New Mathematical approaches in Electrocardiography Imaging inverse problem

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New Mathematical approaches in Electrocardiography Imaging inverse problem

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Context and objectives

Major objectives
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, namely the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

Mathematical model

Forward model
If we know the heart potential we can compute the electrical potential

\[
\begin{align*}
\nabla \cdot (\sigma(x) \nabla u_T) &= 0, \quad \text{in } \Omega_T, \\
\sigma_T \nabla u_T \cdot n &= 0, \quad \text{on } \Gamma_{ext}, \\
u_T &= u_T, \quad \text{on } \Sigma.
\end{align*}
\]

Inverse problem
If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface

\[
\begin{align*}
\nabla \cdot (\sigma(x) \nabla u_T) &= 0, \quad \text{in } \Omega_T, \\
\sigma_T \nabla u_T \cdot n &= 0, \quad \text{and } u_T = T, \quad \text{on } \Gamma_{ext}, \\
u_T &= \gamma, \quad \text{on } \Sigma.
\end{align*}
\]

MFS approach

Solve the linear system

\[
A \tilde{b} = \tilde{f}
\]

Subject to

\[
\begin{align*}
\nabla \cdot (\sigma(x) \nabla u_T) &= 0, \quad \text{in } \Omega_T, \\
u_T(x) &= T, \quad \text{on } \Gamma_{ext}, \\
u_T(x) &= \gamma, \quad \text{on } \Sigma.
\end{align*}
\]

Regularization with CRESO

Optimal control approach

Poincaré–Steklov variational formulation of the inverse problem.

Minimize the following energy functional

\[
\begin{align*}
J(\lambda) &= \frac{1}{2} \sum_{l=1}^{L} (\nabla u(x) - \nabla u(x))^2
\end{align*}
\]

Subject to

\[
\begin{align*}
\nabla \cdot (\sigma(x) \nabla u_T) &= 0, \quad \text{in } \Omega_T, \\
u_T(x) &= T, \quad \text{on } \Gamma_{ext}, \\
u_T(x) &= \gamma, \quad \text{on } \Sigma.
\end{align*}
\]

Descent gradient methods

\[
\nabla J(\lambda) = \sigma_T \nabla u_T(x) - \nabla u(x)
\]

Discretization with Finite elements method.

In silico gold standard

Anatomical data

Computational heart and torso anatomical models + electrodes position

Simulated cases

- 6 single and double stimuli
- 14 entry cases

Results

Relative error and correlation coefficient

<table>
<thead>
<tr>
<th>Cases</th>
<th>metric</th>
<th>MFS + CRESO</th>
<th>O.C integrated</th>
<th>O.C refined data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single and double stimulus (6 cases)</td>
<td>RE</td>
<td>0.81±0.04</td>
<td>0.71±0.02</td>
<td>0.59±0.06</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.57±0.07</td>
<td>0.7a±0.03</td>
<td>0.8a±0.04</td>
</tr>
<tr>
<td>Re-entry (VT) (14 cases)</td>
<td>RE</td>
<td>0.79±0.06</td>
<td>0.67±0.04</td>
<td>0.59±0.05</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.6a±0.08</td>
<td>0.7a±0.04</td>
<td>0.8a±0.04</td>
</tr>
<tr>
<td>All 30 cases</td>
<td>RE</td>
<td>0.79a±0.06</td>
<td>0.69±0.04</td>
<td>0.59±0.05</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.59±0.07</td>
<td>0.72±0.04</td>
<td>0.8±0.04</td>
</tr>
</tbody>
</table>

Remarks

- Introducing the torso heterogeneity is natural with FEM, also anisotropy could be introduced
- The error is more important in the left ventricle

Space distribution of the error

- Space distribution of the RE over time: Left (left) and right (right) ventricles views

Conclusions

Main results and perspectives

- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 30 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79±0.06 to 0.59±0.05
  - CC was improved from 0.59±0.07 to 0.8±0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

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