New Mathematical approaches in Electrocardiography Imaging inverse problem

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New Mathematical approaches in Electrocardiography Imaging inverse problem

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Context and objectives

Major objectives
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

Mathematical model

Forward model
If we know the heart potential we can compute the electrical potential

\[
\begin{align*}
\nabla \cdot (\sigma T \nabla u_T) &= 0, \quad \text{in } \Omega_T, \\
\sigma_T \nabla u_T \cdot n &= 0, \quad \text{on } \Gamma_{int}, \\
u_T &= u_e, \quad \text{on } \Sigma.
\end{align*}
\]

Inverse problem
If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface

\[
\begin{align*}
\nabla \cdot (\sigma_T \nabla u_T) &= 0, \quad \text{in } \Omega_T, \\
\sigma_T \nabla u_T \cdot n &= 0, \quad \text{and } u_T = T, \quad \text{on } \Gamma_{ext}, \\
u_T &= \hat{T}, \quad \text{on } \Sigma.
\end{align*}
\]

MFS approach

Solve the linear system

\[
A \hat{u} = \hat{b}
\]

where

\[
A = \begin{bmatrix}
\frac{1}{\bar{d}} f(1 - x_1) & \cdots & \frac{1}{\bar{d}} f(1 - x_n) \\
\vdots & \ddots & \vdots \\
\frac{1}{\bar{d}} f(1 - x_1) & \cdots & \frac{1}{\bar{d}} f(1 - x_n)
\end{bmatrix},
\]

\[
\hat{b} = \begin{bmatrix}
\hat{u}_1 \\
\vdots \\
\hat{u}_n
\end{bmatrix},
\]

\[
\hat{d} = \begin{bmatrix}
f_1(x_1) \\
\vdots \\
f_1(x_n)
\end{bmatrix}
\]

Regularization with CRESO

Optimal control approach

Poincaré–Steklov variational formulation of the inverse problem.

Minimize the following energy functional

\[
J(\lambda) = \frac{1}{2} \int_{\Omega_T} \left( \nabla u_T - \nabla u_0(\lambda) \right)^2 dx
\]

Subject to

\[
\begin{align*}
\nabla \cdot (\sigma_T \nabla u_T) &= 0, \quad \text{in } \Omega_T, \\
u_T &= u_e, \quad \text{on } \Gamma_{int}, \\
\sigma_T \nabla u_T \cdot n &= 0, \quad \text{on } \Gamma_{ext}, \\
u_T &= \lambda, \quad \text{on } \Sigma.
\end{align*}
\]

Descent gradient methods

\[
\nabla J(\lambda) = \sigma_T \nabla (u_T - u_0(\lambda)) \cdot n_T;
\]

Discretization with Finite elements method.

Results

Relative error and correlation coefficient

<table>
<thead>
<tr>
<th>Cases</th>
<th>metric</th>
<th>Tikhonov</th>
<th>O.C refined</th>
<th>O.C refined</th>
<th>O.C refined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single and double stimulus (6 cases)</td>
<td>RE</td>
<td>0.91 ± 0.04</td>
<td>0.71 ± 0.02</td>
<td>0.59 ± 0.06</td>
<td></td>
</tr>
<tr>
<td>CC</td>
<td>0.6 ± 0.07</td>
<td>0.7 ± 0.03</td>
<td>0.6 ± 0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Re-entry (VT) (14 cases)</td>
<td>RE</td>
<td>0.93 ± 0.06</td>
<td>0.71 ± 0.04</td>
<td>0.6 ± 0.05</td>
<td></td>
</tr>
<tr>
<td>All 20 cases</td>
<td>CC</td>
<td>0.6 ± 0.07</td>
<td>0.71 ± 0.04</td>
<td>0.6 ± 0.05</td>
<td></td>
</tr>
</tbody>
</table>

Remarks
- Introducing the torso heterogeneity is natural with FEM, also anisotropy could be introduced
- The error is more important in the left ventricle

Conclusions

Main results and perspectives
- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 20 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79 ± 0.06 to 0.59 ± 0.05
  - CC was improved from 0.59 ± 0.07 to 0.82 ± 0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution.

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