New Mathematical approaches in Electrocardiography Imaging inverse problem
Nejib Zemzemi, Mark Potse, Laura Bear, Yves Coudière, Rémi Dubois, Jacques Henry, C Dallet, Josselin Duchateau, O Bernus, M Haïssaguerre

To cite this version:
Nejib Zemzemi, Mark Potse, Laura Bear, Yves Coudière, Rémi Dubois, et al.. New Mathematical approaches in Electrocardiography Imaging inverse problem. LIRYC scientific day, Jun 2015, Pessac, France. <hal-01222406>

HAL Id: hal-01222406
https://hal.archives-ouvertes.fr/hal-01222406
Submitted on 29 Oct 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
New Mathematical approaches in Electrocardiography Imaging inverse problem

N. Zemzemi, M. Potse, L. Bear, Y. Coudière, Rémi Dubois, J. Henry, C. Dallet, J. Duchesseau, O. Bernus, M. Haïssaguerre.

Inria Bordeaux Sud-Ouest, IHU-LIRYC, CHU-Bordeaux, Université de Bordeaux.

Context and objectives

Major objectives
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

Mathematical model

Forward model
If we know the heart potential we can compute the electrical potential

\[ \begin{align*}
\text{div}(\sigma_1 \nabla u_T) &= 0, \text{ in } \Omega_T, \\
\sigma_T \nabla u_T \cdot n &= 0, \text{ on } \Gamma_{ext}, \\
u_T &= u_e, \text{ on } \Sigma.
\end{align*} \]

Inverse problem
If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface

\[ \begin{align*}
\text{div}(\sigma_1 \nabla u_T) &= 0, \text{ in } \Omega_T, \\
\sigma_T \nabla u_T \cdot n &= 0, \text{ and } u_T = T, \text{ on } \Gamma_{ext}, \\
u_T &= u_e, \text{ on } \Sigma.
\end{align*} \]

MFS approach

Solve the linear system

\[ A \mathbf{a} = \mathbf{b} \]

\[ \begin{bmatrix}
\frac{1}{f(x_1-x_1)} & \ldots & \frac{1}{f(x_1-x_M)} \\
\vdots & \ddots & \vdots \\
\frac{1}{f(x_M-x_1)} & \ldots & \frac{1}{f(x_M-x_M)}
\end{bmatrix}
\begin{bmatrix}
a_1 \\
\vdots \\
a_M
\end{bmatrix}
= \begin{bmatrix}
b_1 \\
\vdots \\
b_M
\end{bmatrix}
\]

Regularization with CRESO

Optimal control approach

Poincaré–Steklov variational formulation of the inverse problem.

Minimize the following energy functional

\[ J(u) = \frac{1}{2} \int_{\Omega_T} (\nabla u_0(x) - \nabla u_0(x))^2 dx \]

Subject to

\[ \begin{align*}
\text{div}(\sigma_1 \nabla u_0(x)) &= 0, \text{ in } \Omega_T, \\
u_0(x) &= T, \text{ on } \Gamma_{ext}, \\
\sigma_0 \nabla u_0(x) \cdot n &= 0, \text{ on } \Gamma_{ext}, \\
u_0(x) &= u_e, \text{ on } \Sigma.
\end{align*} \]

Descent gradient methods

\[ \nabla J(u) = \nabla \left( \frac{\text{div}(\sigma_1 \nabla u_0(x))}{\sqrt{1 + (\text{div}(\sigma_1 \nabla u_0(x)))^2}} \right) \]

Discretization with Finite elements method.

In silico gold standard

Anatomical data

Computational heart and torso anatomical models + electrodes position

Simulated cases

- 6 single and double stimuli
- 14 entry cases

Relative error and correlation coefficient

<table>
<thead>
<tr>
<th>Cases</th>
<th>metric</th>
<th>MFS + CRESO</th>
<th>O.C integrated</th>
<th>O.C refined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single and double stimuli (6 cases)</td>
<td>RE</td>
<td>0.81 ± 0.04</td>
<td>0.71 ± 0.02</td>
<td>0.59 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.75 ± 0.07</td>
<td>0.76 ± 0.03</td>
<td>0.80 ± 0.04</td>
</tr>
<tr>
<td>Re-entry (VT) (14 cases)</td>
<td>RE</td>
<td>0.78 ± 0.06</td>
<td>0.67 ± 0.04</td>
<td>0.59 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.69 ± 0.08</td>
<td>0.73 ± 0.04</td>
<td>0.83 ± 0.04</td>
</tr>
<tr>
<td>All 30 cases</td>
<td>RE</td>
<td>0.79 ± 0.06</td>
<td>0.69 ± 0.04</td>
<td>0.59 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.69 ± 0.07</td>
<td>0.72 ± 0.04</td>
<td>0.82 ± 0.04</td>
</tr>
</tbody>
</table>

Remarks
- Introducing the torso heterogeneity is natural with FEM, also anisotropy could be introduced
- The error is more important in the left ventricle

Space distribution of the error

Space distribution of the RE over time: Left (left) and right (right) ventricles views

Conclusions

Main results and perspectives
- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 30 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79 ± 0.06 to 0.59 ± 0.05
  - CC was improved from 0.59 ± 0.07 to 0.82 ± 0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

Acknowledgment: This work was partially supported by an ANR grant part of “Investissements d’Avenir” program with reference ANR-10-IHIA-04. It is also supported by the LIRIMA international lab thought the EPICARD team.