PGD driven by the Constitutive Relation Error - Minimal CRE/PGD
Pierre-Eric Allier, Ludovic Chamoin, Pierre Ladevèze

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PGD driven by the CRE
Minimal CRE/PGD

Pierre-Éric Allier, Ludovic Chamoin, Pierre Ladevèze

1 - Problematic
- Numerical models for engineering
  - simulate to design and decide
  - resolution of PDEs with
    - a huge number of degrees of freedom (dofs)
    - a huge number of parameters
    - even nonlinearities …
  - direct approaches impossible due to large computation times and large data sets!

2 - PGD: model reduction
- Principle
  - capture the main features of the behavior
  - separation of variables: approximation of the solution on a reduced basis
  - without prior knowledge of this reduced basis

PDE: \( \mathcal{L}(u(M, t)) = 0 \)
\( u(M, t) = \sum_{i=1}^{\infty} \psi_i(M) \lambda_i(t) \)

- alternative methods : POD / RB
  - drawback: require partial knowledge of the reduced basis

3 - Reduction error
- Different PGD strategies
  - [Ladevèze 99, Ammar 10, Nouy 10]
- Versus a reference reduced solution
  - SVD decomposition of a FEM solution
- Measure of the deviation
  - highly refined space-time meshes: discretization error ignored
  - classic
  - update of \( \lambda \) (LMT method)

4 - Project goal
- Mix of two tools
  - the PGD model reduction
  - the Constitutive Relation Error (CRE) estimation
- Idea: minimization of the error indicator
  \[ \arg\min_{\sigma} ||\sigma - K\epsilon(u)||_E \]

5 - Feasibility study
- Model problem: transient thermal

\[ \begin{cases} \frac{\partial u}{\partial t} - \Delta u = \delta(x - vt) \\ u(0, t) = u(L, t) = u(x, 0) = 0 \end{cases} \]

- Minimizing the CRE indicator
  - checking the equilibrium implies
  \[ \sigma(M, t) \simeq \sum_{i=1}^{m} \frac{\partial \lambda_i(t)}{\partial t} Z_i(M) \]
  \[ \int_{\Omega} Z_i \frac{\partial w}{\partial x} + c_{0} \sqrt{w} \, dx = 0, \forall w \in \Gamma \]
- Greedy algorithm
  - known, computation of a \( m \)th mode (i.e. \( \psi_m, \lambda_m, Z_m \))
- Fixed point algorithm
  - computes alternately \( \lambda_m \) and (\( \psi_m, Z_m \))

6 - First results
- A PGD strategy with its own error indicator

\[ \text{PGD modes m} \]

\[ \text{PGD modes m} \]

7 - Conclusion
- A new PGD strategy with
  - improved PGD solution
  - reliable error reduction estimation
- Outlook: extension to nonlinear problems