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Abstract: To comply with the continually growing demand for multimedia content and higher throughputs, the telecommunication industry has to keep improving the use of the bandwidth resources, leading to the well-known Frequency Assignment Problems (FAP). In this article, we present a new extension of these problems to the case of satellite systems that use a multibeam coverage. With the models we propose, we make sure that for each frequency plan produced there exists a corresponding satellite payload architecture that is cost-efficient and decently complex. Two approaches are presented and compared : a global constraint program that handles all the constraints simultaneously, and a decomposition method that involves both constraint programming and integer linear programming. For the latter approach, we show that the two identified subproblems can respectively be modeled as a multiprocessor scheduling problem and a path-covering problem, and this analogy is used to prove that they both belong to the category of NP-hard problems. We also show that, for the most common class of interference graphs in multibeam satellite systems, the maximal cliques can all be enumerated in polynomial time and their number is relatively low, therefore it is perfectly acceptable to rely on them in the scheduling model that we derived. Our experiments on realistic scenarios show that the decomposition method proposed can indeed provide a solution of the problem when the global CP model does not.

1 INTRODUCTION

A common characteristic of any telecommunication system is that it is bandwidth limited, and one of the main challenges for the system engineers is to optimally use this precious resource. Satellite telecommunications systems are no exception to that rule, and this already difficult task is even more complex when the specific limitations and needs of the satellite payload are taken into consideration. Plenty of literature can be found on the problem of assigning frequencies under the name of “Frequency Assignment Probems” (FAP). For instance, (Aardal et al., 2007) is a very thorough survey on the models and the optimization methods that have been developed over the years to solve the frequency assignment problems that emerged in a lot of different wireless communications systems. The recent litterature proposes more and more sophisticated methods to solve the FAP, such as parallel hyperheuristics (Segura et al., 2011), differential evolution (Salma et al., 2010), population-based heuristics (Luna et al., 2011) (Yang et al., 2014) or considers more and more realistic variants of the FAP according to specific problem characteristics (Koster and Tieves, 2012) (Muoz, 2012) (Wang and Cai, 2014). This article aims at presenting new models and approaches for this extension of the frequency assignment problem to multibeam satellite systems, and promising results on realistic scenarios.

A multibeam satellite system is characterized by a plurality of relatively narrow beams used to provide coverage to its service area as shown in Fig.1, each beam being the representation of an antenna gain loss threshold for the corresponding satellite radio source. Still in Fig.1, the role of the satellite payload (2) is to receive, downconvert, amplify, and retransmit the signals of the uplink (1) in the different beams of the

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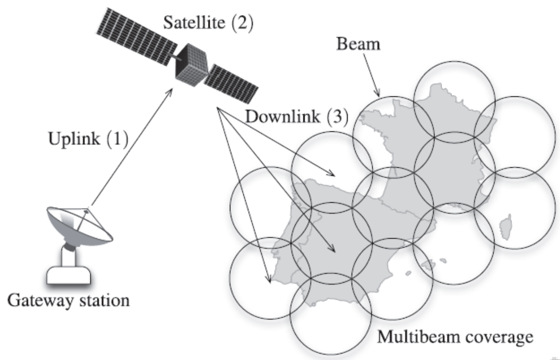


Figure 1: The uplink (1), the satellite payload (2) and the downlink (3) of the forward link of a multibeam satellite system.

downlink (3) where the end-users are located. It is assumed that the system bandwidth is divided into identical frequency channels, the bandwidth of a channel being equal to that of one carrier signal. For each beam, it is either specified by the operator or assessed in advance how much bandwidth is needed and there- fore how many carriers must be transmitted in it. Assuming that the carrier uplink frequencies are known or treated afterwards, system engineers have to define for each carrier of each beam:

- The frequency channel used in the downlink
- The polarization of the signal in the downlink
- The high power amplifier in the payload that will be amplifying the corresponding uplink carrier

These are the variables of the problem presented in this paper. Values must be assigned to them with the goal to minimize the levels of interferences in each beam, the number of high power amplifiers needed in the satellite payload, and the number of hardware needed for the downconversions. More precisely, the approach we have selected is to aim at minimizing the number of high power amplifiers needed in the satellite payload since they are heavy, expensive, and highly power-consuming, while we will be using constraints to limit the interferences and the hardware needed for the downconversions to what is acceptable.

The rest of the article is structured as follows. In section 2, the problem constraints are listed and detailed. Then, section 3 focuses on the different approaches we have devised to actually model the problem. Finally, section 4 provides experimental results and concrete scenario examples, before some concluding remarks in section 5.

2 THE PROBLEM CONSTRAINTS

2.1 Frequency Related Constraints

For the quality of transmission of a signal, the interferences are a determining factor and any frequency assignment procedure should try to minimize them. Let us remind that a frequency and a polarization must be assigned to each carrier of each beam in the downlink. Note that in this work, the isolation of the signals through the time-dimension is not considered. In the end, the frequency related constraints that are taken into account here are the following :

- **Polarization Isolation:**
  A perfect radio antenna transmits and receives waves in a particular polarization and is insensitive to orthogonally polarized signals (Bousquet and Maral, 2009), meaning that the same frequency channel can therefore be used twice in the same area without risking severe interferences. In actual facts, antennas cannot transmit and receive perfectly in one polarization only, it is always a combination of two orthogonal polarizations, one of them being predominant. To take advantage of that property anyway, the choice here has been to consider that two carriers at the same frequency using orthogonal polarizations are allowed to be transmitted in closer zones than two carriers trans- mitted at the same frequency and with the same polarization.
- **Spatial Isolation:**
  Thanks to antenna gain losses, two carriers can use the same color (frequency or frequency-polarization couple) as long as the two corresponding beams are sufficiently distant from each other. This is often turned into a constraint of minimum distance between them, leading the very classic binary interference constraints. The resulting representation is a graph $G = (B, E)$ where each vertex $b \in B$ corresponds to the zone covered by a beam and each edge $e \in E$ is a link between two zones where it is not allowed to use the same color.
- **Limit on the Frequency Channel Reuse Values:**

  Defining an upper-bound for these values allows to balance the number of times each channel is used, which reduces the hardware needs for frequency conversions. Since two uplink carriers can only share a downconverter in the satellite payload if they need the same frequency downconversion, it is interesting to be able to define the uplink frequencies so as to have as many of these situations as possible, and this balance of the frequency reuse factors in the downlink is advantageous on that regard.

2.2 Amplification of the Signals Constraints

A traveling-wave tube (TWT) is a type of high power amplifier for radio frequency signals and a widely used technology for satellite telecommunication payloads (Bousquet and Maral, 2009). A TWT must be assigned to each carrier of each beam under the following constraints:

- **Minimization of the Number of TWT:**
  A TWT is an expensive technology, one should therefore aim at finding a distribution of the carriers in the TWTs that minimizes their number.
- **Frequency Ranges:**
  The TWTs can have a bandwidth narrower than the overall system bandwidth. In that case, payload engineers agree with the equipment manufacturer on a limited number of frequency ranges. Therefore, the assignment of carriers to the TWTs must guarantee that the frequency ranges are supported by the available equipment.
- **Carriers forbidden to use the same TWT:**
  Two carriers cannot be amplified by the same TWT if their amplification requirements are too different, because of the non-linearity of the TWT. These incompatibilities are known in advance.
- **Single Use of the Frequency Channels:**
  A TWT cannot amplify two carriers using the same frequency channel.
- **Limited Number of Carriers per TWT:**
  A TWT is characterized by its output power level. That power is shared by the carriers, therefore the number of carriers per TWT is upper-bounded.
- **Contiguity of the Frequencies:**
  The payload complexity is assumed to be significantly reduced when there are no frequency gaps between the carriers in the same TWT.

3 MODELS

The first model we derived is a global constraint program (section 3.1) that includes all the aforementioned constraints. It has been able to provide really interesting system solutions on some scenarios, however, when the number of variables is set to high realistic values, the global CP model fails at providing solutions or proving unfeasibility in reasonable time. That is why a decomposition method has been developed, with a subdivision of the problem into a multiprocessor scheduling (section 3.2) and a path-covering (section 3.3) problems. The two approaches, the single constraint programming model and the combination of the two submodels, are then compared experimentally in section 4.

3.1 Global Constraint Programming Model

The idea to derive a constraint programming model has been motivated by an analysis of the constraints on the problem variables (frequency, polarization, TWT) that revealed that global constraints could be used to model a large part of the problem. A global constraint (Beldiceanu et al., 2005) is a set of constraints for which it is preferable to treat that set of constraints as a whole than to treat all the constraints of that conjunction of constraints individually. Using global constraints is a way to have a better view on the structure of the problem, which is then exploited with powerful filtering algorithms. On that regard, a very significant example is the all different constraint (van Hoeve, 2001)

$$\text{alldifferent}(X)$$

that forces all the variables of the array X to be different. In the model below, we also use the global cardinality constraint

$$\text{global\_cardinality\_constr}(X,Y,m,M)$$

that allows to bound the number of times some items appear in a list, X being that list, Y the set of sought values, m the array of minimum number of occurrences for each sought value, M the array of maximum number of occurrences for each sought value. Finally, the Gecode convexity global constraint

$$\text{convex}(X)$$

is used to force the integers of an integer set X to be a convex sequence ($\{1,2,3\}$ is one while $\{1,2,4\}$ is not). These global constraints are implemented in the open source solver Gecode (Schulte et al., 2013) that we chose to use.

An instance of this particular frequency assignment problem is defined by a set of N_B beams, each beam $b \in B = \{1, \cdots, N_B\}$ being characterized by the number n_b of carriers transmitted in it, leading to an overall number of carriers

$$N_c = \sum_{b=1}^{N_B} n_b$$

For all $b \in B$ and for all $c \in \{1, \cdots, nb\}$,

$$\text{ind}(b, c) = c + \sum_{\tilde{b}=1}^{b-1} n_{\tilde{b}}$$

defines a 1D sorting of these carriers and for all $b \in B$,

$$Cb = \{\text{ind}(b, c) \mid c \in \{1, \cdots, nb\}\}$$

is the notation for the set of indices of the carriers of the b^{th} beam. Therefore, note that the Cb sets partition the set $C = \{1, \cdots, N_C\}$. The system bandwidth is divided into NF sub-channels indexed by $F = \{1, \cdots, N_F\}$. N_T TWTs are available in the payload, and N_P orthogonal polarizations are considered (typically $N_P = 2$), the corresponding index sets being respectively denoted by T and P. Each carrier $c \in C$ must be assigned a frequency channel $\mathbf{f}_c \in F$, a TWT $\mathbf{t}_c \in T$ and a polarization $\mathbf{p}_c \in P$. These are the problem variables. Two graphs $G = (B, E)$ and $G' = (B, E')$ with $E' \subset E$ are defined: an edge of E' forbids the carriers in the two corresponding beams to use the same frequency channel whatever the polarization, whereas an edge of E only forbids the multiple use of the same frequency-polarization couple. In the following equations, note that card(X) denotes the cardinality of the set X. Here follows the list of the constraints expressed with these variables:

- For a given beam b such that $n_b > 1$, the n_b carriers must be contiguous in frequency, use the same TWT, and have the same polarization. For such b values, the constraints are:

$$\forall i \in \{2, \cdots, n_b\}, \ \ \mathbf{t}_{\text{ind}(b,1)} = \mathbf{t}_{\text{ind}(b,i)} \tag{1}$$

$$\mathbf{p}_{\text{ind}(b,1)} = \mathbf{p}_{\text{ind}(b,i)} \tag{2}$$

$$\mathbf{f}_{\text{ind}(b,i-1)} = \mathbf{f}_{\text{ind}(b,i)} - 1 \tag{3}$$

- As discussed in section 2.1, channel reuse bounds are a tunable parameter in input used to limit hardware needs for the downconversions. Let $\mathbf{R_{min}}$ and $\mathbf{R_{max}}$ be the arrays of size N_F of these bounds (note that in practice the lower-bound array is set to 0, it is just there to fit the definition of the global constraint that use both arrays), then the corresponding corresponding is the following:

$$\text{global\_cardinality\_constr}(\mathbf{f}, F, \mathbf{R_{min}}, \mathbf{R_{max}}) \tag{4}$$

- The binary interference constraints associated to E can be expressed as follows for all $b, b' \in B$ such that $b < b'$ and $(b, b') \in E$:

$$\text{alldifferent}\ (\mathbf{f}_c + N_F(\mathbf{p}_c - 1) \mid c \in C_b \cup C_{b'}) \tag{5}$$

- And for E', for all $b, b' \in B$ such that $b < b'$ and $(b, b') \in E'$:

$$\text{alldifferent}\ (\mathbf{f}_c \mid c \in C_b \cup C_{b'}) \tag{6}$$

- The same frequency cannot be used twice by the carriers of a given TWT :

$$\forall t \in T, \forall f \in F, \text{card}(\mathbf{T}_t \cap \mathbf{F}_f) \leq 1 \tag{7}$$

where $\mathbf{T}_t \subset C$ and $\mathbf{F}_t \subset C$ respectively are the set of carriers using the TWT t and the set of carriers using the frequency channel f, these set variables being linked to the arrays $\mathbf{t}$ and $\mathbf{f}$ by side channeling constraints that we do not provide here for the sake of conciseness.

- The contiguity in the TWTs. Let us denote by F_t the set of frequency channels used in the TWT t, these set variables being easily defined with chan- neling constraints involving the variable arrays $\mathbf{f}$ and $\mathbf{t}$. Then, the global constraint convex does exactly what is sought:

$$\forall t \in T, \text{convex}(\mathcal{F}_t) \tag{8}$$

- The maximum number of carriers in a given TWT that is upper bounded by a tunable parameter n :

$$\forall t \in T, \text{card}(\mathbf{T}_t) \leq n \tag{9}$$

- The incompatibilities between the carriers that cannot use the same TWT. Let $c, c' \in C$ be two carriers forbidden to use the same TWT, then the corresponding constraint is the following:

$$\mathbf{t}_c \neq \mathbf{t}_{c'} \tag{10}$$

- The content of the TWTs must be of a given type. Let $F_1 \subset F$ and $F_2 \subset F$ be two subparts of the system bandwidth such that $F_1 \cup F_2 = F$. These two sets define two types of acceptable frequency contents for the TWTs, which means that the carriers in a given TWT must either all be in F_1 or all be in F_2, which can be expressed as follows:

$$\forall c, c' \in C, \ \mathbf{f}_c \in F \backslash F_2 \ \wedge \ \mathbf{f}_{c'} \in F \backslash F_1 \Rightarrow \mathbf{t}_c \neq \mathbf{t}_{c'} \tag{11}$$

The objective is the minimization of the number of available TWTs actually used. That number n_{used}

is a variable that can be obtained from the array **t** with two successive global counting constraints, the first one generating an array of the number of times each TWT is used, the second counting the number of non- zero values in the latter:

$$\min n_{\text{used}} \tag{12}$$

3.2 Multiprocessor Scheduling Part

3.2.1 The Scheduling Model

An analogy with multiprocessor scheduling problems is possible for the assignment of frequencies and polarizations, that is for the subproblem that only concerns the variable arrays **f**, **p**, and the constraints (2), (3), (4), (5) and (6). That problem, denoted by ($S1$), is an extension of the model proposed in (Kiatmanaroj et al., 2013) where the frequency assignment is addressed regardless of the polarizations. Each beam $b \in B$ is assimilated to a single operation job whose processing time, expressed in time units, is non-preemptive and equal the number of carriers in that beam. Note that such a model is only valid because the frequencies of the carriers in a same beam are constrained by constraint (8) to be contiguous, the contiguousness of frequencies corresponding therefore to the non-preemptiveness of the processing times. Each maximal clique of G' is assimilated to a machine with non-overlapping constraints, while each maximal clique of G is associated to exactly two machines, one for each polarization. For each beam/job $b \in B$, C'_b denotes the set of machines that correspond to the cliques of G' that contain b, while $C_{b,1}$ and $C_{b,2}$ are the sets of machines representing the cliques of G containing b that are respectively associated to the polarizations 1 and 2. For constraint (4), it is assumed that the only restriction here is an upper-bound on the reuse factor $R \in \mathbb{N}^+$ of the channels (same bound for each channel), which leads to the definition of $M = \{m1, \cdots, mR\}$ identical parallel machines. Each job $b \in B$ requires simultaneously multiple machines. More precisely, it must be executed on:

- all the machines of C'_b
- either all the machines of $C_{b,1}$, or all the machines of $C_{b,2}$
- one machine of M

Note that relying on cliques is not necessary to make this analogy with multiprocessor scheduling, another option could be to define a machine for each binary constraint, but relying on cliques allows to take into account several constraints simultaneously, just like global constraints in constraint programming. In the example of Fig.2, for the beam number 1 with the notations $C'_1 = \{c'_{1,1}, c'_{1,2}\}, C_{1,1} = \{c_{1,1,1}, c_{1,1,2}\}$ and $C_{1,2} = \{c_{1,2,1}, c_{1,2,2}\}$, we have:

- $c'_{1,1}$ and $c'_{1,2}$ associated to the cliques/machines $\{1,2\}$ and $\{1,3\}$ of G'
- $c_{1,1,1}$ and $c_{1,1,2}$ associated to the machines of first polarization for the cliques $\{1,2,3\}$ and $\{1,3,4\}$ in G
- $c_{1,2,1}$ and $c_{1,2,2}$ associated to the machines of second polarization for the cliques $\{1,2,3\}$ and $\{1,3,4\}$ in G
- m_1 the machine M used by the beam 1

In the example, the two carriers required in beam 1 use the second and third frequency channels and the first kind of polarization. With a common deadline for all the jobs being equal to the number of frequency channels NF (equal to 4 in Fig.2), one can see that solving this scheduling problem is equivalent to solving the considered subpart of our frequency assignment problem.

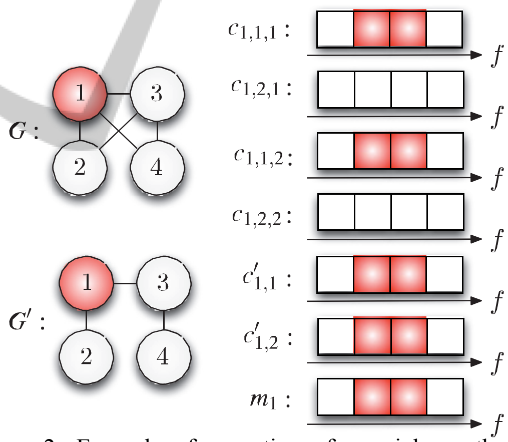


Figure 2: Example of execution of one job on the machines.

Proposition: (S_1) is equivalent to solving a multiprocessor scheduling problem, it is therefore NP-hard.

Proof: The parallel machine problem is a particular case of (S_1). □

3.2.2 Maximal Cliques Enumeration in Multibeam Satellites Interference Graphs

As explained in the previous paragraph, one promising direction to solve efficiently the scheduling part of the frequency assignment problem considered is to use the cliques of the interference graphs. It is thus of interest to study the theoretical and practical complexity of enumerating the maximal cliques. In multibeam systems, the analysis of their exhaustive enumeration differs depending on the type of graphs considered: regular layouts or random interference graphs.

Cliques in Regular Layouts

A regular layout is an organization of the beams that provides a continuous coverage of the zone with overlapping beams that describe an hexagonal lattice, as shown in Fig.1 for instance. It is a very common choice for the system engineer since the contiguous coverage it provides can be a crucial specification of the customer, and also, it requires simpler antenna designs than a non-uniform layout. For a beam $b \in B$, let us denote by c_b the position of its center and by $\Gamma(b)$ the set of its adjacent beams. A common industrial approach for a regular layout with beams of radius r is have $\Gamma(b) = \{\tilde{b} \in B | \tilde{b} \neq b \text{ and } \| c_{\tilde{b}} - c_b \| < d\}$ with d being equal to either $3r$ or $2\sqrt{3}r$ leading to the representations (a) and (c) of Fig.3. They are usually called 3-colors pattern and 4-colors pattern because with such edges in the interference graph, it is possible to partition the set of vertices into respectively 3 and 4 independent sets as shown in figure (b) and (d) of Fig.3. An important property of the regular interference graphs with the edges defined this way is the following:

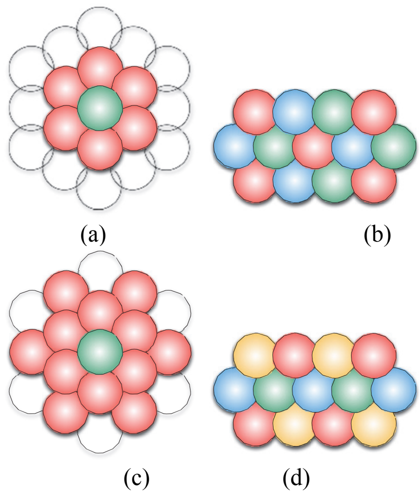


Figure 3: (a) Adjacent beams, $3r$ threshold (b) Independent sets, $3r$ threshold (c) Adjacent beams, $2\sqrt{3}r$ threshold (d) Independent sets, $2\sqrt{3}r$ threshold.

Proposition: The maximal cliques of the interference graphs corresponding to the regular patterns in regular layouts can all be enumerated in polynomial time

Proof: The key idea is that for each exclusion pattern, there exists a finite number m such that for each vertex $b \in B$ there exist m potential cliques that might contain b, m being independent of the size N_B of the graph. For instance, for a graph with the edges of the 4-colors pattern, geometrical considerations allow to understand that, for a given vertex :

- it cannot belong to a clique of size 5 and more,
- the cliques of size 4 that might contain it are those of Fig.4 plus those obtained by rotating of $\frac{\pi}{3}$ around the center of the corresponding beam leading to a total of 20 distinct potential cliques,
- the only way it can belong to a maximal clique of size 1, 2 or 3 is that the corresponding beam is surrounded by less beams than in the full configuration of Fig.4, which can happen either because the beam in question is near the bound of the layout or because there are "holes" in it. Therefore, if such a clique exists, it is a subgraph of what would have been a clique of size 4 if some beams had not been missing. These situations are also in finite number and can be precisely enumerated.

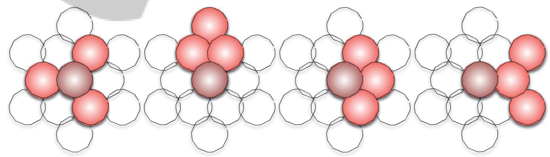


Figure 4: Cliques of size 4 with 4-colors pattern.

Note that in the example of the 4-colors pattern, the number of cliques is therefore upper-bounded by $20N_B$. Each potential clique is characterized by a specific set of adjacent beams and, for the cliques of size less than 4, a set of non-existing beams whose positions are perfectly known in terms of distance to the beam tested and orientation with respect to a given reference direction, say the horizontal direction. The same type of rationale applies for the graphs defined with the 3-colors pattern. Therefore, to enumerate all the maximal cliques in the case of regular layouts, one would only have to iterate on the vertices $b \in B$, that is on the beams, and test each clique possibility to see which ones actually exist for each b. That way, the list of maximal cliques can gradually grow, simple tests allowing to avoid redundancies. In the end, the maximal cliques of the regular layouts are indeed enumerated with a polynomial complexity. ☐

Cliques in Realistic Random Layouts

Even if the standard way to design a layout is to rely on the uniform patterns, it can be interesting to break that regularity in order to match the heterogeneity of the requirements over the service area. One can therefore have to work with a layout that can have beams of differents widths and positions for their centers that do not describe any particular known geometrical pattern. It was therefore necessary in that case to determine whether it was still an acceptable approach to enumerate the cliques before actually solving the frequency assignment problem. To do so, the slightly modified version of the Bron-Kerbosch (Bron and Kerbosch, 1973) algorithm proposed by Tomita et al. (Tomita et al., 2006) has been implemented and used on sets of graphs that were randomly generated with constraints on the vertex degrees. In practice, in multibeam satellite systems interference graphs, these vertex degrees are rarely less than 1 and greater than 12, so this has been specified as the main constraint in the constraint program used to generate these graphs. We generated 10000 different graphs of size $|B| = 200$ (maximum size for a realistic scenario) and observed that the mean number of cliques was 881 and the mean execution time was 14 millisec- onds. These cliques numbers are far from the $3^{\frac{|B|}{3}}$ upper bound of the number of cliques in an undirected graph, which is very interesting in practice because too high numbers of cliques could have made it impossible or unreasonable to rely on a model based on them. But most importantly, the computational times are relatively low, even instantaneous at the time scale of the designing phases of the satellite telecommunication systems. In the end, this means that this preliminary enumeration of the cliques is a pre-processing operation for the frequency assignment problem that is perfectly acceptable, whatever the type of layout.

3.3 Path Covering Part

Let us assume that the frequencies and the polarizations have been assigned somehow to the carriers of a given system, possibly with a scheduling based procedure as the one presented in section 3.2. Then, one can wonder what the problem of assigning the TWTs to these carriers becomes, that problem being denoted by (S_2). The first important remark is that the constraint 11 on the type of TWTs can now be seen as additional incompatibilities in constraint 10 since the frequencies of the carriers are now known. The second is that it is now possible to represent the problem as a path-covering problem of a digraph in which the vertices represent the N_C carriers of the system (see Fig. 5), a path representing a TWT and its content. In this graph, for all $f \in F \setminus \{N_F\}$, the only possible direct successors of the carriers using the frequency f are those using the frequency $f+1$, the in-degrees of the carriers using the frequency 1 being all equal to 0, just like the out-degrees of the carriers using the frequency N_F. As a consequence of these few properties, such graphs are acyclic. The incompatibilities between two carriers that cannot be in the same TWT/path are represented with dotted-line connections. For a given carrier, two situations impact the number of out-arcs : when this carrier is not the last carrier of the beam it belongs to, and when there exist incompatible carriers that use the next frequency. In the former case, only one arc leaves the carrier considered and its head is the next carrier in the

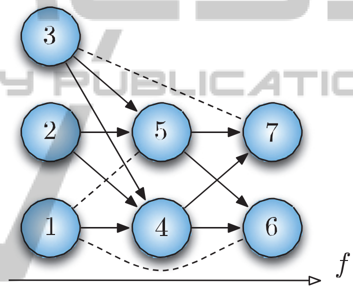


Figure 5: Carrier based graph for TWT assignment.

corresponding beam. In the latter case, the carrier cannot be connected to the carriers with which an incompatibility is shared. Otherwise, for a carrier that is not in any of these two situations, it is connected to all the carriers using the next frequency. One can then see that assigning TWTs to the carriers comes down in that case to finding the minimum number of disjoint paths that cover all the vertices, the contiguity (constraint 8) and the fact that the same frequency cannot be used twice in a TWT (constraint 7) being automatically verified with a graph built that way. But there are also some additional constraints to take into account such as the upper-bound for the length of the paths (constraint 9), the constraint not to use the same TWT for two incompatible carriers (constraint 10), and finally the constraint that the carriers of a block of carriers must use the same TWT (constraint 1). In the end, an instance of the problem considered is entirely defined by : an acyclic digraph D whose vertices can be partitioned into a certain number of ordered “levels” and whose arcs are only between two vertices of a level and the next, an upper bound l for the length of the paths, a set for each carrier of the carriers it must share a TWT with (empty sets being allowed), and a set for each carrier of the carriers incompatible with that carrier (empty sets also allowed).

Proposition: (S_2) is an NP-hard path-covering problem

Proof: Without the additional constraints (1,9,10), the problem of covering a digraph with a minimum number of point-disjoint paths can be solved in polynomial time as shown in (Boesch and Gimpel, 1977). But once they are taken into account, it can be proven that the problem becomes NP-hard. Indeed, let us consider an instance of the problem of finding a minimum cardinality cover of the elements of a partially ordered set (poset) with chains of restricted length, whose NP-completeness has been proven in (Shum and Trotter, 1996). It is common to represent that poset with a digraph partitioned in ordered levels, the edges connecting the comparable elements of the set from one level to the next: this is precisely a Hasse diagram. Then, with the upper bound for the path lengths equal to the maximum length of a chain and with, for each carrier, empty sets for the sets of carriers that must use the same TWT and the sets of incompatible carriers, one can see that solving this poset cover instance is equivalent to solving a particular instance of the path-covering problem considered in this paper. Therefore, it is also NP-complete. □

To solve it, the following integer linear programming model has been derived :

$$min \sum_{t=1}^{N_T} u_t \tag{13}$$

s.t.

$$\forall t \in C, \sum_{t=1}^{N_T} x_{ct} = 1 \tag{14}$$

$$\forall t \in T, \forall f \in F, \sum_{c=1}^{N_C} y_{cf} x_{ct} \leq 1 \tag{15}$$

$$\forall t \in T, u_t \geq \frac{1}{N_C} \sum_{c=1}^{N_C} x_{ct} \tag{16}$$

$$\forall t \in T, \sum_{c=1}^{N_C} x_{ct} \leq n \tag{17}$$

$$\forall c, c' \in C \text{ that are incompatible}, \forall t \in T, \quad x_{ct} + x_{c't} \leq 1 \tag{18}$$

$$\forall c, c' \in C \text{ in the same block of carriers } \forall t \in T, \quad x_{ct} - x_{c't} = 0 \tag{19}$$

$$\forall t \in T, \forall f \in F\{N_F\}, \quad \sum_{c=1}^{N_C} \left(\left[\sum_{f'=f+1}^{N_F} y_{cf'} \right] + N_F y_{cf} - N_F y_{c(f+1)} \right) x_{ct} \leq N_F \tag{20}$$

where $y_{cf} \in \{0, 1\}$ are input Boolean arguments that indicate whether the carrier $c \in C$ uses the frequency $f \in F$, x_{ct} $\{0, 1\}$ are the Boolean variables that indicate if the carrier $c \in C$ uses the TWT $t \in T$, and finally the $u_t \in \{0, 1\}$ are the Boolean variables that indicate whether the TWT t is actually used. Constraint 14 is the constraint to have only one TWT assigned to each carrier, 15 forbids a given TWT to be used by two different carriers using the same frequency channel, 16 is the constraint that forces the u_t to be equal to 1 as soon as the TWT t is used at least once, 17 is the limit on the number of carriers in the same TWT, constraint 18 forbids two incompatible carriers to use the same TWT, 19 forces the carriers in the same block of carriers to use the same TWT, 20 ensures the contiguity of the frequency channels in each TWT, finally 13 is the minimization of the number of TWT actually used.

4 EXPERIMENTAL RESULTS

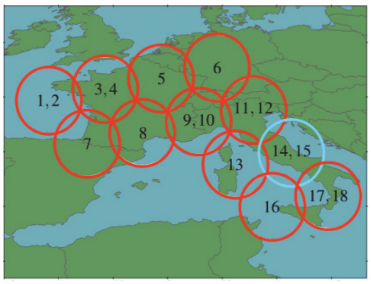
Experiments were needed to assess the performances of the two following approaches:

- Global Approach (GA):
  The global constraint program of section 3.1 solved with a CP solver (Gecode)
- Decomposition Method (DM) :
  Sequential solving of (S_1) of section 3.2 with a CP solver (Gecode) and then of (S_2) of section 3.3 with an ILP solver (Gurobi)

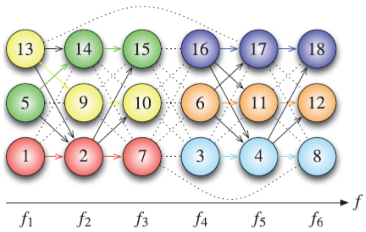
A first detailed example is presented in Fig.6 with a fictitious scenario over France and Italy, with $N_B = 12$ regularly organized beams. The characteristics of the problem solved were the following :

- Each beam $b \in \{1, \cdots, 12\}$ of Fig.6 has a required number of carriers n_b than is either equal to 1 or to 2, the carriers being indexed as shown inside the beams in Fig.6a

- For the beams b with a number of carriers n_b>1, we require contiguous carrier frequencies, same polarization and same TWT
- The system bandwidth is divided into N_F=6 channels
- The acceptable frequency ranges for the TWTs are {1,2,3} and {4,5,6}
- The TWT reuse upper-bound is set to 3, i.e. the width of an admissible frequency range
- The 4-color pattern is used to define binary interference constraints for the reuse of the same frequency-polarization couple (Fig.3c)
- The 3-color pattern is used to define binary interference constraints for the reuse of the same frequency, regardless of the polarization (Fig.3a)
- Carrier n◦5 is incompatible with carriers n◦9 and n◦10, carrier n◦13 is incompatible with carriers n◦17 and n◦18, carrier n◦7 is incompatible with n◦8, which means that they cannot use the same TWT
- Each frequency channel must be used at most third times
- Objective function : number of TWTs used



(a)



(b)

Figure 6: (a) Multibeam coverage and polarizations (b) Frequencies and TWTs.

This is one of the instances for which GA solved with Gecode is unacceptably long to find a solution. On the other hand, with DM, the scheduling part and the subsequent binary linear program are both solved extremely efficiently respectively by Gecode and Gurobi. On Fig.6a, the regular layout is reprensented with a ring color for each polarization, and on Fig.6b, the frequencies of the carriers found in the scheduling part can be read on the horizontal axis, and each color for the carriers represents one TWT. Note that the design of Fig.6 obtained for that example is optimal since the number of TWTs used is exactly equal to the number of carriers divided by the maximum number of carriers in a TWT.

When instances are randomly generated, note that there is no guarantee that they will be feasible. Even if this is true for both approaches, in the case of DM, this risk of infeasibility is even increased since some of the path-covering problem constraints are currently not anticipated in the preceding scheduling problem (the frequency ranges of the TWTs for instance). In practice, infeasibility is significantly harder to detect than actual solutions for feasible instances, at least when Gecode is used, that is in GA and in (S_1) of DM. In the results of this section, the statistic values presented only consider the instances that turned out to be feasible.

For each instance tested with the DM approach, the corresponding (S_1) scheduling problem is solved with Gecode using the corresponding subset of constraints in the global model of section 3.1. Then, the solutions of (S_1) are transformed into (S_2) path-covering instances that are solved with Gurobi thanks to the ILP model we derived in section 3.3. With GA, let us remind that the problem is entirely solved with Gecode. For the first phase of our series of experiments, we generated FAP instances with similar characteristics as the example detailed before, with the following few changes :

- Each beam $b \in \{1, \cdots, 12\}$ of Fig.6 has a now required number of carriers nb than is either equal to 0 or to 1
- The TWT carrier incompatibilities are now ran- domly generated (about 10% of all the possible carrier couples)
- The overall number of required carriers $N_C = \sum_{1 \leq b \leq 12} n_b$ is gradually increased, from 4 to 12, 100 feasible instances being generated at each stage
- Each frequency channel cannot be used more than once when $4 \leq N_C \leq 6$ and more than twice when $7 \leq N_C \leq 12$

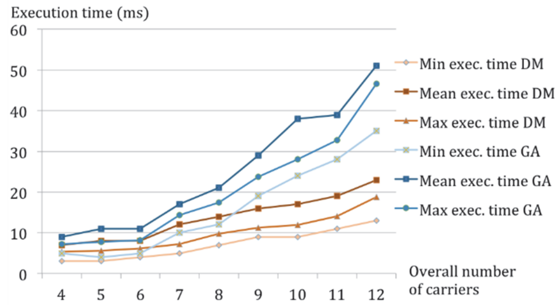


Figure 7: Comparison of GA and DM on 4-carriers to 12-carriers instances with execution time statistics.

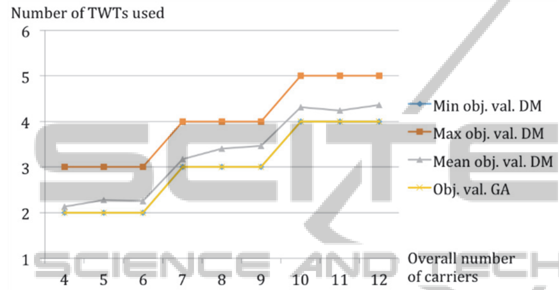


Figure 8: Comparison of GA and DM on 4-carriers to 12-carriers instances with objective value statistics.

Fig. 8 and Fig. 7 allow to compare GA and DM in terms of objective values and execution times. As expected, we can observe that in the the case of a joint assignment of TWT, frequency and polarization to the carriers (GA), the execution times are greater than those of DM but the objective values are better in average. In the particular case of the instances we generated, GA always reaches the theoretical optimal value which is equal to

$$\text{ceiling}\left(\frac{\text{Overall number of carriers}}{\text{Maximum number of carriers in a TWT}}\right)$$

However, the decomposition method often manages to reach that optimal number of TWTs too as shown in Table 1. This is a crucial remark we wanted to emphasize since it is what legitimates the use of DM when GA is not usable in practice.

Table 1: Percentage of times the theoretical optimum is reached with DM for each set of instances of varying number of carriers.

4 carriers	5 carriers	6 carriers
87%	72%	75%
7 carriers	8 carriers	9 carriers
83%	59%	53%
10 carriers	11 carriers	12 carriers
69%	76%	64%

In the next phase of our experiments, the overall number of carriers in the system has been set to be greater than 12 and less than 19, the carrier requirements in each beam being either equal to 1 or 2, and the frequency channel reuse limit being now set to 3. As a result, some new constraints have to be taken into account for the beams b such that $n_b > 1$: contiguity of frequencies, same polarization and same TWT for the carriers belonging to the same beam. In practice, this is the point where GA becomes unusable both for feasible and infeasible instances, because of extremely long execution times even on these instances that are still relatively small compared to the biggest realistic situations. This explains why it has been necessary to develop DM. In Fig. 9, the execution times of $(S1)$ (scheduling) and $(S2)$ (path-covering) are compared on the whole range of instances, from 4-carriers instances to 18-carriers instances. Two main things can be observed in that figure. First, the difference between the instances with at most 12 carriers and those with at least 13 carriers is clear: the new constraints linked to the beams for which the carrier requirement is strictly higher than one slow the search. Also, we see that the computational times grow faster for the scheduling problem than for the path-covering problem. That remark is even more important when we consider the fact that infeasible instances are also re- ally hard to detect for Gecode in the scheduling part. $(S1)$ is therefore the subproblem that deserves more attention for future work, the goal being to solve the highest realistic instances. Our not yet exploited analysis of the cliques in the interference graphs could certainly be an interesting direction.

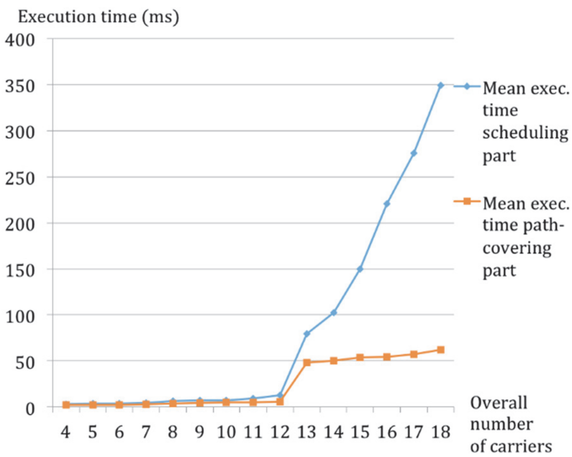


Figure 9: $(S1)$ (scheduling part) and $(S2)$ (path-covering part) execution times.

5 CONCLUSION

The models we proposed for this particular frequency assignment problem applied to the design of multibeam satellite systems allowed to algorithmically solve instances that could not be solved by satellite telecommunications engineers. We showed that the decomposition method we devised could produce solutions and even optimal solutions in reasonable computational times especially compared to the perfor- mances of the global constraint program for that prob- lem. We also showed that relying on the cliques of the interference graphs was an acceptable direction and most likely a way to improve our current algorithms for the scheduling subproblem of our decomposition method. Concerning the path-covering problem, a series of experiments showed that realistic instances where solved almost instantaneously by the solver Gurobi, which tells us that we extracted an interesting subproblem, and we will definitely try to take advantage of this in some way in the next algorithms we will implement. To solve the largest realistic instances, work still has to be done to get faster results and improving the algorithms for the scheduling part might not be enough. Instead of solving the two identified subproblems sequentially, we might aim at more integrated approaches inspired by combinatorial Benders' cuts for instance, or with filtering algorithms solving locally the path covering problem.

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