Base Station Cooperation for Power Minimization in the Downlink: Large System Analysis

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***Abstract*—This work focuses on the downlink of a large-scale multi-cell multi-user MIMO system in which L base stations (BSs) of N antennas each communicate with KL single-antenna user equipments. We consider the design of the linear precoder that minimizes the total power consumption while ensuring target user rates. Two configurations with different degrees of cooperation among BSs are considered: the coordinated beamforming scheme (only channel state information is shared between BSs) and the network-MIMO technology (channel state and data cooperation). The analysis is conducted assuming that N and K grow large with a non trivial ratio K/N. In both configurations, tools of random matrix theory are used to compute, often in closed form, deterministic approximations for: (*i*) the parameters of the optimal precoder; (*ii*) the powers needed to ensure target rates; and (*iii*) the total transmit power. These results are instrumental to get further insight into the structure of the optimal precoder and also to reduce the complexity of its implementation in large-scale networks. Numerical results are used to validate the asymptotic analysis in the finite system regime and to make comparisons among the two different configurations.**

I. Introduction

In this work, we focus on the problem of designing the optimal linear precoder for minimizing the total transmit power while ensuring a set of target user rates [1], [2] in multi-cell networks. This problem has received great attention in the last years [3] and is gaining renewed interest nowadays in particular due to the emerging research area of green multi cellular networks [4]. We specifically consider the downlink of a multi-cell multi-user MIMO system in which L base stations (BSs) equipped with N antennas each communicate with K single-antenna user equipments terminal (UEs). Within this setting, several configurations with different degrees of cooperation can be envisioned [5]. In this work, the following two are considered: (*i*) the coordinated beamforming (CoBF) scheme in [6] in which each BS sends data to its own users only but channel state information (CSI) is shared between the L BSs so that the interference generated in other cells can be taken into account; this has the advantage of not having to distribute all users' data to all BSs, and (*ii*) the fully cooperative scheme widely known in the literature as network-MIMO or coordinated multipoint MIMO (CoMP) [5]. In both scenarios, under the assumption of perfect CSI, the optimal linear precoder is known to be a function of some Lagrange multipliers, the computation of which can be performed using convex optimization tools or solving a fixed-point problem [3]. Although numerically possible, both approaches do not provide any insight into the structure of the optimal precoder. Moreover, the computation must be performed for any new realization of the propagation channels, which might be too computationally cumbersome when the network size becomes large (as envisioned in future 5G networks).

To overcome these issues, we follow the same approach as in other works for single- or multi-cell networks [7]–[13] and resort to the asymptotic regime where N and K grow large with a bounded ratio. The design and analysis of the considered networks is performed under the assumption of imperfect CSI for the UEs. Unlike most previous works [10], [13], the asymptotically optimal values of the Lagrange multipliers are computed using recent results from random matrix theory [14], which provides us with a much simpler means to overcome the technical difficulties arising with the application of standard random matrix theory tools (see e.g., [13]). These results are then exploited to compute explicit expressions for the asymptotic signal-to-interference plus noise ratios (SINRs), which are eventually used to obtain the asymptotic powers needed to ensure target rates as well as the asymptotic total transmit power. As shall be seen, all the aforementioned deterministic approximations are found to depend only on the long-term channel attenuations of the UEs, the relative strength of interference among BSs, the target rates and the quality of the channel estimates. As a notable outcome of this work, the above analysis provides a unified framework that can be used to compare the considered networks under different settings and to eventually get insights on how the different parameters affect their performance. Numerical results are used to show that the asymptotic analysis well approximate the performance of the considered networks in the finite system regime.

The main literature related to this work is represented by [9]–[13], [15]. Specifically, a single-cell setting is considered in [9], [15] while a coordinated beamforming network is investigated in [10]. Unlike [10], we provide closed-form expressions for the Lagrange multipliers, which are instrumental to also compute closed-form expressions for SINRs and transmit powers. In [12], the authors focus on the sum rate of a network MIMO under the assumption of regularized zero-

L. Sanguinetti is funded by the People Programme (Marie Curie Actions) FP7 PIEF-GA-2012-330731 Dense4Green. This research has been also supported by the ERC Starting Grant 305123 MORE and by the HUAWEI project RMTin5G.

forcing precoding. In [13], the authors provide an asymptotic analysis of both present network configurations but for the simplest case in which only two cells are present and CSI is perfect. The analysis is also conducted under the restrictive assumption that the same rate is required by all UEs. We importantly show in the present article that, within our framework, there is no substantial additional difficulty in treating the more general setting of interest here. Due to space limitations, the proofs are omitted but can be found in the extended version [16].

II. System Model

Consider the downlink of a multi-cell multi-user MIMO system composed of L cells, the BS of each cell comprising N antennas to communicate with K single-antenna UEs. As mentioned previously, we consider two different configurations with different degrees of cooperation: the coordinated beamforming scheme in [6] and the coordinated multipoint MIMO [5]. In both scenarios, we are interested in minimizing the total transmit power P_T while satisfying rate constraints at the UEs. In doing so, we assume that the feasibility conditions are satisfied [1]–[3], [10].

A. Coordinated Beamforming

In the CoBF setting, each UE is attached to a specific serving BS while receiving interfering data from other BSs. As such, we shall use a double index notation to refer to each UE as e.g., "user k in cell j". Under this convention, let thus define $\mathbf{h}_{ljk} \in \mathbb{C}^N$ as the channel from BS l to UE k in cell j, given by

$$\mathbf{h}_{ljk} = \sqrt{d_{ljk}}\mathbf{w}_{ljk} \tag{1}$$

where $\mathbf{w}_{ljk} \in \mathbb{C}^N$ is the small-scale fading channel assumed to be Gaussian with zero mean and unit covariance and d_{ljk} accounts for the corresponding large-scale channel fading or path-loss (from BS l to UE k in cell j). Denoting by $\mathbf{g}_{jk} \in \mathbb{C}^N$ the precoding vector of UE k in cell j, its received signal can be written as

$$y_{jk} = \mathbf{h}_{jjk}^H \mathbf{g}_{jk} s_{jk} + \sum_{(l,i)\neq(j,k)} \mathbf{h}_{ljk}^H \mathbf{g}_{li} s_{li} + n_{jk} \tag{2}$$

with $s_{li} \in \mathbb{C}$ being the signal intended to user i in cell l, assumed independent across (l,i) pairs, of zero mean and unit variance, and $n_{jk} \sim \mathcal{CN}(0,\sigma^2)$. Under the assumption of Gaussian codebooks and target UE rates $\{r_{jk}\}$, the power minimization problem can be formulated as:

$$\begin{aligned} \min_{\{\mathbf{g}_{jk}\}} \quad & P_T = \sum_{j=1}^{L}\sum_{k=1}^{K} \mathbf{g}_{jk}^H \mathbf{g}_{jk} \\ \text{s.t.} \quad & \frac{|\mathbf{h}_{jjk}^H \mathbf{g}_{jk}|^2}{\sum\limits_{(l,i)\neq(j,k)} |\mathbf{h}_{ljk}^H \mathbf{g}_{li}|^2 + \sigma^2} \geq \gamma_{jk} \quad \forall j,k \end{aligned} \tag{3}$$

where $\gamma_{jk} = 2^{r_{jk}} - 1$ is the corresponding SINR constraint. As shown in [3], the optimal $\mathbf{g}_{jk}^\star$ (solution of (3)) is found to be $\mathbf{g}_{jk}^\star = \sqrt{\frac{p_{jk}^\star}{N}} \frac{\mathbf{v}_{jk}^\star}{||\mathbf{v}_{jk}^\star||}$ with

$$\mathbf{v}_{jk}^\star = \left(\sum_{l=1}^{L}\sum_{i=1}^{K} \frac{\lambda_{li}^\star}{N} \mathbf{h}_{jli}\mathbf{h}_{jli}^H + \mathbf{I}_N \right)^{-1} \mathbf{h}_{jjk} \tag{4}$$

where $\{\lambda_{li}^\star/N\}$ are the Lagrange multipliers associated to the SINR constraints obtained as the unique fixed point solution of the following set of equations [1]–[3]:

$$\lambda_{jk}^\star = \frac{(1+1/\gamma_{jk})^{-1}}{\mathbf{h}_{jjk}^H \left(\sum\limits_{l=1}^{L}\sum\limits_{i=1}^{K} \lambda_{li}^\star \mathbf{h}_{jli}\mathbf{h}_{jli}^H + N\mathbf{I}_N \right)^{-1} \mathbf{h}_{jjk}} \quad \forall j,k. \tag{5}$$

The optimal $\{p_{jk}^\star\}$ must be computed such that the SINR constraints in (3) are satisfied with equality $\forall j,k$ [3].

B. Coordinated Multipoint MIMO

In the CoMP setting, each UE is jointly served by all BSs. In other words, there exists no cell-user association and thus the UEs can be indexed as k from 1 to KL instead of as a pair (j,k) for $j = 1,\ldots,L$ and $k = 1,\ldots,K$. Let then now $\mathbf{h}_k = [\mathbf{h}_{1k}^T,\ldots,\mathbf{h}_{L,k}^T]^T$ with $\mathbf{h}_{jk} \in \mathbb{C}^N$ being the channel from BS j to user k given by

$$\mathbf{h}_{jk} = \sqrt{d_{jk}}\mathbf{w}_{jk} \tag{6}$$

where $\mathbf{w}_{jk} \in \mathbb{C}^{NL}$ is the small-scale fading channel and d_{jk} accounts for the corresponding path-loss (from BS j to UE k). Denoting by $\mathbf{g}_k \in \mathbb{C}^{NL}$ the joint precoding vector for UE k, its received signal can be written as

$$y_k = \mathbf{h}_k^H \mathbf{g}_k s_k + \sum_{i=1, i\neq k}^{KL} \mathbf{h}_k^H \mathbf{g}_i s_i + n_k \tag{7}$$

with s_i being the signal intended to user i, independent across i, of zero mean and unit variance, and $n_k \sim \mathcal{CN}(0,\sigma^2)$. In the above setting, the power minimization problem takes the form:

$$\begin{aligned} \min_{\{\mathbf{g}_k\}} \quad & P_T = \sum_{k=1}^{KL} \mathbf{g}_k^H \mathbf{g}_k \\ \text{s.t.} \quad & \frac{|\mathbf{h}_k^H \mathbf{g}_k|^2}{\sum\limits_{i\neq k} |\mathbf{h}_k^H \mathbf{g}_i|^2 + \sigma^2} \geq \gamma_k \quad \forall k \end{aligned} \tag{8}$$

where $\gamma_k = 2^{r_k} - 1$ with r_k being the rate constraint of UE k. The solution of (8) is found to be $\mathbf{g}_k^\star = \sqrt{\frac{p_k^\star}{NL}} \frac{\mathbf{v}_k^\star}{||\mathbf{v}_k^\star||}$ with [3]

$$\mathbf{v}_k^\star = \left(\sum_{i=1}^{KL} \frac{\lambda_i^\star}{NL} \mathbf{h}_i\mathbf{h}_i^H + \mathbf{I}_{NL} \right)^{-1} \mathbf{h}_k \tag{9}$$

where $\{\lambda_i^\star/(NL)\}$ are such that [1]–[3]:

$$\lambda_k^\star = \frac{(1+1/\gamma_k)^{-1}}{\mathbf{h}_k^H \left(\sum\limits_{l=1}^{L}\sum\limits_{i=1}^{K} \lambda_i^\star \mathbf{h}_i\mathbf{h}_i^H + NL\mathbf{I}_{NL} \right)^{-1} \mathbf{h}_k} \quad \forall k. \tag{10}$$

As before, the optimal $\{p_k^\star\}$ are computed such that the SINR constraints in (8) are satisfied with equality $\forall k$ [3].

III. Large System Analysis

Let $\boldsymbol{\lambda}^\star$ and $\mathbf{p}^\star$ denote for both settings above the vectors collecting the Lagrange multipliers and power values, respectively. As shown previously, the precoding vectors are parameterized by $\boldsymbol{\lambda}^\star$ and $\mathbf{p}^\star$, where $\boldsymbol{\lambda}^\star$ needs to be evaluated by solving a set of fixed-point equations. This is a computationally demanding task when NL and KL are large since the matrix inversion operation in (5) or (10) must be computed for each new set of channel vectors, with complexity proportional to N^2KL or $(NL)^2KL$. Besides, from a practical standpoint, the operator that manages the evaluation of $\boldsymbol{\lambda}^\star$ needs to be aware of all channels $\mathbf{h}_{ljk}$, thus implying some channel exchange procedure within the network at the rate of the fading channel evolution (hence the "coordinated beamforming" phrase). Finally, computing $\boldsymbol{\lambda}^\star$ as the fixed point of (5) or (10) does not provide any insight into the optimal structure of both $\boldsymbol{\lambda}^\star$ and $\mathbf{p}^\star$. To overcome these issues, we exploit the statistical distribution for $\mathbf{h}_{ljk}$ and $\mathbf{h}_{lk}$ and the large values of N, K to compute deterministic approximations of $\boldsymbol{\lambda}^\star$ and $\mathbf{p}^\star$ [17]. For technical purposes, we assume the following grow rate of the system dimensions:

Assumption 1. *As $N \to \infty$,*

$$0 < \liminf_{N\to\infty} K/N \leq \limsup_{N\to\infty} K/N < \infty.$$

In doing so, we further assume that only imperfect CSI is available at the BSs. Since the optimal linear precoder for both configurations is not known when only imperfect CSI is available, we overcome this issue by simply replacing the true channels with their estimates (which should be an accurate procedure for good CSI quality).

A. Coordinated Beamforming

First assume that $\mathbf{w}_{ljk} \in \mathbb{C}^N$ defined in (6) is a Gaussian vector with zero mean and covariance $\mathbf{I}_N$. We denote by $\widehat{\mathbf{h}}_{ljk}$ an estimate of $\mathbf{h}_{ljk}$ and assume, similar to [7], that

$$\widehat{\mathbf{h}}_{ljk} = \sqrt{d_{ljk}}\left(\sqrt{1-\tau_{ljk}^2}\mathbf{w}_{ljk} + \tau_{ljk}\mathbf{z}_{ljk}\right) \tag{11}$$

where $\mathbf{z}_{ljk} \sim \mathcal{CN}(0, \mathbf{I}_N)$ accounts for the independent channel estimation errors. The parameter τ_{ljk} reflects the accuracy or quality of the channel estimate, i.e., $\tau_{ljk} = 0$ for perfect CSI and $\tau_{ljk} = 1$ for a channel estimate completely uncorrelated to the genuine channel. Replacing $\mathbf{h}_{ljk}$ with $\widehat{\mathbf{h}}_{ljk}$ into (5) yields

$$\lambda_{jk}^\star = \frac{(1+1/\gamma_{jk})^{-1}}{\widehat{\mathbf{h}}_{jjk}^H\left(\sum\limits_{l=1}^{L}\sum\limits_{i=1}^{K}\lambda_{li}^\star\widehat{\mathbf{h}}_{jli}\widehat{\mathbf{h}}_{jli}^H + N\mathbf{I}_N\right)^{-1}\widehat{\mathbf{h}}_{jjk}} \quad \forall j,k. \tag{12}$$

The above formulation for $\lambda_{jk}^\star$ prevents any insightful analysis of the system performance. By a large dimensional analysis, exploiting recent tools from random matrix theory (see notably [14]), we shall subsequently show that $\lambda_{jk}^\star$ gets asymptotically close to a deterministic quantity as N and K grow large as for Assumption 1. This quantity provides clear insights on the behavior of the precoder and the system as a whole.

For technical reasons, the following reasonable assumptions are imposed on the system settings [7]:

Assumption 2. *The $\{d_{ijk}\}$ and $\{\gamma_{jk}\}$ satisfy*

$$\limsup_N \max_{i,j,k}\{d_{ijk}\} < \infty$$
$$\limsup_N \max_{j,k}\{\gamma_{jk}\} < \infty.$$

Our first technical result lies in the following theorem:

Theorem 1. *Let Assumptions 1 and 2 hold. Then, $\max_{jk}|\lambda_{jk}^\star - \overline{\lambda}_{jk}| \to 0$ almost surely with*

$$\overline{\lambda}_{jk} = \frac{1}{\eta_j}\frac{\gamma_{jk}}{d_{jjk}} \tag{13}$$

where $\{\eta_j\}$ are the unique positive solution to the following set of equations

$$\eta_j = \left(\frac{1}{N}\sum_{l=1}^{L}\sum_{i=1}^{K}\frac{\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{1}{\eta_l}}{1+\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{\eta_j}{\eta_l}} + 1\right)^{-1} \quad \forall j \tag{14}$$

or, equivalently,

$$\eta_j = 1 - \frac{1}{N}\sum_{l=1}^{L}\sum_{i=1}^{K}\frac{\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{\eta_j}{\eta_l}}{1+\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{\eta_j}{\eta_l}} \quad \forall j. \tag{15}$$

Proof: The main difficulty lies in the implicit definition of the $\lambda_{jk}^\star$'s. A first step consists in heuristically discarding the implicit structure to retrieve the expression for $\overline{\lambda}_{jk}$. To proceed with an accurate proof, in [16] we follow similar steps as in [14] (in a completely different context though). Similar results are obtained in [10] following a different approach. ■

Some important insights can be readily extracted from Theorem 1. In sharp contrast to (5), the computation of $\overline{\lambda}_{jk}$ in (12) only requires the knowledge of SINR constraints and average channel attenuations. The latter can be accurately estimated and easily exchanged between BSs because they change slowly with time (relative to the small-scale fading). The Lagrange multiplier λ_{jk} is known to act as a user priority parameter that implicitly determines how much interference UE k in cell j may induce to the other UEs. Interestingly, its asymptotic value $\overline{\lambda}_{jk}$ is proportional to the SINR γ_{jk} and inversely proportional to d_{jjk} such that users with weak channels experience larger values. This means that within cell j higher priority is given to UEs that require high performance or have weak propagation conditions.

Observe also that $0 < \eta_j \leq 1$ acts as a cell priority parameter: higher priority is given to cell j if η_j is small [16]. This depends on the ratios $\{d_{jli}/d_{lli}\}$ in (15). In particular, d_{jli}/d_{lli} describes the relative strength of the interference received at UE i in cell l from BS j; it is almost one for cell edge UEs of neighboring cells, while it is almost zero when cell l is very distant from BS j. In other words, higher priority is given to those cells that create high interference, as it should.

We now proceed to computing the asymptotic powers $\{\overline{p}_{jk}\}$ satisfying (at least approximately so, for all large N, K) the

SINR constraints $\{\gamma_{jk}\}$. A known problem with the asymptotic analysis is that the target rates are not guaranteed to be achieved when N is finite and relatively small (e.g., [11]). This is because the approximation errors translate into fluctuations of the resulting SINR values. However, these errors vanish rapidly when N takes large yet finite values as it is envisioned for massive MIMO systems.

To proceed, we first compute the asymptotic values of the SINRs. Replacing $\widehat{\mathbf{g}}_{jk}$ with $\widehat{\mathbf{g}}_{jk}^{\star}$, the SINR of user k in cell j takes the form

$$\mathrm{SINR}_{jk} = \frac{\frac{p_{jk}}{N}\frac{|\mathbf{h}_{jjk}^H\widehat{\mathbf{v}}_{jk}^{\star}|^2}{\|\widehat{\mathbf{v}}_{jk}^{\star}\|^2}}{\sum\limits_{(l,i)\neq(j,k)}^{L}\frac{p_{li}}{N}\frac{|\mathbf{h}_{ljk}^H\widehat{\mathbf{v}}_{li}^{\star}|^2}{\|\widehat{\mathbf{v}}_{li}^{\star}\|^2}+\sigma^2} \tag{16}$$

where

$$\widehat{\mathbf{v}}_{jk} = \left(\sum_{\ell=1}^{L}\sum_{i=1}^{K}\frac{\lambda_{li}}{N}^{\star}\widehat{\mathbf{h}}_{jli}\widehat{\mathbf{h}}_{jli}^H+\mathbf{I}_N\right)^{-1}\widehat{\mathbf{h}}_{jjk}. \tag{17}$$

We then have the following result.

Lemma 1. *Under Assumptions 1 and 2,* $\max_{jk}|\mathrm{SINR}_{jk}-\overline{\mathrm{SINR}}_{jk}|\to 0$ *almost surely with*

$$\overline{\mathrm{SINR}}_{jk} = p_{jk}d_{jjk}\left(1-\tau_{jjk}^2\right)\frac{1-\frac{1}{N}\sum\limits_{l=1}^{L}\sum\limits_{i=1}^{K}\frac{\left(\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{\eta_j}{\eta_l}\right)^2}{\left(1+\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{\eta_j}{\eta_l}\right)^2}}{\overline{I}_{jk}+\sigma^2} \tag{18}$$

where $\overline{I}_{jk}\triangleq\sum_{l=1}^{L}\beta_{ljk}\left(\frac{1}{N}\sum_{i=1}^{K}p_{li}\right)$ *with*

$$\beta_{ljk}\triangleq d_{ljk}\frac{1-\tau_{ljk}^2\left[1-\left(1+\gamma_{jk}\frac{d_{ljk}}{d_{jjk}}\frac{\eta_l}{\eta_j}\right)^2\right]}{\left(1+\gamma_{jk}\frac{d_{ljk}}{d_{jjk}}\frac{\eta_l}{\eta_j}\right)^2}. \tag{19}$$

Proof: Substituting the explicit and deterministic $\bar{\lambda}_{jk}$ for the implicit $\lambda_{jk}^{\star}$, the result follows a classical random matrix approach, as derived in [7] for the single-cell setting. See [16] for more details. ■

For notational convenience, let us now denote by $\mathbf{b}=[b_1,\ldots,b_L]^T$ the vector with entries

$$b_j\triangleq\frac{1}{N}\sum_{i=1}^{K}\frac{\gamma_{ji}}{d_{jji}(1-\tau_{jji}^2)}. \tag{20}$$

The main result of this section unfolds from the previous lemma and provides a large N, K approximation for the minimal transmit power dedicated to each user required to ensure the SINR constraints:

Theorem 2. *Let* $\mathbf{\Gamma}\in\mathbb{C}^{L\times L}$ *be diagonal with entries*

$$[\mathbf{\Gamma}]_{j,j}\triangleq 1-\frac{1}{N}\sum_{l=1}^{L}\sum_{i=1}^{K}\frac{\left(\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{\eta_j}{\eta_l}\right)^2}{\left(1+\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{\eta_j}{\eta_l}\right)^2} \tag{21}$$

and $\mathbf{F}\in\mathbb{C}^{L\times L}$ *such that*

$$[\mathbf{F}]_{j,l}\triangleq\frac{1}{N}\sum_{k=1}^{K}\frac{\gamma_{jk}\beta_{ljk}}{d_{jjk}(1-\tau_{jjk}^2)}. \tag{22}$$

where β_{jli} *is defined in* (19)*. If and only if*

$$\limsup_{K}\left\|\mathbf{\Gamma}^{-1}\mathbf{F}\right\|<1 \tag{23}$$

then under Assumptions 1 and 2, we have that $\max_{jk}|p_{jk}^{\star}-\overline{p}_{jk}|\to 0$ *almost surely with*

$$\overline{p}_{jk}=\frac{\gamma_{jk}}{d_{jjk}\left(1-\tau_{jjk}^2\right)}\frac{\sum\limits_{l=1}^{L}\beta_{ljk}\overline{P}_l+\sigma^2}{1-\frac{1}{N}\sum\limits_{l=1}^{L}\sum\limits_{i=1}^{K}\frac{\left(\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{\eta_j}{\eta_l}\right)^2}{\left(1+\gamma_{li}\frac{d_{jli}}{d_{lli}}\frac{\eta_j}{\eta_l}\right)^2}} \tag{24}$$

where $\overline{P}_l$ *is the* l*th element of the vector* $\overline{\mathbf{P}}=\sigma^2\left(\mathbf{\Gamma}-\mathbf{F}\right)^{-1}\mathbf{b}$ *that collects the total transmit power of each BS with* $\mathbf{b}$ *defined as in* (20)*.*

Proof: The complete proof is given in [16] wherein the minimal transmit powers are set to ensure that the SINR constraints are reached exactly, that is such that $\mathrm{SINR}_{jk}=\gamma_{jk}$. It then suffices to solve the implicit equation $\gamma_{jk}=\overline{\mathrm{SINR}}_{jk}$ in the unknowns $\{p_{jk}\}$ (with $\overline{\mathrm{SINR}}_{jk}$ defined in (18)). This equation turns out to unwrap as an explicit equation for the $\{p_{jk}\}$, which are then readily obtained as in the statement of the theorem. ■

The total transmit power in the asymptotic regime is then simply obtained as follows:

Corollary 1. *Under Assumptions 1 and 2,* $P_T-\overline{P}_T\to 0$ *almost surely with*

$$\overline{P}_T=\mathbf{1}^T\overline{\mathbf{P}}=\sigma^2\mathbf{1}^T\left(\mathbf{\Gamma}-\mathbf{F}\right)^{-1}\mathbf{b}. \tag{25}$$

Proof: The result follows directly from Theorem 2 taking into account that the asymptotic transmit power of BS j is given by $\frac{1}{N}\sum_{k=1}^{K}\overline{p}_{jk}=\overline{P}_j$. ■

B. Coordinated Multipoint MIMO

With a slight abuse of notation, let $\widehat{\mathbf{h}}_{jk}\in\mathbb{C}^{NL}$ be the estimate of the channel from BS j to user k given by

$$\widehat{\mathbf{h}}_{jk}=\sqrt{d_{jk}}\left(\sqrt{1-\tau_k^2}\mathbf{w}_{jk}+\tau_k\mathbf{q}_{jk}\right) \tag{26}$$

where $\mathbf{q}_{jk}\in\mathbb{C}^N$ is Gaussian with zero mean and identity covariance matrix. Letting $\widehat{\mathbf{h}}_k=[\widehat{\mathbf{h}}_{1k}^T,\ldots,\widehat{\mathbf{h}}_{L,k}^T]^T$, we may write

$$\lambda_k^{\star}=\frac{\left(1+1/\gamma_k\right)^{-1}}{\widehat{\mathbf{h}}_k^H\left(\sum\limits_{i=1}^{KL}\lambda_i^{\star}\widehat{\mathbf{h}}_i\widehat{\mathbf{h}}_i^H+NL\mathbf{I}_{NL}\right)^{-1}\widehat{\mathbf{h}}_k}\quad\forall k. \tag{27}$$

Similar to the previous section, we shall require here the following technical setting.

Assumption 3. *The* $\{d_{jk}\}$ *and* γ_k *satisfy*

$$\limsup_N \max_{j,k} d_{jk} < \infty$$
$$\limsup_N \max_k \gamma_k < \infty.$$

For the results below, the techniques are quite similar to those presented in the previous section and are therefore not further discussed. Our first result in this setting is as follows.

Theorem 3. *Under Assumptions 1 and 3,* $\max_k \left|\lambda_k^\star - \overline{\lambda}_k\right| \to 0$ *almost surely with*

$$\overline{\lambda}_k = \frac{\gamma_k}{\frac{1}{L}\sum\limits_{l=1}^{L} d_{lk}\mu_l} \tag{28}$$

where $\{\mu_l\}$ *is the unique positive solution to the following set of equations:*

$$\mu_l = \left(\frac{1}{NL}\sum_{i=1}^{KL} \frac{d_{li}}{\frac{1}{L}\sum\limits_{j=1}^{L} d_{ji}\mu_j}\frac{\gamma_i}{1+\gamma_i} + 1\right)^{-1} \quad \forall l. \tag{29}$$

Unlike (13), in the CoMP configuration the Lagrange multiplier of UE k is found to be inversely proportional to a weighted priority parameter given by

$$\epsilon_k = \frac{1}{L}\sum_{l=1}^{L} d_{lk}\mu_l \tag{30}$$

which basically takes into account the effort of each cell for jointly serving user k [16]. Replacing $\mathbf{g}_k$ with $\widehat{\mathbf{g}}_k = \sqrt{\frac{p_k}{NL}}\frac{\mathbf{v}_k^\star}{\|\mathbf{v}_k^\star\|}$, the SINR of user k takes the form

$$\mathrm{SINR}_k = \frac{\frac{p_k}{NL}\frac{|\mathbf{h}_k^H\widehat{\mathbf{v}}_k|^2}{\|\widehat{\mathbf{v}}_k\|^2}}{\sum\limits_{i=1,i\neq k}^{KL}\frac{p_i}{NL}\frac{|\mathbf{h}_k^H\widehat{\mathbf{v}}_i|^2}{\|\widehat{\mathbf{v}}_i\|^2} + \sigma^2} \tag{31}$$

where $\widehat{\mathbf{v}}_k = \left(\sum_{i=1}^{KL}\lambda_i^\star\widehat{\mathbf{h}}_i\widehat{\mathbf{h}}_i^H + NL\mathbf{I}_{NL}\right)^{-1}\widehat{\mathbf{h}}_k$. To proceed further, we call $\boldsymbol{\epsilon}_k' = [\epsilon_{1k}', \ldots, \epsilon_{KLk}']^T$ the vector obtained as $\boldsymbol{\epsilon}_k' = (\mathbf{I}_{KL} - \mathbf{J})^{-1}\mathbf{c}_k'$ where $\mathbf{c}_k' \in \mathbb{C}^{KL}$ is such that

$$[\mathbf{c}_k']_i = \frac{\gamma_k^2}{\left(\frac{1}{L}\sum\limits_{l=1}^{L} d_{lk}\mu_l\right)^2}\left(\frac{1}{L}\sum_{l=1}^{L} d_{li}d_{lk}\mu_l^2\right) \tag{32}$$

and $\mathbf{J} \in \mathbb{C}^{KL\times KL}$ has entries given by

$$[\mathbf{J}]_{i,k} = \frac{[\mathbf{c}_k']_i}{NL(1+\gamma_k)^2}. \tag{33}$$

Mimicking the derivations of the previous section, we then have the following SINR approximation.

Lemma 2. *Under Assumptions 1 and 3,* $\max_k |\mathrm{SINR}_k - \overline{\mathrm{SINR}}_k| \to 0$ *almost surely with*

$$\overline{\mathrm{SINR}}_k = p_k\frac{\epsilon_k^2}{\epsilon_k'}\frac{1-\tau_k^2}{\overline{I}_k + \sigma^2} \tag{34}$$

where

$$\overline{I}_k \triangleq \frac{1-\tau_k^2\left[1-(1+\gamma_k)^2\right]}{(1+\gamma_k)^2}\left(\frac{1}{NL}\sum_{i=1}^{KL} p_i\frac{\epsilon_{ik}'}{\epsilon_i'}\right) \tag{35}$$

and $\boldsymbol{\epsilon}' = [\epsilon_1', \ldots, \epsilon_{KL}']^T = (\mathbf{I}_{KL} - \mathbf{J})^{-1}\mathbf{c}$ *where* $\mathbf{c} \in \mathbb{C}^{KL}$ *has elements* $[\mathbf{c}]_i = \frac{1}{L}\sum_{l=1}^{L} d_{li}\mu_l^2$.

Our main result is then as follows:

Theorem 4. *Let* $\mathbf{Z} \in \mathbb{C}^{KL\times KL}$ *be such that*

$$[\mathbf{Z}]_{k,i} \triangleq \frac{1}{NL}\frac{\gamma_i}{1-\tau_i^2}\frac{\epsilon_{ik}'}{\epsilon_i^2}\frac{1-\tau_i^2\left[1-(1+\gamma_i)^2\right]}{(1+\gamma_i)^2}. \tag{36}$$

If and only if

$$\limsup_K \|\mathbf{Z}\| < 1 \tag{37}$$

then under Assumptions 1 and 3, $\max_k |p_k^\star - \overline{p}_k| \to 0$ *almost surely with*

$$\overline{p}_k = \frac{\gamma_k}{1-\tau_k^2}\frac{\epsilon_k'}{\epsilon_k^2}\left(\overline{\Omega}_k\frac{1-\tau_k^2\left[1-(1+\gamma_k)^2\right]}{(1+\gamma_k)^2} + \sigma^2\right) \tag{38}$$

where $\overline{\boldsymbol{\Omega}} = [\overline{\Omega}_1, \overline{\Omega}_2, \ldots, \overline{\Omega}_{KL}]^T$ *is obtained as* $\overline{\boldsymbol{\Omega}} \triangleq \sigma^2(\mathbf{I}_{KL} - \mathbf{Z})^{-1}\mathbf{z}$ *and* $\mathbf{z} = [z_1, \ldots, z_{KL}]^T$ *with*

$$z_k \triangleq \frac{1}{NL}\sum\nolimits_{i=1}^{KL}\frac{\gamma_i}{1-\tau_i^2}\frac{\epsilon_{ik}'}{\epsilon_i^2}. \tag{39}$$

Corollary 2. *Under Assumptions of Theorem 4, we have that* $P_T - \overline{P}_T \to 0$ *almost surely where*

$$\overline{P}_T = \frac{1}{NL}\sum_{i=1}^{KL}\overline{p}_i \tag{40}$$

with $\{\overline{p}_i\}$ *given by Theorem 4.*

IV. Numerical results

Monte-Carlo (MC) simulations are now used to validate the above asymptotic analysis for a network with finite size. The results are obtained for 1000 different channel realizations and UE distributions. We consider a multi-cell network composed of L square cells distributed in a square region of side length $D = 500$ m. The pathloss function d_{ljk} is obtained as [9] $d_{ljk} = 2L_{\bar{x}}\left(1 + \|\mathbf{x}_{ljk}\|^\kappa/\bar{x}^\kappa\right)^{-1}$ where $\mathbf{x}_{ljk} \in \mathbb{R}^2$ is the position of user k in cell j with respect to BS l, $\kappa > 2$ is the pathloss exponent, $\bar{x} > 0$ is some cut-off parameter and $L_{\bar{x}}$ is a constant that regulates the attenuation at distance $\bar{x}$. We assume that $\kappa = 3.5$ and $L_{\bar{x}} = -86.5$ dB [9]. Similarly, we have that $d_{jk} = 2L_{\bar{x}}\left(1 + \|\mathbf{x}_{jk}\|^\beta/\bar{x}^\beta\right)^{-1}$ with $\mathbf{x}_{jk}$ being the position of UE k with respect to BS j. The transmission bandwidth is $W = 10$ MHz and the total noise power $W\sigma^2$ is -104 dBm. In all subsequent simulations, we assume that the same data rate must be guaranteed to each UE and the accuracy of CSI is the same for all UEs in both settings. Moreover, we assume that $K = 8$ and $N = 32$.

Fig. 1 illustrates the average transmit power in Watt vs. target rate in bps/Hz/UE when $L = 4$. The error bars indicate

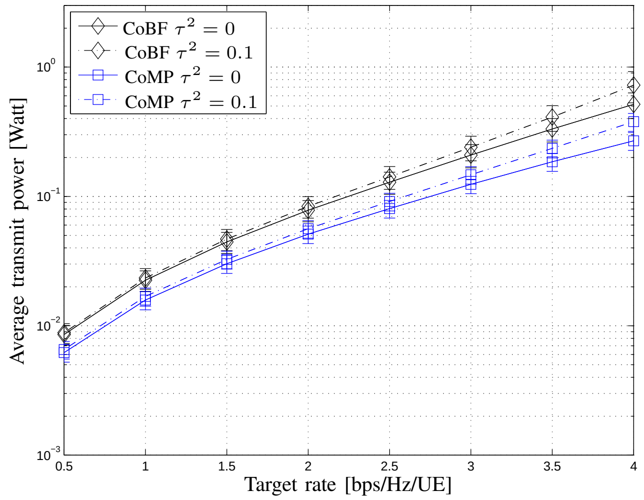

Fig. 1. Average transmit power in Watt vs. target rate when $L = 4, K = 8$ and $N = 32$.

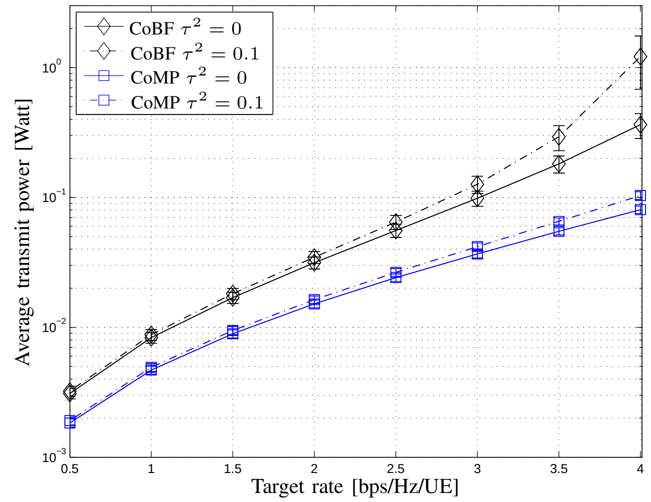

Fig. 2. Average transmit power in Watt vs. target rate when $L = 16, K = 8$ and $N = 32$.

the standard deviation of the MC results. Clearly, $\tau^2 = 0$ corresponds to the perfect CSI case. Compared to CoMP, an increase of power is required by CoBF. As seen, the deterministic approximation lies roughly within one standard deviation of the MC simulations and thus we may conclude that the analysis is accurate even for networks of finite size.

Fig. 2 plot the average transmit power in Watt vs. target rate when $L = 16$. Similar conclusions as for Fig. 1 can be drawn with the only difference that the average transmit power for $\tau^2 = 0$ is smaller for both schemes due to the shorter distances of UEs from their serving BSs. A slight increase of the transmit power is observed for CoBF when $\tau^2 = 0.1$ if the target rate is larger than 3.5 [bps/Hz/UE]. This is due to the larger interference residual coming from the imperfect CSI.

V. Conclusions

In this work, we analyzed the structure of the optimal linear precoder for minimizing the total transmit power when BSs cooperates through coordinated beamforming or network-MIMO. Stating and proving new results from large-scale random matrix theory allowed us to give concise approximations of the Lagrange multipliers, the powers needed to ensure target rates and the total transmit power. Numerical results indicated that these approximations are very accurate even for small system dimensions. Applied to practical networks, such approximations may lead to important insights into the system behavior, especially with respect to target rates, CSI quality and induced interference. More details and insights on these aspects are given in the extended version [16].

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