New Mathematical approaches in Electrocardiography Imaging inverse problem

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Context and objectives

Major objectives
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

Mathematical model

Forward model
If we know the heart potential we can compute the electrical potential
\[
\begin{aligned}
\Delta u &= \vec{b} \\
\text{in } \Omega, \\
\sigma_T \nabla u - n &= 0, \quad \text{on } \Gamma_{ext}, \\
u_T &= u_e, \quad \text{on } \Sigma.
\end{aligned}
\]

Inverse problem
If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface
\[
\begin{aligned}
\Delta u &= 0, \quad \text{in } \Omega, \\
\sigma_T \nabla u - n &= 0, \quad \text{on } \Gamma_{ext}, \\
u_T &= u_e, \quad \text{on } \Sigma.
\end{aligned}
\]

In silico gold standard

Anatomical data

 Computational heart and torso anatomical models + electrodes position

Simulated cases
- 6 single and double stimuli
- 14 entry cases

Results

Relative error and correlation coefficient

<table>
<thead>
<tr>
<th>Cases</th>
<th>metric</th>
<th>MFS + CRESO</th>
<th>O.C integrated</th>
<th>O.C refined data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single and double stimuli</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6 cases)</td>
<td>RE</td>
<td>0.81±0.04</td>
<td>0.71±0.02</td>
<td>0.59±0.06</td>
</tr>
<tr>
<td>Re-entry (VT) (14 cases)</td>
<td>CC</td>
<td>0.57±0.07</td>
<td>0.72±0.03</td>
<td>0.68±0.04</td>
</tr>
<tr>
<td>All 20 cases</td>
<td>CC</td>
<td>0.58±0.07</td>
<td>0.72±0.04</td>
<td>0.67±0.04</td>
</tr>
</tbody>
</table>

Remarks
- Introducing the torso heterogeneity is natural with FEM, also anisotropy could be introduced
- The error is more important in the left ventricle

Optimal control approach

Poincaré–Steklov variational formulation of the inverse problem.

Minimize the following energy functional
\[
J(\lambda) = \frac{1}{2} \int_{\Omega_T} (\nabla u(\lambda) - \nabla u_0(\lambda))^2
\]

Subject to
\[
\begin{align*}
\text{div}(\sigma_T \nabla u(\lambda)) &= 0, \quad \text{in } \Omega_T, \\
u_0(\lambda) &= T, \quad \text{on } \Gamma_{ext}, \\
u_0(\lambda) &= \lambda, \quad \text{on } \Sigma,
\end{align*}
\]

Descent gradient methods
\[
\nabla J(\lambda) = \sigma_T \nabla u(\lambda) - \nabla u_0(\lambda), n_{\Sigma}
\]

Space distribution of the error

Comparison of the optimal control solution for heterogeneous (blue) and homogeneous (green) torso conductivities

Conclusions

Main results and perspectives
- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 20 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79±0.06 to 0.59±0.05
  - CC was improved from 0.59±0.07 to 0.82±0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

Acknowledgment: This work was partially supported by an ANR grant part of "Investissements d’Avenir" program with reference ANR-10-IAIH/04. It is also supported by the IRyM international lab thought the EPICARD team.