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A Framework for Computing Power Consumption Scheduling Functions Under Uncertainty

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Abstract—One of the goals of this paper is to make a step further towards knowing how an electrical appliance should exploit the available information to schedule its power consumption; mainly, this information corresponds here to an imperfect forecast of the non-controllable (exogenous) load or electricity price. Reaching this goal led us to three key results which can be used for other settings which involve multiple agents with partial information: 1. In terms of modeling, we exploit the principal component analysis to approximate the exogenous load and show its full relevance; 2. Under some reasonable but improvable assumptions, this work provides a full characterization of the set of feasible payoffs which can be reached by a set of appliances having partial information; 3. A distributed algorithm is provided to compute good power consumption scheduling functions. These results are exploited in the numerical analysis, which provides several new insights into the power consumption scheduling problem. We provide first results for the standard cost functions, transformer aging in particular, where we compare our method with iterative water filling algorithm (IWFA). We test our proposed algorithm on real data and show that it is more robust with respect to noise in the signals received. We also observe that our proposed method becomes even more relevant when the proportion of appliances with smart counters increase.

I. INTRODUCTION

An important problem for modern electrical networks is to design intelligent strategies for use of electrical appliances (Electric Vehicles (EV) being an example) which are able to exploit the knowledge they have about the non local demand or the electricity price to reach a certain objective. The objective can be to reduce the impact of electricity consumption on the distribution network or to minimize the monetary charging cost paid by the consumer. The most standard approach is to design a charging scheme which assumes a perfect knowledge of the non local demand (or price) and evaluate the performance of the corresponding algorithm by feeding the latter with a forecast or noisy version of the non local demand. Illustrative and recent examples of this approach are given e.g., by [1] [2], [3], [4]. In the quoted references, the energy need of a given user is computed by assuming perfect knowledge of the electricity price or the exogenous demand namely, the part of the demand which is not controlled by the smart consuming devices. The obtained power scheduling schemes have essentially or exactly the water-filling structure i.e., that holes in terms of price or demand are exploited in the first place. One of the drawbacks of this approach is the potential lack of robustness of the designed algorithm to imperfect forecast.

Among existing works which take uncertainty into account in the design of power scheduling scheme we find in particular [5]. Therein the authors propose a threshold-based scheduling policy which accounts for past and current prices and the statistics of future prices; each appliance consumes according to a rectangular profile and starts consuming when the price is below a time-varying threshold ("threshold policy"). In [5] the price values are assumed independent from the load level in each time slot while this assumption is relaxed in [6]. Additionally, in the latter reference the authors also consider uncertainty in the algorithm design part but the uncertainty concerns load and user energy consumption needs, and not prices as in [5]. Another example of relevant work where price uncertainty is considered in real-time demand response model is [7]; therein robust optimization is exploited. In [8], the problem is addressed by stochastic gradient based on the statistical knowledge of future prices.

To our knowledge, there is no contribution in the literature related to the present work which treats the problem of optimality of a power consumption scheduling scheme under given arbitrary imperfect observation or forecast. In fact, to characterize the performance achievable when exploiting optimally the available information, we resort to a very recent result in information theory [9] (Sec. III). In Sec. IV, we exploit the corresponding theorem and the model proposed in Sec. II to build a convergent algorithm which allows suboptimal but typically good power consumption scheduling schemes to be determined numerically. This algorithm is exploited in Sec. V to conduct the provided numerical analysis. In Sec. VI, the main assets of the proposed approach are summarized and several extensions to address its limitations are provided.

II. PROPOSED SYSTEM MODEL

We consider a set of $K \geq 1$ smart electrical appliances. Each appliance aims at scheduling its power consumption to maximize a certain payoff function, which is provided further. To this end, it exploits the available knowledge about a state which affects its payoff. In this paper, this state is the *exogenous load* namely, the part of the load which is not controlled by the smart consuming devices but the proposed model and derived results can be directly used for other types of states such as the electricity price. To define the exogenous load and other key quantities such as the power consumption vectors, we need to specify the

timing aspect. Time is assumed to be slotted in stages $t \in \{1, \dots, T\}$. Typically, a stage may represent a day and T may represent the number of days over which the payoff is averaged. At the beginning of *stage* t , appliance k has to choose a power consumption vector¹ $\underline{x}_k = (x_{k,1}, \dots, x_{k,N})$ by exploiting perfect observations of the past exogenous load vectors $\underline{x}_0(1), \dots, \underline{x}_0(t-1)$ (note that $\underline{x}_0(t)$ is a vector of size N) and a signal which is an image or forecast of the system state and appliances actions at stage t ; this signal is denoted by $\underline{s}_k \in \mathcal{S}_k$. For example, such a signal may be a forecast of the exogenous load or the total load, the total load being equal to the sum $x_0(t) + \sum_{k=1}^K x_k(t)$. For $k \in \{0, 1, \dots, K\}$, the set in which \underline{x}_k lies is assumed to be discrete and is denoted by \mathcal{X}_k . This choice is not only motivated by the fact that both power and time can be discrete in real systems such as electric vehicle battery charging systems but also to obtain a solution which is robust against forecasting noise; the latter issue has been addressed recently in [10] where rectangular consumption profiles typically perform better than continuous profiles.

The computational complexity of the algorithm proposed in Sec. IV depends on the cardinality of the set \mathcal{X}_0 (the set in which the exogenous load lies). In general, assuming \mathcal{X}_0 to be discrete amounts to approximating the exogenous load vectors. To obtain a good approximation, we propose to apply the principal component analysis (PCA) [11] on exogenous load vectors. The exogenous load vector for stage t is approximated as follows:

$$\hat{\underline{x}}_0(t) = \hat{\underline{\mu}}_L + \sum_{i=1}^M a_i(t) \underline{v}_i \quad (1)$$

where $\hat{\underline{\mu}}_L$ is defined by

$$\hat{\underline{\mu}}_L = \frac{1}{L} \sum_{t \in \mathcal{L}} \underline{x}_0(t), \quad (2)$$

\mathcal{L} being a set of L samples which is available to estimate $\hat{\underline{\mu}}_L$; typically, it may correspond to data measured during the preceding year. The vectors \underline{v}_i are the eigenvectors of the following matrix

$$\hat{\mathbf{R}}_L = \frac{1}{L} \sum_{t \in \mathcal{L}} \left[\underline{x}_0(t) - \hat{\underline{\mu}}_L \right] \left[\underline{x}_0(t) - \hat{\underline{\mu}}_L \right]' \quad (3)$$

where the notation $[\cdot]'$ stands for transpose. One of the advantages of using such a decomposition is that for a given number of basis vectors M (the basis is then $(\underline{v}_1, \dots, \underline{v}_M)$), the quality of approximation is maximized; more specifically, the expected distortion $\mathbb{E} \left\| \hat{\underline{X}}_0 - \underline{X}_0 \right\|^2$ is minimized. To minimize the latter quantity we will exploit the Lloyd-Max algorithm [12] in the numerical analysis; it will be applied to the vector $\underline{a} = (a_1, \dots, a_M)$.

To find a 'good' value for M , the average normalised distortion $\frac{1}{L} \sum_{t \in \mathcal{L}} \|\underline{x}_0(t) - \hat{\underline{x}}_0(t)\|^2 / \|\underline{x}_0(t)\|^2$ against the

¹For example, $N = 24$ if a stage is a day and comprises 24 time-slots whose duration is one hour.

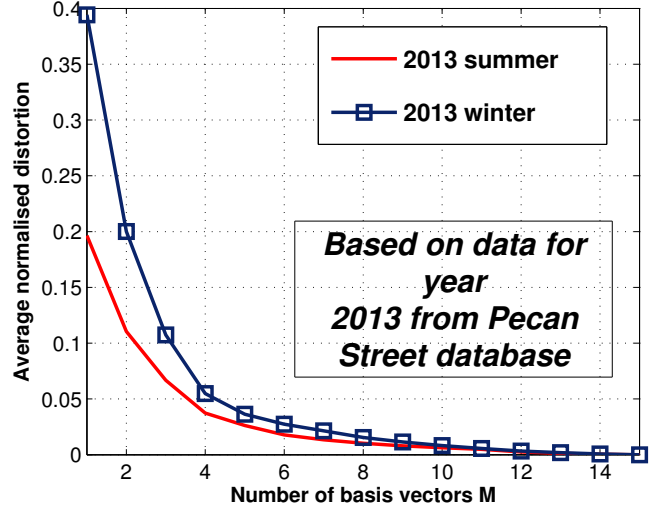


Fig. 1. Average normalised distortion against number of basis vectors chosen. We see that 4 vectors are enough to approximate \underline{X}_0 for an error tolerance level of 5%

number of basis vectors M that we use in our modeling of X_0 is traced. $\hat{\underline{x}}_0(t) = \hat{\underline{\mu}}_L + \sum_{i=1}^M a_i \underline{v}_i$ where $\underline{a} = \arg \min_{\underline{a}} \|\underline{x}_0(t) - \hat{\underline{\mu}}_L - \sum_{i=1}^M a_i \underline{v}_i\|^2$ using convex optimisation algorithms. As we see from Fig 1, the average distortion is more sensible to a few basis vectors \underline{v}_i . Thus, by accepting a distortion of 5%, we can reduce the basis space from 15 to 4 which enables us to model X_0 with fewer coefficients \underline{a} and reasonable accuracy.

The *stage or instantaneous payoff* function of appliance k is denoted by $u_k(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K)$. In the simulations we will assume that u_k can be written as

$$u_k(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K) = \sum_{n=1}^N u_{k,n} \left(x_{0,1} + \sum_{k=1}^K x_{k,1}, \dots, x_{0,n} + \sum_{k=1}^K x_{k,n} \right) \quad (4)$$

where n is the index for the hours of the day and $u_{k,n}$ is the consumption. Note that the cost function is not instantaneous inside a stage, but for a given day as seen from the equation 4. The function $u_{k,n}$ can e.g., represent the price charged to the consumer, Joule losses, battery aging, or distribution transformer aging at time-slot n . To define the average payoff of appliance k which is the function to be maximized by appliance k , the key notion of power consumption scheduling strategies needs to be defined. A *strategy* for appliance k is a sequence of mappings which is defined by:

$$\sigma_{k,t} : \begin{array}{l} \mathcal{X}_0^{t-1} \times \mathcal{S}_k \rightarrow \mathcal{X}_k \\ (\underline{x}_0(1), \dots, \underline{x}_0(t-1), \underline{s}_k(t)) \mapsto x_k(t) \end{array} \quad (5)$$

The *average payoff* of appliance k is then defined by

$$U_k(\sigma_1, \dots, \sigma_K) = \mathbb{E} \left[\frac{1}{T} \sum_{t=1}^T u_k(X_0(t), X_1(t), \dots, X_K(t)) \right]. \quad (6)$$

The expectation operator is used since the load is typically considered as a random process, which implies that in full

mathematical generality, the appliances decisions have also to be considered as random processes. The two main technical problems we address in this paper are the characterization of achievable average payoffs when T grows large and the determination of a practical scheme which provides good performance. A key technical point here is that we will assume a general structure for the knowledge \underline{s}_k . Indeed, we assume that $(\underline{s}_1, \dots, \underline{s}_K)$ are the outputs of a general discrete memoryless channel (see e.g., [13] for more details) whose conditional probability is $\Gamma(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0, \underline{x}_1, \dots, \underline{x}_K)$.

III. LIMITING PERFORMANCE CHARACTERIZATION

Theorem 1. *Assume the random process $\underline{X}_0(t)$ to be i.i.d. with a probability distribution ρ and the available load forecast \underline{S}_k to be the output of a discrete memoryless channel whose conditional probability is $\Gamma(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0, \underline{x}_1, \dots, \underline{x}_K)$. An expected payoff vector $(\bar{U}_1, \dots, \bar{U}_K)$ is achievable in the limit $T \rightarrow \infty$ if and only if it can be written as:*

$$\bar{U}_k = \sum_{\substack{\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K, \underline{w}, \underline{s}_1, \dots, \underline{s}_K \\ \Gamma(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0, \underline{x}_1, \dots, \underline{x}_K)}} \rho(\underline{x}_0) P_{\underline{W}}(\underline{w}) \times \left(\prod_{k=1}^K P_{\underline{X}_k | \underline{S}_k, \underline{W}}(\underline{x}_k | \underline{s}_k, \underline{w}) \right) u_k(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K) \quad (7)$$

where \underline{W} is an auxiliary variable which can be optimized.

The proof of this theorem is omitted here. It consists of a generalization of a result given in [14]. Therein, the authors characterize implementable distributions for the case $\sigma_{k,t}(S_k)$, S_k being the output of the simplified channel $\Upsilon(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0)$ namely, the output signal only depends on the state, whereas here it could also depend on the decisions chosen by others. The auxiliary variable \underline{W} is an external lottery known to players, which in general can help the appliances coordinate better.

One of the merits of Theorem 1 is to provide the best performance achievable in terms of average payoffs when the appliances have an arbitrary information structure (as long as it is of the form given by (5)). This requires the knowledge of the distribution of the exogenous load i.e., ρ and the conditional distribution Γ . However, Theorem 1 does not provide practical strategies which would allow a given payoff vector to be reached. Finding "optimal" scheduling strategies consists of finding good sequences of functions structured according to (5), which is an open and promising direction to be explored. More pragmatically, we restrict our attention to finding stationary strategies which are merely scheduling functions of the form $f_k : \mathcal{S}_k \rightarrow \mathcal{X}_k$. This choice is motivated by practical considerations such as computational complexity and it is also coherent with the current state of the literature. The water-filling solution is a special instance of this class of strategies. To find good scheduling functions, the idea we propose is to exploit the expected payoff given by Theorem 1. This is the purpose of the next section.

IV. PROCEDURE FOR DETERMINING POWER CONSUMPTION SCHEDULING FUNCTIONS

The first observation we make is that the best performance only depends on the vector of conditional probabilities

$(P_{\underline{X}_1 | \underline{S}_1, \underline{W}}, \dots, P_{\underline{X}_K | \underline{S}_K, \underline{W}})$. It is therefore relevant to try to find an optimum vector of lotteries for every action possible and use it to take decisions. Since this task is typically computationally demanding, a suboptimal approach consists in applying a distributed algorithm to maximize the average payoff. The procedure we propose here is to use the sequential best response dynamics (see e.g., [15]). This consists of each player choosing his best action while the others' strategies remain the same and all the players update their strategies sequentially. The key observation we make is then to see that when the lotteries of the other appliances are fixed, the best lottery for appliance k boils down to a function of \underline{s}_k and \underline{w} . The choice of power scheduling functions we propose precisely corresponds to running the algorithm provided below several times, convergence being obtained after a few iterations typically, with the iterations being linearly proportional to the number of players.

A sufficient condition for convergence of the algorithm is that the stage payoff function writes as $u_k(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K) = \alpha_k u(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K)$. This has been shown to be very relevant in the smart grid literature. For instance, in [1] u is a function which represents the total cost associated with operations such as charging operations and $\alpha_k = \frac{C_k}{\sum_{j=1}^K C_j}$ is a constant which represents the way this cost is shared among the consumers, C_j being the energy need of user j . However, for our algorithm, we minimise u , the total cost. Clearly minimising the total cost also minimises the individual costs as they are shared.

From Theorem 1, we can rewrite the expected payoff in the following manner (steps have been omitted for space constraints):

$$\bar{U}_k = \sum_{\substack{\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K, \underline{w}, \underline{s}_1, \dots, \underline{s}_K \\ \Gamma(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0, \underline{x}_1, \dots, \underline{x}_K)}} \rho(\underline{x}_0) P_{\underline{W}}(\underline{w}) \times \quad (8)$$

$$\Gamma(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0, \underline{x}_1, \dots, \underline{x}_K) \times \quad (9)$$

$$\left(\prod_{k=1}^K P_{\underline{X}_k | \underline{S}_k, \underline{W}}(\underline{x}_k | \underline{s}_k, \underline{w}) \right) u_k(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K) \\ = \sum_{i_k, j_k, w} \delta_{i_k, j_k, w} P_{\underline{X}_k | \underline{S}_k, \underline{W}}(\underline{x}_k | \underline{s}_k, \underline{w}) \quad (10)$$

where i_k, j_k, w are the respective indices of $\underline{x}_k, \underline{s}_k, \underline{w}$ and

$$\delta_{i_k, j_k, w} \triangleq \left[\sum_{i_0} \rho(x_{i_0}) \Gamma_k(s_k | x_{i_0}) \sum_{i_{-k}} u_k(x_{i_0}, x_{i_1}, \dots, x_{i_K}) \times \right. \\ \left. \sum_{j_{-k}} \prod_{k' \neq k} \Gamma_{k'}(s_{j_{k'}} | x_0) \prod_{k' \neq k} P_{\underline{X}_{k'} | \underline{S}_{k'}, \underline{W}}(x_{k'} | \underline{s}_{k'}, \underline{w}) \right] P_{\underline{W}}(\underline{w}) \quad (11)$$

where i_{-k}, j_{-k} are the indices which represent i_k, j_k being constant, while all the other indices are summed over. We have also assumed the independence of the observation channels as well as independence of the signal with the strategies chosen by the appliances, i.e.

$\Gamma(\underline{s}_1, \dots, \underline{s}_K | \underline{x}_0, \underline{x}_1, \dots, \underline{x}_K) = \Gamma_1(\underline{s}_1 | \underline{x}_0) \times \dots \times \Gamma_K(\underline{s}_K | \underline{x}_0)$. Note that we do this to correspond to our system model, but one can solve the general problem too.

Written in this form, for every player, optimising the cost in a distributed manner implies giving a probability 1 for the optimal coefficient $\delta_{i_k, j_k, w}$, and every player does that turn by turn. This is another way of seeing why $P_{\underline{X}_k | \underline{S}_k, W}(\underline{x}_k | \underline{s}_k, w)$ becomes just a function for the distributed optimisation algorithm. $f_k : \mathcal{S}_k \times \mathcal{W} \rightarrow \mathcal{X}_k$.

Algorithm 1: Proposed decentralized Algorithm for finding an optimal point

inputs: $\mathcal{X}_k \quad \forall k \in \{0, \dots, K\}, w(x_0, x_1, \dots, x_K) \quad \forall \underline{x}$,
 $\rho(x_0), \Gamma_{\underline{S} | \underline{X}_0}(\underline{s} | \underline{x}_0) \quad \forall x_0, f_k^{init} \quad \forall k \in \{1 \dots K\}, \epsilon$
output: $f_k^*(s_k) \quad \forall k \in \{1 \dots K\}$

Initialization - $f_k^0 = f_k^{init}, \quad iter = 0$

```

while  $\forall k \|\hat{f}_k^{(iter-1)} - f_k^{iter}\|^2 \leq \epsilon$  OR  $iter=0$  do
    iter = iter+1;
    forall the  $k \in \{1, \dots, K\}$  do
        forall the  $s_k \in \mathcal{S}_k$  do
            forall the  $w \in \mathcal{W}$  do
                forall the  $x_k \in \mathcal{X}_k$  do
                    Find the optimal coefficient  $\delta_{i_k, j_k, w}$ ;
                    Update the function
                     $f_k^{iter}(s_k) \in \arg \min_{i_k} \delta_{i_k, j_k, w}$ ;
                end
            end
        end
    end
end
end
end
end

```

Create \hat{X}_0 - For good modeling of the given data to create accurate \hat{X}_0 , we do PCA analysis of $\hat{\mathbf{R}}_L$ as explained in section II. We thus obtain M 'significant' vectors. We then find the continuous vectors $\underline{a}^C(t)$ which lies in the set \mathbb{R}^M and minimises the distortion $\|\hat{X}_0(t) - \underline{X}_0(t)\|^2$, for every day t . We discretize the continuous alphabet using vectorial lloyd max algorithm, with the $\underline{a}^C(1), \dots, \underline{a}^C(t)$ and specifying the number of points for quantization. The representatives thus generated is our alphabet \mathcal{X}_0 .

Create noise model - Since we evaluate the average payoffs for white Gaussian noise $\hat{\underline{x}}_0 + z$, where $Z \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$, whereas our algorithm uses a DMC channel Γ , the probability of error due to the noise should be taken into account in the algorithm. To reduce the complexity, it is assumed as a first approximation that *only* the closest neighbour of \hat{X}_0 in terms of the Euclidean norm $\|\cdot\|_2$ can be gotten by error as the approximation of exogenous profiles which should have been approximated by \hat{X}_0 . To estimate the probability of error, 100 draws of a white gaussian noise are added to the realizations of a training set of the exogenous load over $\{1, \dots, T\}$ and the number of times when a realization leads

to the closest neighbour instead of the right approximation are counted. Then, the probability of error is estimated as the ratio of this number of errors over the number of times when a given element of \mathcal{X}_0 should have been chosen.

V. NUMERICAL ANALYSIS

A. Simulation setup

The simulation setup assumed here by default corresponds to a Texan district in 2013 in which the smart electrical appliances are electric vehicles. A stage is a period from 8 am a given day to 8 am the day after: there are $N = 24$ time-slots of 1 hour each. The power consumption of the smart electrical devices will be scheduled only during the "night-time" corresponding here to the period 5 pm - 8 am the next day, i.e. $n \in \{10, \dots, 24\}$. The whole time period is then $\{1, \dots, T = 365\}$. Data corresponding to the exogenous profiles $(\underline{x}_0(1), \dots, \underline{x}_0(365))$ are obtained from the Pecan Street database [16]. To get a realistic profile at the scale of a district, we sum the hourly consumption data available for Texan households and normalize the aggregated profile as explained a little further. We verified using clustering algorithms of Matlab (function kmeans) that on an average over 3 years, the real consumption of the aggregated profile in summer and winter were considerably different. Winter corresponds to $1, 120] \cup [301, 365]$ and summer to $[121, 300]$. Thus we chose to only run the simulations for summer, with some tests for winter to see if the results of summer conform with those of winter.

In the case of EV charging, the strategy sets \mathcal{X}_k are constrained by the mobility parameters and the charging need. Based on a recent French² survey [17], the arrival μ_k^a , departure time μ_k^d and number of time-slots needed to charge C_k are taken to be the closest integers of realizations of Gaussian random variables $\tilde{\mu}_k^a \sim \mathcal{N}(2, 0.75)$, $\tilde{\mu}_k^d \sim \mathcal{N}(14.5, 0.375)$ and $\tilde{C}_k \sim \mathcal{N}(2.99, 0.57)$. Motivated by battery aging consideration [18], it is furthermore assumed that charging profiles are rectangular: charging is done at a constant rate, here 3kW, and cannot be stopped. Note also that profiles without interruption are even required in some important scenarios encountered with home energy management [19][20]. A strategy is then define only by the time to start charging and the strategy set is reduced to $\mathcal{X}_k = \{\mu_k^a, \dots, \mu_k^d - C_k + 1\}$, the potential time to start charging.

One important assumption in our analysis is the memorylessness of the payoff function. Regarding the instantaneous payoff function, two cases will be distinguished. The first considers a "memoryless" payoff with $u_k(\underline{x}_0, \underline{x}_1, \dots, \underline{x}_K) = -\sum_{n=1}^N (x_{0,n} + \sum_{k=1}^K x_{k,n})^2$. This is convenient to express Joule losses or a price charged to the consumer depending on the total load at the scale of the district as in [1]. The second corresponds to payoffs "with memory", which means that $u_{k,n}$ depends on the whole past of the total

²A more accurate study with mobility data from Texas could constitute an interesting extension of this work but as a first step, we assumed that the French ones could be applied also in the context described here.

load($x_{0,1} + \sum_{k=1}^K x_{k,1}, \dots, x_{0,n} + \sum_{k=1}^K x_{k,n}$) and not only on the current load $x_{0,n} + \sum_{k=1}^K x_{k,n}$. In the context of a distribution network, such a cost can be transformer aging. To be very brief, the most influential parameter for the transformer aging is known to be the hot-spot (HS) temperature [21]. Indeed, the transformer isolation damage is directly related to the HS temperature: aging is exponentially accelerated (decelerated) when the HS temperature is above (resp. below) its nominal value [21]. This is mathematically translated as

$$u_{k,n}(x_{0,1} + \sum_{k=1}^K x_{k,1}, \dots, x_{0,n} + \sum_{k=1}^K x_{k,n}) = e^{0.12 \times F_t^{\text{HS}}(x_{0,1} + \sum_{k=1}^K x_{k,1}, \dots, x_{0,n} + \sum_{k=1}^K x_{k,n}) - 11}, \quad (12)$$

which provides the instantaneous aging relatively to the nominal case. The transformer HS temperature evolution law F_t^{HS} , which has a memory, is assumed to follow the ANSI/IEEE linearized Clause 7 top-oil-rise model, which is described in [22]. To make the simulations reproducible we provide the values of the different parameters of F_t^{HS} : $\Delta_t = 0.5$ h; $T^0 = 2.5$ h (thermal inertia) for all simulations concerning the transformer, $\gamma = 0.83$; $R = 5.5$; $\Delta\theta_{\text{PL}}^0 = 55$ °C; $\Delta\theta_{\text{PL}}^{\text{HS}} = 23$ °C. The initial parameters for this dynamics with memory are assumed to be the same from one stage to another³ in a given season. These are set to $\theta_0^{\text{HS}} = 37$ °C and $x_{0,9} = 30$ W (resp. $\theta_0^{\text{HS}} = 75$ °C and $x_{0,9} = 87$ W) in winter (resp. summer). Defining a larger number of periods than the two (winter and summer) seasons with initial parameters for each period could improve the accuracy of the estimation of the transformer lifetime but would need more data to approximate the exogenous load. With these parameters, the exogenous load is normalized such that without EV the transformer lifetime (inversely proportional to the average aging) is 40 years (standard value for nominal conditions).

The functions found using our algorithm and iterative Water Filling algorithm are applied to a noisy version of data for exogenous load (Summer 2013) and real costs are calculated assuming that the vehicles use the decision functions given by the respective algorithms. Note however, that we suppose that the vehicles already know the statistics for the future. The knowledge of future statistics was also assumed in [5]. We use some typical values for Forecasting Signal to Noise Ratio FSNR, namely FSNR = 7dB as used in [5] to be coherent with the literature. FSNR is defined as

$$\text{FSNR} = 10 \log_{10} \left(\frac{1}{\sigma_{\text{day}}^2} \frac{1}{NT} \sum_{t=1}^T \sum_{n=1}^N X_{0,n}(t)^2 \right)$$

B. Simulation Results

In our results, we distinguish 2 kinds of vehicles, *informed vehicles* and *uninformed vehicles*. By informed vehicles, we mean vehicles who receive a signal corresponding to the exogenous demand $X_0(t)$ and take their decisions accordingly, whereas the uninformed vehicles receive no signal, and

³Given that the transformer temperature has enough time to converge to the same value at 5 pm for different values of this temperature at the beginning of the day at 8 am.

thus follow a 'plug and charge' policy, since they have no information. The total number of vehicles for the simulations were kept constant at 10.

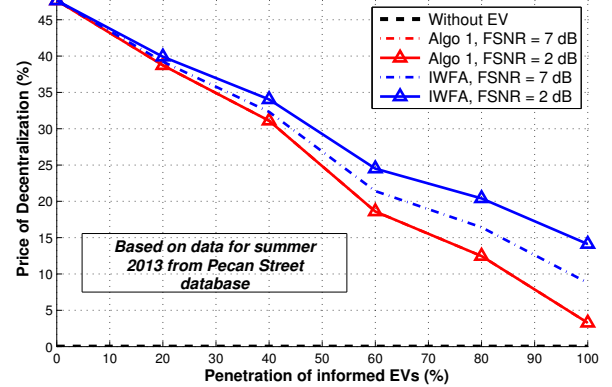


Fig. 2. Price of Decentralization ($(u_{\text{total}}/u_{\text{noEV}}) - 1$ in percent) against penetration percentage. For more discussion on the choice of this metric refer to [10]. This figure illustrates the robustness of our approach as well as increased importance at higher penetration rates.

In Fig. 2, there are 2 key messages: 1) Algo 1 is much more robust to noise, in fact in Fig. 2, the curves for 2 different noise level merge. IWFA however is sensible to noise, which is not surprising. Since we impose rectangular charging profiles on the vehicles, they will not react to noise since it might just be a transient optimal period to charge. 2) We see that as the number of informed players increase the difference between our algorithm and iterative Water filling algorithm (IWFA) increases.

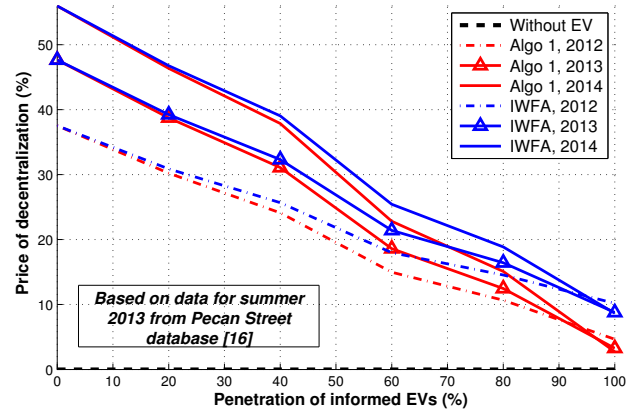


Fig. 3. Price of decentralization ($(u_{\text{total}}/u_{\text{noEV}}) - 1$ for different years for both Algos in comparison. We see that Algo 1 does considerably better every year.

In Fig 3 we see that the general trend discussed for 2 is true irrespective of the year. Moreover all the 3 years show similar patterns, thus showing that the good performance at higher penetration is not by chance. We could not test for more years as that information is not available in a similar format.

Number of Vehicles	1	2	3	4	5	6	7	8
Transformer Aging	0.99	0.98	0.96	0.96	0.94	0.93	0.91	0.89
Joules Loss	0.99	0.98	0.96	0.96	0.94	0.94	0.92	

TABLE I

RATIO OF u_{Algo1}/u_{IWFA} FOR DIFFERENT NUMBER OF VEHICLES, SUPPOSING THAT ALL THE VEHICLES ARE INFORMED (IN POSSESSION OF A SIGNAL)

We see in Table I that the ratio between the average costs of Algo 1 and IWFA decreases as number of vehicles increases. This implies that as number of vehicles increase, Algo 1 does increasingly better than IWFA. Moreover, the same pattern is observed for the cost function taken to be Joules losses. This vindicates our claim that the Algo proposed by us is generic and could be applied to any cost function to provide 'good' strategies.

Since the cost also depends on the need d which represents the needs of the electric vehicles, we generated d using the general statistics for demand observed. We generated 3 d vectors independently with the same distribution, and found that all the results discussed above hold. Same is true for the winter periods as defined in the simulation setup. All the following tests of robustness show the general nature of our approach.

VI. CONCLUSION

Numerical results show the full relevance of the proposed PCA-based model. Remarkably, an accurate approximation can be obtained by using only a few eigenvectors and applying the Lloyd-Max algorithm on the weights to be applied to these eigenvectors. The proposed framework to characterize the best performance of power consumption scheduling exploits a very recent result in information theory and also allows to derive robust scheduling functions. Simulations show that in the presence of uncertainty on the exogenous load forecast, the obtained functions outperform iterative Water-Filling based schemes. Most importantly, we provide a general framework which can be applied to various scenarios, and guarantees electric vehicle using functions achieving good performance for every scenario, thanks to the general information theoretical result derived.

The proposed framework can be extended. While the discrete alphabet assumption seems like a good assumption to obtain robust scheduling schemes, the i.i.d assumption on the exogenous load would need to be relaxed e.g., into a milder assumption such as a Markovian process, the i.i.d. assumption being made for the performance characterization theorem. Another direction to further improve performance is maximize the expected payoff jointly and not by using a distributed algorithm. The obtained schemes would still be distributed decision-wise.

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