New Mathematical approaches in Electrocardiography Imaging inverse problem

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New Mathematical approaches in Electrocardiography Imaging inverse problem

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Context and objectives

Major objectives
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

Mathematical model

Forward model
If we know the heart potential we can compute the electrical potential
\[ \text{div} (\sigma \nabla \psi) = 0, \text{in } \Omega, \]
\[ \sigma \nabla \psi \cdot n = 0, \text{on } \Gamma_{ext}, \]
\[ \psi = u_t, \text{on } \Sigma. \]

Inverse problem
If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface
\[ \text{div} (\sigma \nabla u) = 0, \text{in } \Omega, \]
\[ \sigma \nabla u \cdot n = 0, \text{and } u = T, \text{on } \Gamma_{ext}, \]
\[ u = u_r, \text{on } \Sigma. \]

MFS approach

Solve the linear system
\[ \tilde{A} \tilde{a} = \tilde{b} \]
\[ \begin{pmatrix} f(y_{(1)} - x) & \cdots & f(y_{(N)} - x) \\ f(y_{(1)} - y) & \cdots & f(y_{(N)} - y) \\ \vdots & \ddots & \vdots \\ f(y_{(1)} - y) & \cdots & f(y_{(N)} - x) \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_N \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_N \end{pmatrix} \]

Regularization with CRESO

Optimal control approach

Poincaré–Steklov variational formulation of the inverse problem.

Minimize the following energy functional
\[ J(a) = \frac{1}{2} \int_{\Omega} (\nabla \psi(a) - \nabla \psi_0(a))^2 \]

Subject to
\[ \text{div} (\sigma \nabla \psi_0(a)) = 0, \text{in } \Omega, \]
\[ \psi_0(a) = u_t, \text{on } \Gamma_{ext}, \]
\[ \psi_0(a) = \lambda, \text{on } \Sigma. \]

Descent gradient methods
\[ \nabla J(a) = \sigma \nabla \psi_0(a) - \nabla \psi_0(a) \cdot n_n \]

Discretization with Finite elements method.

In silico gold standard

Anatomical data

Computational heart and torso anatomical model + electrodes position

Simulated cases
- 6 single and double stimuli
- 14 entry cases

Relative error and correlation coefficient

<table>
<thead>
<tr>
<th>Cases</th>
<th>metric</th>
<th>MFS + CRESO</th>
<th>O.C. integrated</th>
<th>O.C. refined data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single and double stimuli (6 cases)</td>
<td>RE</td>
<td>0.81±0.04</td>
<td>0.71±0.02</td>
<td>0.59±0.06</td>
</tr>
<tr>
<td>Re-entry (VT) (14 cases)</td>
<td>CC</td>
<td>0.97±0.07</td>
<td>0.76±0.03</td>
<td>0.84±0.04</td>
</tr>
<tr>
<td>All 20 cases</td>
<td>CC</td>
<td>0.99±0.07</td>
<td>0.72±0.04</td>
<td>0.84±0.04</td>
</tr>
</tbody>
</table>

Remarks
- Introducing the torso heterogeneity is natural with FEM, also anisotropy could be introduced
- The error is more important in the left ventricle

Conclusions

Main results and perspectives
- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 20 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79±0.06 to 0.59±0.05
  - CC was improved from 0.59±0.07 to 0.84±0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

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