New Mathematical approaches in Electrocardiography Imaging inverse problem

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New Mathematical approaches in Electrocardiography Imaging inverse problem

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Context and objectives

Major objectives
- Improve ECGI inverse problem reconstruction
- Introduce new mathematical approaches to the field of the ECGI inverse problem
- Compare the performance of the new mathematical approaches to the state-of-the-art methods, mainly the MFS method used in commercial devices.
- In silico validation of the new approaches.
- Assessment of some simplification hypothesis: Torso inhomogeneity
- Propose some uncertainty quantification approaches to deal with measurements errors

Mathematical model

Forward model
If we know the heart potential we can compute the electrical potential
\[
\begin{align*}
\text{div}(σT \nabla u_T) &= 0, \text{ in } Ω_T, \\
σT \nabla u_T \cdot n &= 0, \text{ on } Γ_{ext}, \\
u_T &= u_e, \text{ on } Σ. 
\end{align*}
\]

Inverse problem
If we know the electrical potential and the current density at the outer boundary of the torso and we look for the electrical potential at the heart surface
\[
\begin{align*}
\text{div}(σT \nabla u_T) &= 0, \text{ in } Ω_T, \\
σT \nabla u_T \cdot n &= 0, \text{ and } u_T = T, \text{ on } Γ_{ext}, \\
u_T &= 7, \text{ on } Σ. 
\end{align*}
\]

MFS approach

Solve the linear system
\[
\begin{align*}
\mathbf{A} \mathbf{a} &= \mathbf{b} \\
\mathbf{a} &= \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{pmatrix}, \quad \mathbf{b} &= \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix}, \\
\mathbf{f}(r) &= \begin{pmatrix} f_1(r) \\ f_2(r) \\ \vdots \\ f_N(r) \end{pmatrix}, \\
x_1, \ldots, x_T &\text{: Torsos points} \\
y_1, \ldots, y_M &\text{: Heart points}.
\end{align*}
\]

Regularization with CRESO

Optimal control approach

Poincaré–Steklov variational formulation of the inverse problem.

Minimize the following energy functional
\[
J(λ) = \frac{1}{2} \int_{Ω_T} (|∇u_T(λ) − ∇u_{\text{ref}}(λ)|^2)
\]

Subject to
\[
\begin{align*}
\text{div}(σT \nabla u_T(λ)) &= 0, \text{ in } Ω_T, \\
u_T(λ) &= T, \text{ on } Γ_{ext}, \\
u_{\text{ref}}(λ) &= λ, \text{ on } Σ. 
\end{align*}
\]

Descent gradient methods
\[
\nabla J(λ) = σT \nabla u_T(λ) − ∇u_{\text{ref}}(λ), \eta_λ,
\]

Discretization with Finite elements method.

In silico gold standard

Anatomical data

Computational heart and torso anatomical models + electrodes position

Simulated cases
- 8 single and double stimuli
- 14 entry cases

Results

Relative error and correlation coefficient

<table>
<thead>
<tr>
<th>Cases</th>
<th>metric</th>
<th>MFS + CRESO</th>
<th>O.C integrated</th>
<th>O.C refined data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single and double stimulus (6 cases)</td>
<td>RE</td>
<td>0.81±0.04</td>
<td>0.71±0.02</td>
<td>0.59±0.06</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.57±0.07</td>
<td>0.7a±0.03</td>
<td>0.8a±0.04</td>
</tr>
<tr>
<td>Re-entry (VT) (14 cases)</td>
<td>RE</td>
<td>0.7a±0.06</td>
<td>0.67±0.04</td>
<td>0.59±0.05</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.6a±0.08</td>
<td>0.7a±0.04</td>
<td>0.8a±0.04</td>
</tr>
<tr>
<td>All 30 cases</td>
<td>RE</td>
<td>0.7a±0.06</td>
<td>0.69±0.04</td>
<td>0.59±0.05</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>0.59±0.07</td>
<td>0.72±0.04</td>
<td>0.8±0.04</td>
</tr>
</tbody>
</table>

Remarks
- Introducing the torso heterogeneity is natural with FEM, also anisotropy could be introduced
- The error is more important in the left ventricle

Conclusions

Main results and perspectives
- New mathematical approaches for solving the inverse problem in electrocardiography imaging based on optimal control
- Over all the 30 cases used in this study the optimal control method performs better than the MFS both in terms of relative error and correlation coefficient:
  - RE was improved from 0.79±0.06 to 0.59±0.05
  - CC was improved from 0.59±0.07 to 0.8±0.04
- Our results show that the heterogeneity in the torso has an impact on the accuracy of the solution both in terms of RE and CC.
- We are working on other new approaches for solving ECGI problem and also quantifying the effect of the torso conductivity uncertainties on the ECGI solution

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