New Mathematical approaches in Electrocardiography Imaging inverse problem

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New Mathematical approaches in Electrocardiography Imaging inverse problem

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Context and objectives

Major objectives
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

Mathematical model

Forward model
If we know the heart potential we can compute the electrical potential
\[
\begin{align*}
\text{div}(\sigma_T \nabla u_T) &= 0, \text{in } \Omega_T, \\
\sigma_T \nabla u_T \cdot n &= 0, \text{on } \Gamma_{ext},
\end{align*}
\]

Inverse problem
If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface
\[
\begin{align*}
\text{div}(\sigma_T \nabla u_T) &= 0, \text{in } \Omega_T, \\
\sigma_T \nabla u_T \cdot n &= 0, \text{and } u_T = \mathbf{T}, \text{on } \Gamma_{ext},
\end{align*}
\]

MFS approach

1. Solve the linear system
\[
\mathbf{A}\mathbf{a} = \mathbf{b}
\]
\[
\mathbf{A} = \begin{bmatrix}
\frac{1}{f_0} & \cdots & \frac{1}{f_{N-1}} \\
\vdots & \ddots & \vdots \\
\frac{1}{f_{N-2}} & \cdots & \frac{1}{f_0}
\end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix}
\frac{1}{f_0}(s_0) \\
\vdots \\
\frac{1}{f_{N-1}}(s_{N-1})
\end{bmatrix}
\]
2. Regularization with CRESDO

Optimal control approach

- Poincaré–Steklov variational formulation of the inverse problem.
- Minimize the following energy functional
\[
J(\lambda) = \frac{1}{2} \int_{\Gamma} (\nabla u_0(\lambda) - \nabla u_0(\lambda))^2 d\Gamma
\]

Subject to
\[
\begin{align*}
\text{div}(\sigma_T \nabla u_0(\lambda)) &= 0, \text{in } \Omega_T, \\
u_0(\lambda) &= \mathbf{T}, \text{on } \Gamma_{ext},
\end{align*}
\]

Descent gradient methods
\[
\nabla J(\lambda) = \sigma_T \nabla u_0(\lambda) - \nabla u_0(\lambda) n_T
\]

Discretization with Finite elements method.

In silico gold standard

Anatomical data

Computational heart and torso anatomical models + electrodes position

Simulated cases
- 6 single and double stimuli
- 14 entry cases

Relative error and correlation coefficient

<table>
<thead>
<tr>
<th>Cases</th>
<th>metric</th>
<th>MFS + CRESDO</th>
<th>O.C integrated</th>
<th>O.C refined data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single and double stimuli (6 cases)</td>
<td>RE</td>
<td>0.81 ± 0.04</td>
<td>0.71 ± 0.02</td>
<td>0.59 ± 0.06</td>
</tr>
<tr>
<td>Re-entry (VT) (14 cases)</td>
<td>CC</td>
<td>0.57 ± 0.07</td>
<td>0.74 ± 0.03</td>
<td>0.85 ± 0.04</td>
</tr>
<tr>
<td>All 20 cases</td>
<td>CC</td>
<td>0.58 ± 0.07</td>
<td>0.72 ± 0.04</td>
<td>0.82 ± 0.04</td>
</tr>
</tbody>
</table>

Remarks
- Introducing the torso heterogeneity is natural with FEM, also anisotropy could be introduced
- The error is more important in the left ventricle

Conclusions

Main results and perspectives
- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 20 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79 ± 0.06 to 0.59 ± 0.05
  - CC was improved from 0.59 ± 0.07 to 0.82 ± 0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

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