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A possible generalization of quantum mechanics to special and general relativity

O. Chavoya Aceves

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Abstract

Based on a reinterpretation of Hamilton-Jacobi equation, a generalization of Madelung's hydrodynamic model of quantum mechanics is proposed, which is valid in the realm of special relativity and can be extended to study gravitational fields with quantum effects. As an additional result, we show that there are elements to presume that, in fact, quantum mechanics is not a complete theory of motion.

1 Introduction

In classical analytical mechanics, the solution of the Hamilton-Jacobi equation appears as means to the end of finding a canonical transformation under which the corresponding Hamilton function is zero and, as a consequence, the new phase coordinates are constant. If \( H(q, p) \) and \( K(Q, P) \) are the Hamilton functions of a mechanical system, for two systems of canonical coordinates, the equations of motion can be obtained from the conditions:

\[
\delta \int_{t_0}^{t_1} p_i dq_i - H dt = 0 \quad \text{or} \quad \delta \int_{t_0}^{t_1} -q_i dp_i - H dt = 0. \tag{1}
\]

and

\[
\delta \int_{t_0}^{t_1} P_i dQ_i - K dt = 0 \quad \text{or} \quad \delta \int_{t_0}^{t_1} -Q_i dP_i - K dt = 0. \tag{2}
\]

There are thus four possibilities:

\[
\begin{align*}
p_i dq_i - H dt &= P_i dQ_i - K dt + dF_1, \\
p_i dq_i - H dt &= -Q_i dP_i - K dt + dF_2, \\
-q_i dp_i - H dt &= P_i dQ_i - K dt + dF_3, \\
-q_i dp_i - H dt &= -Q_i dP_i - K dt + dF_4.
\end{align*} \tag{3}
\]

where \( F_1, F_2, F_3 \) and \( F_4 \) are known as generating functions of the first, second, third, and fourth kind, respectively.
For a generating function $S(q_i, P_i)$, of the second kind

$$p_i = \frac{\partial S}{\partial q_i}; \quad Q_i = \frac{\partial S}{\partial P_i} \quad \text{and} \quad K = H + \frac{\partial S}{\partial t},$$

(4)

which suggests to look for a generating function satisfying the equation

$$\frac{\partial S}{\partial t} + H\left(q_i, \frac{\partial S}{\partial q_i}\right) = 0;$$

(5)

because, then, $K \equiv 0$ and each of the phase coordinates $(Q_i, P_i)$ are constants of motion, or integrals of the canonical equations.

A complete solution of (5) has the form

$$S = S(q_1, ..., q_n, t, P_1, ..., P_n) \quad \text{and},$$

from this, an implicit solution of the canonical equations can be obtained, in terms of $2 \cdot n$ constants $(Q_i, P_i)$, as follows from (4).

In this paper we look at the Hamilton-Jacobi equation from the perspective of a hydrodynamic analogy to classical mechanics. We obtain some results which we consider as evidence that quantum mechanics does not reduce to classical mechanics in the limit $h \approx 0$ but to a particular case of it, which we take as an indication that it is not a complete theory of motion.

As we have stressed in a previous paper [2] we consider a theory complete if:

1. The representation used by the theory is complete and the corresponding interpretation is unambiguous;
2. the axioms are logically consistent;
3. the theory does not leave any sensitive question without a sensitive answer;
4. the theory is true.

Those results allow us to propose a generalization of classical quantum mechanics, that abandons the concept of a wave function. This extension can, in turn, be generalized to special relativity, providing an example of a possible way to understand the interaction of matter and radiation in a self-consistent fashion. Though conservation of mass energy is here expressed in terms of a stress energy tensor and the formalism makes not use of wave functions—but only the mathematical apparatus of tensor calculus—and therefore this formalism can be translated to the language of general relativity, we do not pretend to have a self consistent general field theory of matter, radiation, and gravitation, because the results here presented refer to a single kind of material particles.
2 Hydrodynamic analogy of classical mechanics

Let’s consider a classical Hamiltonian system with \( n \) degrees of freedom. The equations of motion in the phase space \((q_i, p_i)\) are

\[
\dot{q}_i = \frac{\partial H}{\partial p_i}; \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} \tag{6}
\]

We focus our attention on a region \( R(0) \) of the configuration space \( q = (q_1, ..., q_n) \) where a vector field \( p = p_i(q_1, \ldots, q_n) \) is defined. We assume this field to be continuous and differentiable as much as required to ensure the validity of our arguments. As times passes and the phase points with the initial conditions \((q, p(q))\) at \( t = 0 \) move—here \( q \in R(0) = R \) changes, as well as the field \( p = p(q, t) \). The value of Hamilton’s function as a function of position in \( R(t) \) is

\[
H(q, p(q, t)).
\]

The time derivative of \( p_i \) along the phase trajectory that passes through the point \((q, p(q))\)—a sort of convective derivative—is

\[
\frac{dp_i}{dt} = \frac{\partial p_i}{\partial t} + \sum_{j=1}^{n} \dot{q}_j \frac{\partial p_i}{\partial q_j} = \frac{\partial p_i}{\partial t} + \sum_{j=1}^{n} \frac{\partial H}{\partial p_j} \frac{\partial p_i}{\partial q_j}.
\]

Using (6):

\[
\frac{\partial p_i}{\partial t} + \sum_{j=1}^{n} \frac{\partial H}{\partial p_j} \frac{\partial p_i}{\partial q_j} = -\frac{\partial H}{\partial q_i} + \sum_{j=1}^{n} \frac{\partial H}{\partial p_j} \frac{\partial p_j}{\partial q_i}.
\]

The reason to include the second term in the right side is that, in (6), all the phase variables are treated as independent, whilst in the situation we are considering the momenta are functions of time and position in the configuration space.

By rearranging the terms we get to the equation

\[
\frac{\partial p_i}{\partial t} + \frac{\partial H}{\partial q_i} = \sum_{j=1}^{n} \frac{\partial H}{\partial p_j} \left( \frac{\partial p_j}{\partial q_i} - \frac{\partial p_i}{\partial q_j} \right). \tag{7}
\]

If it happens that there is a function \( \phi(q) \) such that at \( t = 0 \)

\[
p_i(q_i) = \frac{\partial \phi}{\partial q_i}, \tag{8}
\]

the right side of (7) will be zero at \( t = 0 \); and this will not change as time passes, because of HELMHOLTZ’S CIRCULATION THEOREM [6, p. 180]. Therefore, if (8) is true, (7) can be simplified to

\[
\frac{\partial \phi}{\partial t} + H(q, \frac{\partial \phi}{\partial q_i}) = 0.
\]
This equation is essentially (5), now presented as a particular case of (7), which is valid in general, no matter what the initial conditions are on the field \( p_i \).

Notice that

\[
\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \sum_{j=1}^{j=n} \dot{q}_j \frac{\partial \phi}{\partial q_j} = \sum_{j=1}^{j=n} p_j \dot{q}_j - H = L,
\]

where \( L \) is the Lagrange function, which in this scenario is a function of position in the configuration state and time. This identifies \( \phi \) as the mechanical action \( S \)—but for a function of time, which is of no consequence for the determination of the momenta using (8).

3 Classical particle acted by a conservative force

If the system is a single particle acted by an external conservative field, the Hamilton function, in Cartesian coordinates, assumes the well-known form

\[
H(r, p) = \frac{p^2}{2m} + V(r); \tag{9}
\]

and (7) can be written as

\[
\frac{\partial p_i}{\partial t} + \frac{\partial}{\partial x_i} \left( \frac{p^2}{2m} + V \right) = \frac{p_i}{m} \left( \frac{\partial p_i}{\partial x_i} - \frac{\partial p_j}{\partial q_j} \right)
\]

or

\[
\frac{\partial p}{\partial t} + \nabla \left( \frac{p^2}{2m} \right) - \frac{p}{m} \times (\nabla \times p) = -\nabla V, \tag{10}
\]

and

\[
m \left( \frac{\partial v}{\partial t} + (v \cdot \nabla) v \right) = -\nabla V. \tag{11}
\]

(Notice the analogy to a perfect incompressible fluid, but for the condition \( \nabla \cdot v = 0 \) that follows from the continuity equation.)

If \( \nabla \times p = 0 \) at \( t = 0 \) then—as aforementioned—there is a function \( S = S(r, t) \) such that \( p = \nabla S \) and

\[
\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(r) = 0. \tag{12}
\]

The last equation is a particular case of (10) and it poses no difficulty to imagine initial conditions \( p = p(r, 0) \), where \( \nabla \times p \neq 0 \). It follows then that—in principio—for the solution of any problem of the sort we are studying, (10) must be considered as the fundamental law and any theory—or description of motion—that is based on (12) must be understood as applicable to particular cases and, subsequently, insufficiently general.
3.1 Madelung substitution in quantum mechanics

In 1927, one year after Erwing Schrödinger published his paper reducing the problem of quantization to the eigenvalue problem \[8\], Erwin Madelung published a paper revealing an analogy between classical hydrodynamics and quantum mechanics \[7\].

Madelung used the substitution

\[
\Psi = \sqrt{\rho} e^{i\phi}
\]

in Schrödinger equation for a single particle

\[
i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + V\Psi.
\]

(14)

For reasons we have explained somewhere else \[3\]—because \(\sqrt{\rho}\) can fail to be differentiable at points where \(\rho = 0\), even if \(\rho\) is differentiable at those points—we will use Landau’s substitution (\[5, p. 52\])

\[
\Psi = Ae^{i\phi},
\]

(15)

where \(A\) and \(\phi\) are real functions.

Working on the terms in (14) one by one, and considering the real and imaginary parts

\[
hA \frac{\partial \phi}{\partial t} + \frac{\hbar^2}{2m} A(\nabla \phi)^2 + AV - \frac{\hbar^2}{2m} \Delta A = 0,
\]

and

\[
\frac{\partial A}{\partial t} + \frac{\hbar}{m} \nabla A \nabla \phi + \frac{\hbar}{2m} A \Delta \phi = 0.
\]

By making \(S = h\phi\), these equations can be rewritten as

\[
\frac{\partial S}{\partial t} + \frac{(\nabla \phi)^2}{2m} + V - \frac{\hbar^2}{2m} \frac{\Delta A}{A} = 0.
\]

(16)

and

\[
\frac{\partial \rho}{\partial t} + \frac{1}{m} \nabla \cdot (\rho \nabla S) = 0.
\]

(17)

Equation (16) corresponds to (12) which, as we have seen, is only a particular case of equation (10), which casts doubts about the generality of classical quantum mechanics as it is formulated and therefore about its completeness.

Whether quantum mechanics should lead to (12) or the more general equation (10) in the classical limit, is a sensitive question wanting a sensitive answer, as required by the principle of correspondence. In regards to this there are two possibilities to consider:

I If quantum mechanics should lead to (10) in the classical limit, the representation used by quantum mechanics—the wave function—is not complete, in the sense that it cannot be used to describe all physical possibilities. Correspondingly, quantum mechanics as it is currently formulated is not a complete theory of motion.
If quantum mechanics should lead to (12) in the classical limit, the fact that this requires $\nabla \times \mathbf{p} = 0$ requires an explanation which is not found in Quantum mechanics and, accordingly, quantum mechanics is not a complete theory.

We do not see any a priori reason to prefer (12) instead of (10) as the classical limit of quantum mechanics. Eq. (5) is a mathematical contrivance to find solutions to the canonical equations of motion, while (10) has a physical meaning in and of itself. For these reasons, we feel compelled to consider what seems to be the simplest generalization of the equations of quantum mechanics that is compatible with (10) in the classical limit:

$$\frac{\partial \mathbf{p}}{\partial t} + \nabla \left( \frac{\mathbf{p}^2}{2m} + V \right) - \frac{\mathbf{p}}{m} \times (\nabla \times \mathbf{p}) = \nabla \left( \frac{\hbar^2}{2m} \frac{\Delta A}{A} \right),$$  \(18\)

$$\frac{\partial A^2}{\partial t} + \frac{1}{m} \nabla \cdot (A^2 \mathbf{p}) = 0,$$  \(19\)

Those equations were obtained by E. Madelung [7], who then abandoned them because he noticed that they do not generalize to the case of many particles. However, we still have (7) of which (5) is only a particular case. Therefore, our question makes sense for a system of two or more particles.

Equation (18) is singular at those points where $A = 0$. This singularity is removed by multiplication of both sides of it with $\rho = A^2$ because

$$\rho \frac{\partial}{\partial x_i} \left( \frac{\Delta A}{A} \right) = -\frac{\partial A}{\partial x_i} \frac{\partial^2 A}{\partial x_j^2} + A \frac{\partial^3 A}{\partial x_j^2 \partial x_i}$$

$$= \frac{\partial}{\partial x_j} \left( A \frac{\partial^2 A}{\partial x_i \partial x_j} - \frac{\partial A}{\partial x_i} \frac{\partial A}{\partial x_j} \right).$$

Equation (18) takes then the form

$$\rho \left[ \frac{\partial \mathbf{p}}{\partial t} + \nabla \left( \frac{\mathbf{p}^2}{2m} + V \right) - \frac{\mathbf{p}}{m} \times (\nabla \times \mathbf{p}) \right] = -\nabla \cdot \mathbf{T},$$  \(20\)

where

$$\mathbf{T}_{ij} = \frac{\hbar^2}{2m} \left( \frac{\partial A}{\partial x_i} \frac{\partial A}{\partial x_j} - A \frac{\partial^2 A}{\partial x_i \partial x_j} \right).$$

Equation (20) simplifies to:

$$\rho \frac{\partial \mathbf{p}}{\partial t} + \left( \frac{\rho \mathbf{p}}{m} \cdot \nabla \right) \mathbf{p} + \nabla V = -\nabla \cdot \mathbf{T}.$$

Using (19) we get:

$$\frac{\partial \rho \mathbf{p}_i}{\partial t} + \frac{\partial \Pi_{ij}}{\partial x_j} = 0,$$  \(21\)

$$\Pi_{ij} = \frac{\rho p_i p_j}{m} + \mathbf{T}_{ij} + V \delta_{ij},$$  \(22\)
\[ T_{ij} = \frac{\hbar^2}{2m} \left( \frac{\partial A}{\partial x_i} \frac{\partial A}{\partial x_j} - A \frac{\partial^2 A}{\partial x_i \partial x_j} \right), \]  
\[ \frac{\partial \rho}{\partial t} + \frac{1}{m} \frac{\partial (\rho p_j)}{\partial x_j} = 0, \]  
and
\[ \rho = A^2. \]  

Though those equations are non-linear and certainly more complex than the equations of classical quantum mechanics they have a very attractive feature: that they can be generalized to special relativity without too much effort or mathematical complications.

4 Relativistic particle in a external electromagnetic field

The equations of motion of a relativistic charged particle in an external electromagnetic field can be obtained from the Lagrange function
\[ L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} + \frac{q}{c} A \cdot v - qA_0, \]  
where \((A_0, A)\) is the four-potential, which we assume satisfies the Lorentz condition:
\[ \frac{\partial A^\mu}{\partial x^\mu} = 0. \]  
The linear momentum is
\[ p = \frac{\partial L}{\partial \dot{v}} = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{q}{c} A \]  
and the energy:
\[ \mathcal{E} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} + qA_0 \]  
From those expressions we see that
\[ \left( \frac{\mathcal{E} - qA_0}{c} \right)^2 - \left( p - \frac{q}{c} A \right)^2 = m^2 c^2, \]  
and, in consequence
\[ H = c \sqrt{\left( p - \frac{q}{c} A \right)^2 + m^2 c^2 + qA_0} \]
Considering again $p = p(r)$, if $p = \nabla \phi$, we get
\[
\frac{\partial \phi}{\partial t} + c\sqrt{\left(\nabla \phi - \frac{q}{c} A\right)^2 + m^2c^2 + qA_0} = 0.
\] (31)

However, in general, (7) takes the form:
\[
\frac{\partial p}{\partial t} + c\sqrt{\left(p - \frac{q}{c} A\right)^2 + m^2c^2 + q\nabla A_0} = \frac{c\left(p - \frac{d}{c} A\right)}{\sqrt{\left(p - \frac{d}{c} A\right)^2 + m^2c^2}} \times (\nabla \times p).
\] (32)

It is not difficult to prove that the field of velocities is given by
\[
v = \frac{c\left(p - \frac{q}{c} A\right)}{\sqrt{\left(p - \frac{q}{c} A\right)^2 + m^2c^2}}.
\] (33)

By use of this in (32) we get
\[
\frac{\partial p}{\partial t} + c\sqrt{\left(p - \frac{q}{c} A\right)^2 + m^2c^2 + q\nabla A_0} = v \times (\nabla \times p).
\]

Then, after a few transformations equation (32) is written as
\[
\frac{\partial (p - \frac{d}{c} A)}{\partial t} + (v \cdot \nabla) \left(p - \frac{d}{c} A\right) = qE + \frac{q}{c} v \times H,
\] (34)

where
\[
E = -\nabla A_0 - \frac{1}{c} \frac{\partial A}{\partial t} \quad \text{and} \quad H = \nabla \times A.
\]

Equation (34) can be further simplified to:
\[
\frac{\partial (p - \frac{d}{c} A)}{\partial t} + (v \cdot \nabla) \left(p - \frac{d}{c} A\right) = qE + \frac{q}{c} v \times H.
\] (35)

Considering the components of the four-velocity field:
\[
\mu^a = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{v}{c\sqrt{1 - \frac{v^2}{c^2}}}\right)
\]
we can see that (35) is equivalent to the spatial part of the covariant equation
\[
mc\mu^a \frac{\partial \mu^a}{\partial x^b} = \frac{q}{c} F^{\alpha\beta} \mu_\beta
\] (36)

where
\[
F^{ab} = \frac{\partial A^b}{\partial x^a} - \frac{\partial A^a}{\partial x^b},
\] (37)
is Faraday’s tensor. By replacing (23) with
\[
\frac{\partial (\rho \mu^b)}{\partial x^b} = 0
\]
and making
\[
\rho = \alpha^2,
\]
we get
\[
\tau_Q^{ab} = \frac{\hbar^2}{2mc} \left( \frac{\partial \alpha}{\partial x_a} \frac{\partial \alpha}{\partial x_b} - \frac{\partial^2 \alpha}{x_a x_b} \right),
\]
we get
\[
mc\rho \mu^b \frac{\partial \mu^a}{\partial x^b} = \frac{q}{c} \rho \mu_b F^{ab} - \frac{\partial \tau_Q^{ab}}{\partial x^b}.
\]
In the classical limit of low speeds and macroscopic objects, equations (38-41) reduce to the equations of classical mechanics for this kind of problem. In the limit of low speeds and small masses they reduce to a set of equations including classical quantum mechanics as a special case, but more general, because they are not subject to the restriction that \( \nabla \times p = 0 \).

Using equation (38), (41) can be written as
\[
\frac{\partial}{\partial x^b} \left( mc\rho \mu^a \mu^b + \tau_Q^{ab} \right) = \frac{q}{c} \rho \mu_b F^{ab}.
\]
In this equation we see that
\[
\mathcal{j}_b = q \rho \mu_b
\]
appears as a current density, which suggests us to consider Maxwell equations [4, p. 67 eq. (26.5) & p. 74 eq. (30.2)]:
\[
\frac{\partial F^{ab}}{\partial x_c} + \frac{\partial F^{bc}}{\partial x_a} + \frac{\partial F^{ca}}{\partial x_b} = 0,
\]
and
\[
\frac{\partial F_{ab}}{\partial x_b} = -\frac{4\pi}{c} \mathcal{j}_a
\]
Using (44) the right side of (42) can be written as
\[
\frac{\mathcal{j}_b}{c} F^{ab} = -\frac{1}{4\pi} F^{ab} \frac{\partial F_{bc}}{\partial x_c}.
\]
Then
\[
-\frac{1}{4\pi} F^{ab} \frac{\partial F_{bc}}{\partial x_c} = -\frac{1}{4\pi} \left[ \frac{\partial}{\partial x_c} (F_{bc} F^{ab}) - F_{bc} \frac{\partial F^{ab}}{\partial x_c} \right].
\]
Using (43)
\[
-F_{bc} \frac{\partial F^{ab}}{\partial x_c} = F_{bc} \left( \frac{\partial F^{bc}}{\partial x_a} + \frac{\partial F^{ca}}{\partial x_b} \right).
\]
\[\begin{align*}
&= 1 \frac{\partial}{\partial x_c} \left( \delta^a_b F_{de} F^{de} \right) + F_{bc} \frac{\partial F^{ca}}{\partial x_b} \\
&= 1 \frac{\partial}{\partial x_c} \left( \delta^a_b F_{de} F^{de} \right) + \frac{\partial F_{bc} F^{ca}}{\partial x_b} - F^{ca} \frac{\partial F_{bc}}{\partial x_b}
\end{align*}\]

Interchanging the indexes \(b\) and \(c\) and considering that Faraday’s tensor is antisymmetric:

\[
\frac{\partial F_{bc} F^{ca}}{\partial x_b} - \frac{\partial F^{ca} F_{bc}}{\partial x_b} = 0
\]

Equation (45) can then be written as

\[
- \frac{1}{4\pi} F_{ab} \frac{\partial F_{bc}}{\partial x_c} = - \frac{1}{4\pi} \frac{\partial}{\partial x^b} \left( - F^{ac} F_{bc}^{\phantom{c}} + \frac{1}{4} \delta^{ab} F_{cd} F^{cd} \right) = - \frac{\partial \tau^{ab}_{\mathcal{E}}}{\partial x^b}
\]

where

\[
\tau^{ab}_{\mathcal{E}} = - \frac{1}{4\pi} \left( - F^{ac} F_{bc}^{\phantom{c}} + \frac{1}{4} \delta^{ab} F_{cd} F^{cd} \right)
\]

is the stress-energy tensor of the electromagnetic field [4, p.81 eq. 33.1].

### 5 Summary

To summarize we include here a whole set of equations of a covariant hydrodynamic reformulation of quantum electrodynamics, or a possible self-consistent description of the interaction of matter and radiation:

\[
\frac{\partial \rho \mu^a}{\partial x^a} = 0,
\]

(Expressing conservation of electric charge.)

\[
\rho = \alpha^2,
\]

\[
\tau^{ab}_{\mathcal{Q}} = \frac{\hbar^2}{2mc} \left( \frac{\partial \alpha}{\partial x_a} \frac{\partial \alpha}{\partial x_b} - \frac{\partial^2 \alpha}{\partial x_a \partial x_b} \right),
\]

(First pair of Maxwell equations.)

\[
\frac{\partial F_{ab}}{\partial x_c} + \frac{\partial F_{bc}}{\partial x_a} + \frac{\partial F^{ca}}{\partial x_b} = 0,
\]

(Expressing conservation of electric charge.)

\[
\frac{\partial F_{ab}}{\partial x_b} = - \frac{4\pi}{c} q \rho \mu_a
\]
\[ \tau_{ab}^g = \frac{1}{4\pi} \left( -F^{ac}F_{c}^{b} + \frac{1}{4} \delta_{ab} F_{cd}F^{cd} \right) \] (52)

\[ \frac{\partial}{\partial x^b} \left( mc^\mu a^\mu b + \tau_{ab}^g + \tau_{ab}^e \right) = 0. \] (53)

(Expressing conservation of mass energy.)

Thought the concept a wave function is not mentioned here, nor probabilities, this won’t be a causal theory, because the electrical current density, as well as the stress-energy tensor depend on the density \( \rho \). Also, a conspicuous feature of those equations is that spin and spinors do not play any role in them. This might appear as a strong reason to reject these ideas. However, as we have explained in a previous paper, there are also good reasons to believe that spin is not a fundamental property of matter [1].

Finally, those equations and, in particular, the stress tensor in (53) can be used to describe in a self-consistent manner the interaction of matter, radiation, and gravitational field, including quantum effects. Of course, those equations refer to a single kind of particle, which means that they are, at most, one step towards a general field theory of matter, radiation, and gravitation.

References


