New Mathematical approaches in Electrocardiography Imaging inverse problem

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New Mathematical approaches in Electrocardiography Imaging inverse problem

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Context and objectives

Major objectives
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

Mathematical model

Forward model
If we know the heart potential we can compute the electrical potential
\[
\begin{align*}
\nabla \cdot (\sigma r \nabla u_T) &= 0, \text{ in } \Omega_T, \\
\sigma r \nabla u_T \cdot n &= 0, \text{ on } \Gamma_{ext}, \\
u_T &= u_e, \text{ on } \Sigma.
\end{align*}
\]

Inverse problem
If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface
\[
\begin{align*}
\nabla \cdot (\sigma r \nabla u_T) &= 0, \text{ in } \Omega_T, \\
\sigma r \nabla u_T \cdot n &= 0, \text{ and } u_T = T, \text{ on } \Gamma_{ext}, \\
u_T &= 7, \text{ on } \Sigma.
\end{align*}
\]

MFS approach

Solve the linear system
\[
\Delta \tilde{A} = \tilde{b}
\]

Optimal control approach

Poincaré–Steklov variational formulation of the inverse problem.

Minimize the following energy functional
\[
J(\lambda) = \frac{1}{2} \int_{\Omega} (\nabla u_{\lambda}(x) - \nabla u_{\lambda}(x))^2 dx
\]

Subject to
\[
\begin{align*}
\nabla \cdot (\sigma r \nabla u_{\lambda}(x)) &= 0, \text{ in } \Omega_T, \\
u_{\lambda}(x) &= 7, \text{ on } \Gamma_{ext}, \\
u_{\lambda}(x) &= \lambda, \text{ on } \Sigma.
\end{align*}
\]

Descent gradient methods
\[
\nabla J(\lambda) = \sigma r \nabla u_{\lambda}(x) - \nabla u_{\lambda}(x)
\]

Discretization with Finite elements method.

In silico gold standard

Anatomical data

Computational heart and torso anatomical models + electrodes position

Simulated cases
- 8 single and double stimuli
- 14 entry cases

Results

Relative error and correlation coefficient

<table>
<thead>
<tr>
<th>Cases</th>
<th>metric</th>
<th>MFS + CRESO</th>
<th>O.C. integrated</th>
<th>O.C. refined data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single and double stimulus (6 cases)</td>
<td>RE</td>
<td>0.81±0.04</td>
<td>0.71±0.02</td>
<td>0.59±0.06</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.75±0.07</td>
<td>0.77±0.03</td>
<td>0.84±0.04</td>
</tr>
<tr>
<td>Re-entry (VT) (14 cases)</td>
<td>RE</td>
<td>0.79±0.06</td>
<td>0.67±0.04</td>
<td>0.59±0.05</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.68±0.08</td>
<td>0.73±0.04</td>
<td>0.83±0.04</td>
</tr>
<tr>
<td>All 20 cases</td>
<td>RE</td>
<td>0.79±0.06</td>
<td>0.69±0.04</td>
<td>0.59±0.05</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.69±0.07</td>
<td>0.72±0.04</td>
<td>0.84±0.04</td>
</tr>
</tbody>
</table>

Torso Heterogeneity effect

Anatomical data

CT scan of a 43 years old women (left) and computational mesh obtained after segmentation (right) 

Comparison of the optimal control solution for heterogeneous (blue) and homogeneous (green) torso conductivities

Space distribution of the error

Space distribution of the RE over time: Left (left) and right (right) ventricles views

Remarks
- Introducing the torso heterogeneity is natural with FEM, also anisotropy could be introduced
- The error is more important in the left ventricles

Conclusions

Main results and perspectives
- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 20 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79±0.06 to 0.59±0.05
  - CC was improved from 0.59±0.07 to 0.82±0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

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