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Decentralized diagnosis in a spacecraft attitude
determination and control system

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Abstract. In model-based diagnosis (MBD), structural models can provide useful information for fault diagnosis and fault-tolerant control design. In particular, they are known for supporting the design of analytical redundancy relations (ARRs) which are widely used to generate residuals for diagnosis. On the other hand, systems are increasingly complex whereby it is necessary to develop decentralized architectures to perform the diagnosis task. Decentralized diagnosis is of interest for on-board systems as a way to reduce computational costs or for large geographically distributed systems that require to minimizing data transfer. Decentralized solutions allow proper separation of industrial knowledge, provided that inputs and outputs are clearly defined. This paper builds on the results of [1] and proposes an optimized approach for decentralized fault-focused residual generation. It also introduce the concept of Fault-Driven Minimal Structurally-Overdetermined set (FMSO) ensuring minimal redundancy. The method decreases communication cost involved in decentralization with respect to the algorithm proposed in [1] while still maintaining the same isolation properties as the centralized approach as well as the isolation on request capability.

1. Introduction

With increasing complexity of industrial processes, the requirement for reliability, availability and security is growing significantly. Fault detection and isolation (FDI) are becoming a major issue in industry.

The structural approach constitutes a general framework to provide information when the system becomes complex. The main aim of the structural approach application is to identify the subsets of equations which include redundancy.

The system structure analysis, originally developed for the decomposition of large systems of equations for their hierarchical resolution, was adopted by the Fault Detection and Isolation (FDI) community [2, 3]. Structural concepts are used for analysis of system monitor ability using the concept of complete matching on a graph.

Decentralized diagnosis has received considerable attention to deal with distributed systems or with systems that may be too large to be diagnosed by one centralized site. In the same way, the decentralized solution allows proper separation of industrial knowledge, provided that inputs and outputs are clearly defined.
Decomposition has been recognized as an important leverage to manage architectural complexity of the systems to be diagnosed. Most of the approaches, however, focus on hierarchical decomposition [4], while decentralization has been explored less frequently. The majority of decentralized diagnosis methods concern discrete event systems [5, 6, 7]. In [6], the purpose of the method is to provide efficient online diagnosis to detect and isolate faults in large discrete event systems. [5] uses a decentralized approach to deal with the size of the model and to get a tractable representation of diagnosis. Along the same idea, [7] proposes a hierarchical framework that exploits different local decisions.

These ideas are taken into account for our approach of decentralization using the idea of isolation on request. If local diagnosis is not sufficient, a hierarchical architecture is built that allows for less ambiguous hierarchical diagnosis tests. Moreover, our proposal to use fault-driven residual generation targets decreasing computational effort.

Decentralized diagnosis methods have been applied only recently on continuous or hybrid systems. [4] presents a decentralized architecture for systems modeled in the qualitative framework. The architecture proposed is similar to that which is presented in this paper: the model of each subsystem is private and the supervisor needs no a-priori knowledge about the subsystems and their interactions. In addition, a multi-layered hierarchy of diagnosers is possible.

This paper presents the design, in a decentralized architecture, of a fault-driven residual generation scheme bridging the algorithm for computing a Fault-Driven Minimal Structurally-Overdetermined (FMSO) set of subsystems. This paper is organized as follows: Section 2 introduces the basic concepts of analytical redundancy and presents the structural approach. Section 3 presents an approach for residual focus generation. Section 4 proposes an algorithm to generate the Decentralized Diagnoser based on FMSO. Section 5 illustrates the method proposed on the Attitude Determination and Control System (ADCS) of a LEO satellite. Finally, Section 6 discusses some conclusions and brings in current work.

2. Structural approach

In spite of their simplicity, structural models can provide much useful information for fault diagnosis and fault-tolerant control design. Structural analysis is able to identify those components of the system which are or are not able to be monitored, to provide design approaches for analytic redundancy based residuals, to suggest alarm filtering strategies, as well as to identify those components whose failure can be tolerated through reconfiguration [8].

The system structural model is an abstraction of its behavior in a sense that the structure of the constraints only considers links between variables and parameters and does not consider links between the constraints themselves. The links are represented by a bipartite graph, which is independent of the nature of the constraint (e.g. quantitative, qualitative, equations, rules, etc.), variables and parameters values [9]. It is important to summarize some structural concepts.

**Definition 1 (Model):** The behavior model of a system is denoted by $M(z, x)$ or $M$ for short, to be any set of equations relating the known variables $z$, i.e. measured variables, and the unknown variables $x$. The equations $e_i(z, x) \subseteq M(z, x)$, $i = 1, ..., n$ are assumed to be differential or algebraic equations in $z$ and $x$, and the model is considered a differential-algebraic equation system. By example shown in Table 1, we illustrate the concepts introduced in this section. This is composed of five equations, $e_1$ to $e_5$, relating the unknown variables $x = \{x_1, x_2, x_3, x_4, x_5\}$ and the known variables $z = \{u, y, z\}$.

**Definition 2 (ARR for $M(z, x)$):** The structure of a system is an abstract representation of which variables are involved in different equations and compose the model of the system. The structural abstraction allows us to derive redundancies disregarding the actual analytical expressions of equations making up the system model.

Let $M(z, x)$ be a model. Then, a relation $r(z, \dot{z}, \ddot{z}, ...) = 0$ is an Analytical Redundancy Relation (ARR) for $M(z, x)$ if for each $z$ consistent with $M(z, x)$ the equation is fulfilled.
Table 1. Expressions and biadjacency matrix of an illustrative example.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
<th>Unknown</th>
<th>Known</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$x_1$</td>
<td>$x_2$</td>
</tr>
<tr>
<td>$e_1$</td>
<td>$e_1^{x_1} = au$</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>$e_2$</td>
<td>$e_2^{x_1} = bx_2$</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>$e_3$</td>
<td>$e_3^{y} = x_2$</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>$e_4$</td>
<td>$e_4^{z} = x_2 + x_3^2$</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>$e_5$</td>
<td>$e_5^{x_3} = x_4 + x_5$</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Obtaining ARRs for a model $M(z,x)$ involves the elimination of unknown variables, which can be inferred from the bipartite graph. The bipartite graph actually represents which unknown variables are involved in the equations models of the system.

### 3. Fault-driven Residual Generation

In graph theory, a classical result from bi-partite graph states that any finite-dimensional graph can be decomposed into three subgraphs with specific properties by using the Dulmage-Mendelson (DM) canonical decomposition [10] as can be seen in Figure 1. The structurally over-determined part, represented by $M^+$, has more equations than variables; the structurally just-determined part, represented by $M^0$, has many equations than variables and the structurally under-determined part represented by $M^-$ has more variables than equations.

The biadjacency matrix in Figure 1, shows a DM canonical decomposition of a bipartite graph $G(M \cup X, A)$. The light blue-shaded areas contain ones and zeros, while the white areas only contains zeros. The thick line represents a maximal matching in the graph, where rows and columns are rearranged. For the example, by rearranging rows and columns, the DM decomposition is:

The bold $X$ denotes a complete matching. The subsets for this example are: $M^- = \{e_5\}$, $M^0 = \{e_4\}$, $M^+ = \{e_1, e_2, e_3\}$, $X^- = \{x_4, x_5\}$, $X^0 = \{x_3\}$ and $X^+ = \{x_1, x_2\}$.

The structural redundancy is used to define the ideas of structurally overdetermined (SO) set, proper structurally overdetermined (PSO) set and minimal structurally overdetermined (MSO) set. These ideas are therefore also compared to analytical redundancy [11].

Figure 1. Dulmage-Mendelson decomposition of $M$. 
Table 2. DM decomposition of illustrative example.

<table>
<thead>
<tr>
<th>Equation</th>
<th>x_4</th>
<th>x_5</th>
<th>x_3</th>
<th>x_1</th>
<th>x_2</th>
<th>u</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>e_5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e_4</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e_1</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e_2</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e_3</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Definition 3 (Structurally overdetermined, SO): A set $M$ of equations is SO if $M$ has more equations than unknowns.

Definition 4 (Proper structurally overdetermined, PSO): A set of equations $M$ is a PSO set if $M = M^+$. A PSO set is generically a testable subsystem but may contain smaller PSO subsets that are also testable subsystems [12]. The minimal PSO sets, called the MSO sets, are of special interest since these have the attractive properties of giving good isolation.

Definition 5 (Minimal structurally overdetermined, MSO): An SO set is a MSO set if no proper subset is also an SO set. Note that an MSO set is also a PSO set.

Structural methods have been used to perform isolation capability analysis and to find testable submodels (MSO sets). However, the number of MSO sets grows exponentially with the degree of redundancy making the task of computing MSO sets intractable for systems with high degree of redundancy. [13] introduces the concept of Test Equation Supports (TES), which is a set of equations expressing redundancy specific to a set of considered faults. The influence of faults is taken into account in the TES.

Let $F(M)$ denote the set of faults that influence any of the equations in $M$. Then, since a PSO set exactly characterizes a set of equations that can be used to form a test, a formal definition is then given by:

Definition 6 (Test Support, TS): Given a model $M$ and a set of faults $F$, a subset of faults $\zeta \subseteq F$ is a test support if there exists a PSO set $M' \subseteq M$ such that $F(M) = \zeta$. Of special interest are minimal test supports which are naturally defined as:

Definition 7 (Minimal Test Support, MTS): Given a model, a test support is a minimal test support (MTS) if no proper subset is a test support.

Definition 8 (Test Equation Support, TES): An equation set $M$ is a Test Equation Support (TES) if: $F(M) \neq 0$, $M$ is PSO set and for any $M' \not\subseteq M$ where $M'$ is a PSO set it holds that $F(M') \not\supseteq F(M)$.

Also here it is interesting to consider minimal such sets of equations.

Definition 9 (Minimal Test Equation Support, MTES): A TES $M$ is a minimal TES (MTES) if there exists no subset of $M$ that is a TES.

Here it is clear that there is a one to one correspondence between a TES $M$ and TS $\zeta$ given by the relation $\zeta = F(M)$.

An MSO set or MTES signifies the theoretical presence of a structural redundancy which can be used as a consistency check for a part of the system. It is important to notice that, whereas an MSO gives rise to one single residual generator, an MTES gathers all the equations leading to all the possible residual generators for the associated set of faults. Besides the fact that an MTES has a non empty TS, it represents a compact form for the whole set of MSOs sharing the
Definition 10 (Fault Driven Minimal structurally overdetermined, FMSO): A FMSO is a minimal redundancy structurally overdetermined set of $M$ focused on a minimal set of fault of interest. minimal($F(M)$)

The elements of a set of MTES can become FMSOs if they have a minimal redundancy. MTES does not guarantee the minimal redundancy sets so these sets will not always be the optimal sets to develop diagnosability.

4. Decentralized Diagnoser
This section introduces the ideas required to formalize decentralized ARR generation in a structural framework. In the following, the global level refers to no decentralization and, without loss of generality, we consider two hierarchical levels, the so-called local level and hierarchical level.

4.1. Fault-driven Decentralized Ideas
A decomposition of the system $M$, with associated bipartite graph $G(M \cup X \cup Z, A)$, into several subsystems $M_i$ is defined as a partition of its equations.

Let $M = \{M_1, M_2, \ldots, M_n\}$ with $M_i \subseteq M$: $M_i \neq \emptyset$, $\bigcup_{i=1}^{n} M_i = M$, $M_i \cap M_j = \emptyset$ if $i \neq j$.

The decomposition of the global system leads to $n$ subsystems denoted $M_i(x_i^{local}, z_i)$, with associated subgraphs $G(M_i \cup X_i^{local} \cup Z_i, A_i), i = 1, \ldots, n, X_i^{local}$ is defined below.

Definition 11 (Local Variables): The set of local variables of the $i^{th}$ subsystem, denoted as $X_i^{local}$, is defined as the subset of vertices of $X_i$ that are adjacent only to vertices in $M_i$ and not to vertices in any other subsystem $M_j, j \neq i$.

\[
X_i^{local} = \{ u \in X_i : \exists j (j \neq i), v \in M_j, (u, v) \in A \}
\]

Definition 12 (Shared Variables): The set of shared variables, denoted as $X^{shared}$ are defined as the subset of vertices of $X$ that can not be considered local variables for any subsystem:

\[
X^{shared} = X \setminus \bigcup_{i=1}^{n} X_i^{local}
\]  

Consequently: $\forall i \in \{1, \ldots, n\}, X_i^{local} \cap X^{shared} = \emptyset$.

The extensions of the ideas of FMSO and MSO at the local and the hierarchical level are first given below.

Definition 13 (Local FMSO): The set of local FMSO, denoted as $Local FMSO_i$, is defined as the subset of FMSO of the $i^{th}$ subsystem that are adjacent only to local variables in $M_i$ and not to shared variables in any other subsystem $M_j, j \neq i$.

\[
Local FMSO_i = \{ FMSO \in FMSO_i : \exists j (j \neq i), x^{shared} \in M_j \}
\]

Definition 14 (Shared FMSO): The set of Shared FMSO of the $i^{th}$ subsystem, is defined as the subset of FMSO that can not be considered Local FMSO:

\[
Shared FMSO_i = FMSO_i \setminus \left( \bigcup_{i=1}^{n} Local FMSO_i \right)
\]

Consequently: $\forall i \in \{1, \ldots, n\}, Local FMSO_i \cap Shared FMSO_i = \emptyset$.

Definition 15 (Stored MSO): The set of Stored MSO, denoted as $ Stored MSO_i$, is defined as the subset of MSO of the $i^{th}$ subsystem whose vertices $X^m$ are adjacent to vertices $X^n$ in Shared FMSO of the $j^{th}$ subsystem.
Definition 16 (Hierarchical FMSO): A set of hierarchical FMSO is a set of top level which includes the relations in $M_i$ of the $i^{th}$ subsystem which are adjacent to the relations in Shared FMSO$_i$ and Stored FMSO$_i$ sets for all corresponding subordinates subsystems.

Figure 2 shows the scheme of the decentralized diagnoser proposed in this paper, for subsystems at level $i$ of the diagnoser hierarchy. We consider two stages: the first one is offline for calculating analytical residual generators, and the second one is online and communicates directly with the system through a residual generator bank. In the following, we present the algorithm for calculating analytical residual generators in the first stage.

4.2. Algorithm
The diagnoser design is done offline and consists of the 6 steps below. These steps are performed for each subsystem $M_{i,j}$, with a nested loop. Here $i$ is the level in the hierarchy, and $j$ the enumeration of subsystems at the level. Figure 2 illustrates the proposed algorithm for level $i$.

**Figure 2. Architecture of diagnoser offline**

1) At the local level (level $i$), calculate the FMSO of the structural model for each subsystem $M_{i,j}$, considering shared variables as observed. Branch the results in two subsets.

**Outputs:**
- *Local FMSO*: for the subsystem $M_{i,j}$ at level $i$.
- *Shared FMSO*: for the subsystem $M_{i,j}$ at level $i$.

2) Still at local level $i$, compute the FMSO sets of the structural model for each subsystem $M_{i,j}$ and store the set of equations (included in FMSO set) that share variables with *Shared FMSO* of the other subsystem $M_{i,j}$.

**Output:**
- *Stored MSO* set for the subsystem $M_{i,j}$ at level $i$ to be sent at level $i+1$.

3) Send the *Shared FMSO* and *Stored MSO* sets to the hierarchical level.

4) Store the set of relations included in the *Shared FMSO* and the *Stored MSO* set for each subsystem $M_{i,j}$.

**Output:**
- $M_{i,j}$ included Rel : relations included in the *Shared FMSO* and the *Stored MSO* set for the each subsystem $M_{i,j}$ as input at level $i$.

5) At the hierarchical level: compute the FMSO with the $M_{i,j}$ included Rel of the all subordinate local diagnosers of level $i-1$.

**Output:**
- **Hierarchical FMSO**: FMSO of the relations included in the *Shared FMSO* and the *Stored MSO* for all subsystem subordinate local diagnosers of level *i*-1.

6) (optional) As a check, join the *local FMSO* and the *hierarchical FMSO* to find the FMSO computed for the global system in the centralized way.

**Output:**

- **Global FMSO**: *local FMSO* plus *hierarchical FMSO*.

In step one, the aim is find the *Shared FMSO*. *Local FMSO* are only involved in local redundancies and do not need to be analyzed in the upper hierarchical level. After the offline structural analysis, the diagnoser is implemented as a residual generator bank. With the system inputs/outputs, hierarchical and local residual generators are used online to detect and isolate faults at each level. Notice that, if a *Shared FMSO* or *Stored MSO* set is not sent at hierarchical level, it is possible that all MTES are not obtained.

The offline / online FDI process is illustrated in Figure 3. Considering the system inputs and outputs, the hierarchical and local residual generators are used online to detect and isolate faults each level.

![Figure 3. Scheme of a decentralized diagnoser for a subsystem at level i](image)

### 4.3. The equivalence of centralized and decentralized diagnosis

It is desirable that properties such as detectability and isolation of faults are not altered by decentralization. This can be ensured if the set of FMSO derived with global and decentralized architectures are identical [1].

**Proposition 1**: The set of FMSO computed in a centralized way and the set of MTES computed in a decentralized way from the *Stored MSO* is the same.

**Proof**: Given a system decomposed into *n* subsystems denoted as \(\{M_1, M_2, \ldots, M_n\}\). Let \(\{\text{Local FMSO}_i, \text{Shared FMSO}_i, \text{Stored MSO}_i\}\), *i* = 1..*n* it is not necessary to send all MSO because the necessary redundant information is in relationships that contain shared variables; i.e., the FMSO sets that share information with all *Shared FMSO* of the other subsystems. In the same way, the set of MTES computed in a centralized way and the set of MTES computed in a decentralized way ends the same, resulting in both case [1].

### 5. Application to the Attitude Determination and Control System of a LEO Satellite

The decentralized diagnoser has been tested on the Attitude Determination and Control System (ADCS) of a low earth-orbit satellite. The composition of the ADCS of a typical satellite is represented in figure 4:

The attitude determination subsystem (ADS) is composed of sensors which sense the rate and angular position of the satellite. An attitude estimate is achieved using a sensor fusion (rate
and vector sensors) [14], which is provided as input to the attitude control subsystem (ACS). The ACS is composed of the control signal calculation and the actuators which provide the stabilizing and/or control torque to the satellite. The satellite under study is assumed to be a three-axis stabilized satellite in orbit around the earth. Here, reaction wheels and magnetorquers are considered as actuators [15].

5.1. Dynamics of the satellite
To analyze the motion of the satellite, two sets of coordinate systems are defined: a Sun-centered inertial frame with its origin at the center of the Sun and its third axis ($z_i$) normal to the ecliptic plane of the rotation of the Earth around the Sun ($x_i, y_i$) and a body-fixed frame which has its origin at the center of mass and its axes aligned with the principal axes of the satellite inertia.

The satellite is modeled as a rigid body having the moments of inertia matrix along the principal axes of rotation, $I=\text{Diag}_{3\times3}\{I_x, I_y, I_z\}$. Assuming that $x_b$, $y_b$ and $z_b$ are the principal axes of inertia, the rotational motion of the satellite can be described in the body frame as follow [14]:

$$\begin{bmatrix} I \dot{w} \\ 0 \\ 0 \\ \end{bmatrix} = I \dot{w} = T - w \times (Iw) \tag{3}$$

where the angular velocity vector $w$ has components $w_x$, $w_y$ and $w_z$, each along the body axes $x_b$, $y_b$ and $z_b$ of the satellite, $\dot{w}$ is the angular acceleration, and $T$ is the total torque acting along the body axes. The system of differential equations describing the vehicle attitude is

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \frac{1}{c_\theta} \begin{bmatrix} 0 & s_\phi & c_\phi \\ 0 & c_\phi c_\theta & -s_\phi c_\theta \\ -s_\phi & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} \tag{4}$$

where $\psi$, $\theta$ and $\phi$ denote yaw, pitch and roll angles, respectively. The state vector of the satellite is $X = [\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}]$. They are the angles by which the body frame is rotated relative to a reference frame.

5.2. Attitude Determination and Control System modelling
The rate sensors of the satellite are three gyroscopes and the vector sensors are sun and star sensors. It is assumed that sensing axes of the rate gyro are aligned with each of the body axes of the satellite. The angular rate measurements from the gyro are used to solve the set of differential equations described by 4. $w_x, w_y$ and $w_z$ represent outputs of the three orthogonal rate gyros with their sensing axes aligned with the roll, pitch and yaw axes, respectively.

The attitude measurement from vector sensors is bounded and used to aid gyro to eliminate attitude drift error. The sensitive axes of the rate gyro are aligned with each of the body axes of the satellite. The modelling of the AD system is described in [14]. The vector and rate sensor
outputs are used to estimate the state vector both independently and merged together. These preliminary estimates are then fused together to arrive at the estimate which is feedback to the ACS. This redundancy can be used to check consistency [15].

The ACS is equipped with a three reaction wheels for 3-axis control. Another external torque source is necessary to unload the wheel’s angular momentum. For this satellite, magnetic torques are selected instead of thrusters that consume a large amount of fuel [16].

5.3. Fault scenarios
Faults are introduced in the system model equations. We consider faults occurring in the rate and vector sensors of the ADS and in the reaction wheels of the ACS. Each of these faults has three components corresponding to the three axes. The faults are summarized in Table 3.

<table>
<thead>
<tr>
<th>Component</th>
<th>Subsystem</th>
<th>Fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector sensors</td>
<td>ADS</td>
<td>$f_{vs}(f_{vxs}, f_{vys}, f_{vzs})$</td>
</tr>
<tr>
<td>Rate sensors</td>
<td>ADS</td>
<td>$f_{rs}(f_{rsx}, f_{rsy}, f_{rsz})$</td>
</tr>
<tr>
<td>Reaction wheel</td>
<td>ACS</td>
<td>$f_{rw}(f_{rwx}, f_{rwy}, f_{rwz})$</td>
</tr>
</tbody>
</table>

5.4. Structural Model of the ADCS
The structure of the ADCS is abstracted as a set of constraints on a set of variables. Related information of such modelling can be founded in [16, 14]. Most constraints are composed of three behavioral relations corresponding to three axes. From the set of variables of the system, the sensed quantities form the set of observed variables with all the rest assumed to be unobserved. The general procedure for the diagnoser design starts with assuming a small set of observed quantities, and can be optionally expanded to fulfill diagnosis and isolation capability specifications.

The bi-adjacency matrix of the ADCS is shown in figure 5. The unobserved faults and observed variables are separated along the X-axis. The constraints that describe the behavior of the system components are described on the Y-axis. The structural models of the ADCS, ADS and ACS are represented as $(C_{ADCS}, X_{ADCS}, Z_{ADCS}), (C_{ADS}, X_{ADS}, Z_{ADS})$ and $(C_{ADC}, X_{ADC}, Z_{ADC})$ respectively. The structural model of the ADCS is composed of 42 constraints with 42 unobserved variables, 15 observed variables and 9 faults (modeled as variables in the bi-adjacency matrix).

The structural model of the satellite ADCS, is considered to demonstrate the proposed decentralized architecture while still maintaining the same isolation capability power as the centralized approach and the advantageous isolation on request capability.

5.5. Diagnoser Design
For reference, first we use the algorithm to determine MTS, MSO and FMSO sets to the ADCS considered globally. Constraints $e_i$ from 1 to n are denoted $e_1...e_n$.

ADCS global diagnoser
Max fault isolability: $[f_{rwx}, f_{rwy}, f_{rwz}, f_{rsx}, f_{rsy}, f_{rsz}, f_{vsx}, f_{vsy}, f_{vzs}]$
FMSO sets: $[e_4, e_3, e_{10}, e_{13}], [e_5, e_8, e_{11}, e_{14}], [e_6, e_9, e_{12}, e_{15}], [e_7...e_{21}, e_{25}], [e_7...e_{21}, e_{26}], [e_7...e_{21}, e_{27}], [e_7...e_{21}, e_{22}, e_{28}], [e_7...e_{21}, e_{23}, e_{29}], [e_7...e_{21}, e_{24}, e_{30}]$
Figure 5. ADCS structure of a LEO satellite.

Num. MSO sets: 2448

ACS local diagnoser ($X^{\text{SHARED}}$ unobserved)
Max fault isolability: $[frw_x], [frw_y], [frw_z]$
Local FMSO sets: $[e_4, e_7, e_{10}, e_{13}], [e_5, e_8, e_{11}, e_{14}], [e_6, e_9, e_{12}, e_{15}]$

ADS local diagnoser ($X^{\text{SHARED}}$ unobserved)
Max fault isolability: $[frs_x, fvs_x], [frs_y, fvs_y], [frs_z, fvs_z]$
Local FMSO sets: $[e_{22}, e_{25}, e_{28}], [e_{23}, e_{26}, e_{29}], [e_{24}, e_{27}, e_{30}]$

According to these results, it can be seen that all faults can be isolated with a centralized global diagnoser for the ADCS. However, for the local diagnoser ADS, faults on the rate and vector sensors can not be isolated locally. It also shows that between 2448 MSO sets and 9 FMSO sets there is a large computational advantage. Because MTES does not guarantee minimum redundancy, it has not been considered. It is more efficient to consider FMSO introduced in the previous section.

The proposed decentralized architecture will now be applied to the ADCS by designing the local and hierarchical diagnosers. It will be demonstrated that the isolation capability of such a decentralized diagnoser is equivalent to the global diagnoser above.

For each local diagnoser, the shared variables $X^{\text{SHARED}}$ are assumed to be observed. It can be assumed that for each subsystem, shared variables are calculated in other local systems. Shared FMSO can be derived using the algorithm with this assumption.

ADS local diagnoser ($X^{\text{SHARED}}$ observed)
Max fault isolability: $[frs_x], [frs_y], [frs_z], [fvs_x], [fvs_y], [fvs_z]$
Shared FMSO: $[e_{19}, e_{20}, e_{21}, e_{22}, e_{28}], [e_{19}, e_{20}, e_{21}, e_{23}, e_{29}], [e_{19}, e_{20}, e_{21}, e_{24}, e_{30}], [e_{19}, e_{20}, e_{21}, e_{25}], [e_{19}, e_{20}, e_{21}, e_{26}], [e_{19}, e_{20}, e_{21}, e_{27}]$

ACS local diagnoser ($X^{\text{SHARED}}$ observed)
Max fault isolability: $[frw_x], [frw_y], [frw_z]$
Shared FMSO: $[e_4, e_7], [e_4, e_{16}], [e_5, e_8], [e_5, e_{17}], [e_6, e_9], [e_6, e_{18}]$
In the case of ACS local diagnoser, the results demonstrate the efficiency of FMSO sets to ensure minimal redundancy \((sr = 1)\) compared with MTES sets: using the algorithm, as it discussed in [1], the structural redundancy for this MTES set is equal to 10. Now, we can calculate the Hierarchical diagnoser with relations included in Shared FMSO and Stored MSO sets for every subsystem, this calculation its necessary to disambiguate faults.

**ADCS Hierarchical diagnoser**

\[
\text{Rel in Shared FMSO and Store MSO (ADCS)} = \begin{bmatrix} e_1, e_2, e_3, \ldots, e_{18}, & e_{19}, e_{20}, e_{21}, \ldots, e_{30} \end{bmatrix}
\]

**Fault isolability:**  
\[
\begin{bmatrix} frw_x, frw_y, frw_z, frs_x, frs_y, frs_z, fus_x, fus_y, fus_z \end{bmatrix}
\]

**Hierarchical FMSO sets:**  
\[
\begin{bmatrix} e_4, e_3, e_{10}, e_{13}, e_5, e_8, e_{11}, e_{14}, e_6, e_9, e_{12}, e_{15}, e_{7}, e_{21}, e_{25}, e_{7}, \ldots, e_{21}, e_{26}, e_{7}, \ldots, e_{21}, e_{27}, \ldots, e_{21}, e_{22}, e_{28}, e_{7}, \ldots, e_{21}, e_{23}, e_{29}, e_{7}, \ldots, e_{21}, e_{24}, e_{30} \end{bmatrix}
\]

Num. MSO sets: 2448

If we calculate the Hierarchical MTES sets, we can find the same sets because they have the minimal redundancy \((sr = 1)\). However, they do not always have the same results because the MTES may have no minimum structural redundancy, which does not make them fully effective.

5.6. Simulation of the decentralized ADCS diagnoser

Figure 6 and 7 show the performance of the proposed decentralized diagnoser. In 6, there is a fault injection between 100 and 105 seconds for reaction wheel 3 during reference tracking. Simultaneously, as shown in Figure 7, there is a fault injection in the rate sensor (gyro 2) between 80 and 85 seconds during reference tracking.

Faults with a 7.5 % range around their mean values were injected into the reaction wheels and rate sensors in the simulation. A much smaller set of threshold parameters is required compared to the case of the conventional diagnoser, as the residual generators are structurally decoupled from the design phase. It is noted that decentralized diagnoser correctly identifies both faults.

**Figure 6.** Reaction wheel residuals with fault injection in reaction wheel 3.  
**Figure 7.** Rate sensor residuals with fault injection in rate gyro 2.
6. Conclusion
This paper proposes an approach for the decentralization of model-based diagnosis through structural redundancy analysis. Decentralized diagnosis is justified by many applications such as on-board systems that need to reduce the calculations required in each time cycle or large systems that require minimal communication between elements. The first contribution of the paper is the design of a fault-driven residual generation method and corresponding algorithm in a decentralized case. The second contribution is the introduction of the Fault-Driven MSO concept which can be directly used to construct an ARR or residual generator, as compared to MTES that by definition may involve several subsets leading to RRAs. Consequently, FMSO sets represent a more practical solution for generating RRAs focused on faults. The algorithm has been implemented and tested on the Attitude Determination and Control System of a LEO Satellite. This implementation illustrates the advantages of decentralized diagnosis architecture, which offers lower complexity and isolation on request capabilities with the same isolation capability power as centralization architecture.

The fault-driven residual generation based on FMSO drastically reduces the number of residual generators to be computed. However, as in the case of the ACS local diagnoser studied, all the generated FMSOs may not be necessary to achieve required isolability. Thus, it is necessary to use optimization criteria to select the better sets to develop RRAs. Current work focuses on that issue.

References
[8] Blanche M et al. 2006 Diagnosis and Fault-tolerant Control (Berlin: Springer)
[16] Zuliana I, Renuganth V A study of reaction wheel configurations for a 3-axis satellite attitude control (Advances in Space Research) vol 45 no 6 p 750 - 759