fig. 3, the experimental tire characteristic presents approximately two slopes, then, we define two values $C_{(f,r)_1}$ and $C_{(f,r)_2}$. According to the characteristic of Fig. 4 the following switching rule can be considered:

$$\begin{cases} C_{(f,r)} = C_{(f,r)1} & \text{If } \beta < \beta^* \\ C_{(f,r)} = C_{(f,r)2} & \text{If } \beta > \beta^* \end{cases} \tag{24}$$

with β^* is the switching threshold on the sideslip angle.

For this, a switched controller can then be designed with unmeasurable premise variable. Moreover, the system must be robust again the premise variables (Kiss et al.

and vertical V_z), three rotational motions (roll ϕ, pitch θ and yaw ϕ) and dynamical models of the four wheels. The forces of NLFWM are computed using nonlinear tire models Pacejka [2005] in order to simulate the realistic behavior of vehicle. Moreover, such tire models take into account the coupling of vertical, longitudinal and lateral motions.

[2011], Ichalal et al. [2010]). In our case, the premise variables is β which can be estimated online (see for example Villagra et al. [2008])

Consequently, two local models are defined, then, $M = 2$ and $(i\ , j) \in= \{1\ , 2\}^2$. Consequently, the stability of switched H_∞ optimal control for any switching signal is guaranteed by the resolution of 4 LMI constraints of theorem 1 to find two Lyapunov matrices X_1 and X_2.

Notice that the tire characteristic plotted on the upper part of Fig. 3, is obtained using the measured lateral force and the sideslip angle. These measurements are acquired by a laboratory vehicle during a trial performed on a real race track under high dynamic loads (lateral acceleration $-5\ m/s^2 \leq a_y \leq 5\ m/s^2$ see Fig. 5). All measurements used for our simulation are shown by grey curves of Figs. 5, 6 and 7. We can observe that the obtained results are similar to the measured ones. The dynamic variables plotted on these Figs. are: lateral accelerations, yaw rates, longitudinal accelerations, longitudinal speed. The performances results in terms of tracking trajectories are illustrated on Fig. 8, which are less than $1\,deg$ on the yaw angle and $0.1\,m$ on the lateral deviation. Let us emphasize that these errors are quite small.

The steering angle computed by the controller is similar to the measured one, this, for any maneuver as illustrated in Figs. 6, 7 and 8. Notice that the steering angle of switched control remains stable even during the switching phases at the positions $630\,m$ and $830\,m$ (see the switching rule depicted by the lower part of Fig. 3).
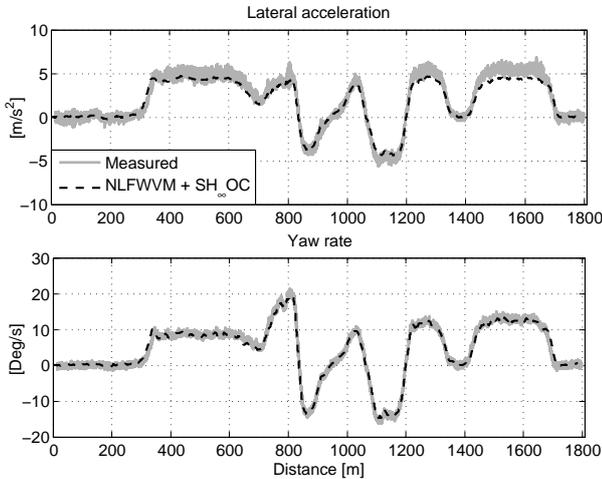

Fig. 5. lateral acceleration and yaw rate: measurements and those computed by L2WVM + SH_∞OC and NLFWVM + SH_∞OC
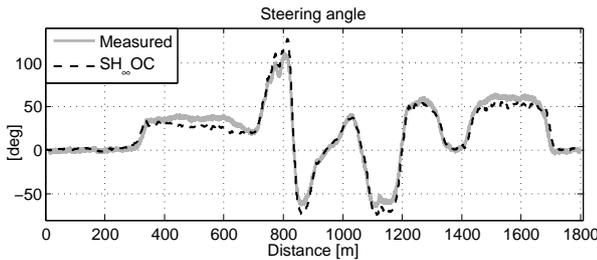

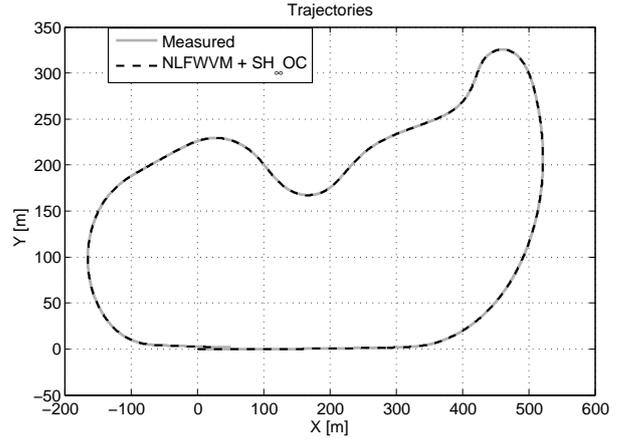Fig. 6. Steering angle: Measured and those computed by SH_∞OC + NLFWVM

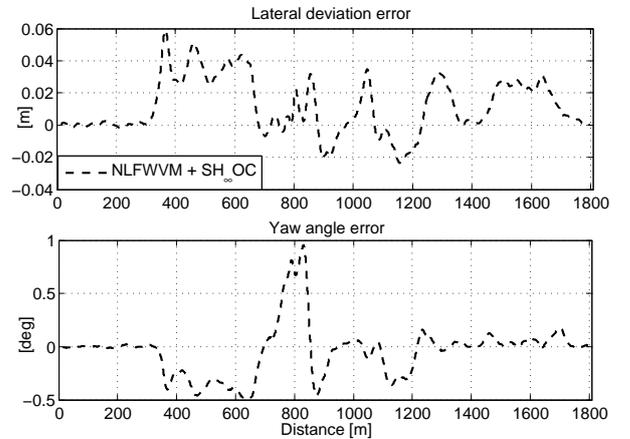Fig. 7. Trajectories: Reference and those of NLFWVM + SH_∞OC

Fig. 8. Roll angle and roll rate: estimated and measured

For the subsystems $((\bar{A}_i - \bar{B}_i\bar{K}_i), \bar{F}_i, \bar{C}_i)$, the UI attenuation properties, between ω to y can be observed on Fig. 9.

In this work, the switching strategy has been constructed on cornering stiffnesses C_f and C_r (see (24)). This strategy uses the measured sideslip angle. However, this measurement is provided by expensive sensors. This problem can be solved using estimators from low cost sensors available on the vehicle (see Menhour et al. [2012] for lateral forces estimation approach).

5. CONCLUSIONS AND FUTURE WORK

In this work, a switched H_∞ optimal control is presented. Conditions of global convergence of this control problem are established and proved. All conditions are presented in terms of linear matrix inequalities (LMI) using switched Lyapunov function approach. An application on steering vehicle control is given. Simulation results under high lateral acceleration are presented. These simulations are conducted using the experimental data. By measured lateral force and sideslip angle, a switching strategy on cornering stiffnesses C_f and C_r is constructed.

For future works, estimation methods of sideslip angle, lateral and vertical forces will be introduced in order to establish a new switching rule on C_f and C_r.
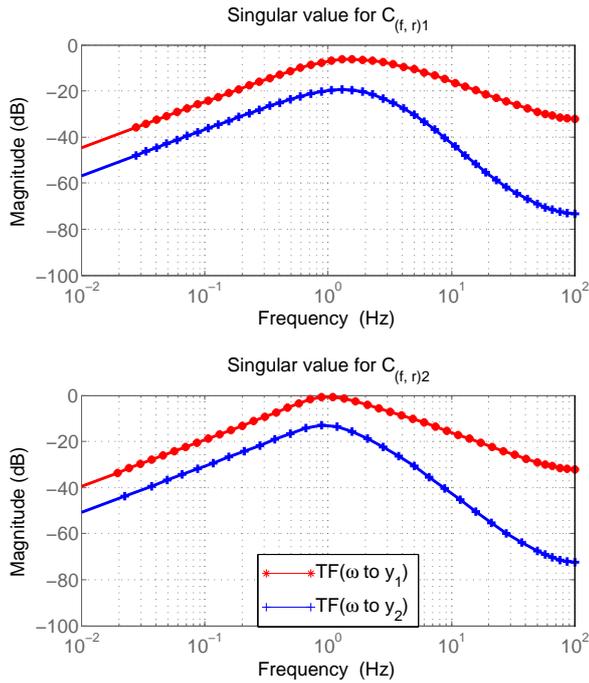
Fig. 9. Transfer function of system $((\bar{A}_i - \bar{B}_i \bar{K}_i), \bar{F}_i, \bar{C}_i)$ for two values of cornering stiffnesses $C_{(f,r)1}$ and $C_{(f,r)2}$ coefficients, between ω to $\tilde{y}$

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