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To cite this version:
Rajae Aboulaich, Najib Fikal, El Mahdi El Guarmah, Nejib Zemzemi. Sensitivity of the electrocardiography inverse solution to the torso conductivity uncertainties. LIRYC scientific day, Jun 2015, Pessac, France. hal-01222404

HAL Id: hal-01222404
https://hal.archives-ouvertes.fr/hal-01222404
Submitted on 2 Nov 2015

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Sensitivity of the electrocardiography inverse solution to the torso conductivity uncertainties

A simulation study

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Abstract
Electrocardiography imaging (ECGI) is a non-invasive technology used for heart diagnosis. It allows to estimate the electrical potential on the heart surface only from measurements on the body surface and some geometrical informations of the torso. The purpose of this work is twofold. First, we propose a new formulation to calculate the distribution of the electric potential on the heart, from measurements on the torso surface. Second, we study the influence of the errors and uncertainties on the conductivity parameters, on the ECGI solution. We use an optimal control formulation for the mathematical formulation of the problem with a stochastic diffusion equation as a constraint. The discretization is done using stochastic Galerkin method allowing to separate random and deterministic variables. The optimal control problem is solved using a conjugate gradient method where the gradient of the cost function is computed with an adjoint technique. The efficiency of this approach to solve the inverse problem and the ability to quantify the effect of conductivity uncertainties in the torso are demonstrated through a number of numerical simulations on a 2D geometrical model.

Main Objectives
1. Propose a new method for solving the ECGI problem.
2. Introduce the uncertainty of the conductivity in the ECGI problem
3. Evaluate the effect of uncertainties on the forward and inverse solutions.

Methods

Stochastic forward problem of electrocardiography
We denote by $D$ the space domain and $\Omega$ the probability space.

\begin{equation}
\begin{aligned}
&\Delta u(x,\omega) + \beta(x,\omega) u(x,\omega) = 0 \quad \text{in } D \times \Omega, \\
&w(x,\omega) \cdot n(x) = 0 \quad \text{on } \Gamma_{\text{ext}} \times \Omega, \\
&w(x,\omega) \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_{\text{int}} \times \Omega,
\end{aligned}
\end{equation}

where $\Gamma_{\text{int}}$ and $\Gamma_{\text{ext}}$ are the epicardial and torso boundaries respectively, $\xi \in \mathbb{R}$ is the stochastic variable (it could also be a vector) and $w$ is the potential at the epicardial boundary.

Numerical discretization of the stochastic forward problem
We use the stochastic Galerkin method to solve equation (1). The stochastic conductivity and solution are projected on the probability density functions $\{\phi_i\}_{i=1}^N$.

\begin{equation}
\begin{aligned}
&\int_{\Omega} \beta(x,\omega) \phi_i(x) u(x,\omega) \, d\omega = 0, \\
&\int_{\Omega} \mathbf{n} \cdot \nabla_x \phi_i(x) \cdot \mathbf{n} \cdot u(x,\omega) \, d\omega = 0, \\
&\int_{\Omega} w(x,\omega) \cdot \mathbf{n} \cdot \phi_i(x) \, d\omega = 0.
\end{aligned}
\end{equation}

The elliptic equation (1) projected in the stochastic basis could be solved

\begin{equation}
\begin{aligned}
&\sum_{j=1}^N J_{ij} \phi_i(x) \phi_j(x) = 0, \\
&\sum_{j=1}^N J_{ij} \phi_i(x) \mathbf{n} \cdot \phi_j(x) \cdot \mathbf{n} \cdot \phi_j(x) = 0, \\
&\sum_{j=1}^N J_{ij} \phi_i(x) \phi_j(x) \mathbf{n} \cdot \phi_j(x) = 0,
\end{aligned}
\end{equation}

where $J_{ij} = \int_{\Omega} \beta(x,\omega) \phi_i(x) \phi_j(x) \, d\omega$.

Stochastic ECGI Inverse Problem

Mathematical formulation
We look for the current density and the value of the potential on the epicardial boundary $\{u_1\} \in L^2(\Omega)^N$ by minimizing the following cost function under a stochastic constraint on $v$

\begin{equation}
\begin{aligned}
&\min_{\mathbf{J}} \int_{\Omega} \left[ \frac{1}{2} \beta(\mathbf{J},\omega) \mathbf{J}^T \mathbf{J} \, d\omega + \frac{1}{2} \alpha^2(\mathbf{J},\omega) \mathbf{J}^T \mathbf{J} \, d\omega \right] \\
&\text{subject to} \quad \int_{\Omega} \beta(x,\omega) \phi_i(x) u(x,\omega) \, d\omega = 0, \\
&\int_{\Omega} w(x,\omega) \cdot \mathbf{n} \cdot \phi_i(x) \phi_j(x) \, d\omega = 0, \\
&\int_{\Omega} w(x,\omega) \cdot \mathbf{n} \cdot \phi_i(x) \phi_j(x) \phi_j(x) \, d\omega = 0.
\end{aligned}
\end{equation}

Main Remarks
1. The mean value of the stochastic solution matches with the exact forward solution. This comes from the linearity of the forward problem.
2. For each organ, the uncertainty on the conductivity is reflected by a high uncertainty of the solution at its boundary.
3. The direction of the standard deviation iso-value are are modified when they cross the the organ for which we introduce the uncertainty.
4. The magnitude of the uncertainty does not exceed ±2% of the magnitude of the forward solution.

References

Acknowledgements
We would like to thank the LIRIMA laboratory which financially supported the teams ANO and EPICARD to perform this work. This work was partially supported by an ANR grant part of “Investissements d’Avenir” program with reference ANR-10-IAIHU-04.