FEEL++: A Versatile High Performance Finite Element Embedded Library into C++

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Some features

- A Domain Specific Language for PDEs embedded in C++ providing a syntax very close to the mathematical language in order to describe the variational formulation:
  - Supports generalized arbitrary order Galerkin methods (CG, DG) in 1D, 2D, 3D
  - Supports simplex, hypercube and high order meshes
  - Supports finite elements: Lagrange, Hermite, Nédélec, Raviart-Thomas
  - Supports seamless parallel computing
  - Supports seamless interpolation between grids/function spaces
  - Supports symbolic computation thanks to GiNaC
  - Supports large scale parallel linear and non-linear solvers (PETSc/SLEPC)
  - Supports hybrid computing: MPI, multi-thread, GPU (HAIPTS)
  - Supports in-situ visualization with ParaView

Some advanced methods

- Fluid-Structure Interaction
- Levelset method
- High order ALE
- Non-conforming methods
- h-p mortar methods
- Certified Reduced Basis methods
- Fast evaluation of functional outputs
- Certifiable accuracy of functional outputs
- Applicable to non-linear multi-physics problems

Preconditioners and HPC

- Stokes problem:
  \[
  \begin{align*}
  \text{Find } (u,p) \text{ such that} \\
  & -\text{div}(u) = 0 \quad \text{in } \Omega, \\
  & \text{div}(p) = 0 \quad \text{in } \Omega, \\
  & u = 0 \quad \text{on } \partial \Omega, \\
  \end{align*}
  \]

- Mortar: Laplacian
  \[
  \begin{align*}
  \text{Preconditioner} & = \begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix}^{-1} \\
  \end{align*}
  \]

- BDD-GenEO: Linear Elasticity

- PCG: 2D Backward-facing step flow

Question: What are the effects of different model and flow conditions on the quantification of hemodynamic variables?

- Influence of the inflow boundary condition (order of magnitude).
- Influence of the outflow boundary conditions:
  - Free outlet condition vs. mortar methods
  - Mixed and Seidel, Schur
- Influence of the non-Newtonian characteristics of blood flow.

Strategy: computational modeling + sensitivity analysis approach

Preconditioner used:

- \( A^{1/2}(\mu) \): finite element solution, \( A^{1/2} \) reduced basis solution

\[
W^{\mu} = \sum_{i=1}^{n} w^{\mu}(\mu_i) v^{\mu}(\mu_i) \quad \text{and} \quad u^{\mu} = \sum_{i=1}^{n} u^{\mu}(\mu_i) v^{\mu}(\mu_i)
\]

\[
A^{1/2}(\mu) v^{\mu}(\mu) = F^{1/2}(\mu) \quad \text{system } N \times N \text{ with } N \ll N
\]

Hemodynamic simulations in the cerebral venous network

- LNCMI: Laboratory of high magnetic fields
  - Application domains: chemistry, magneto-science, superconductors
  - Continuous magnetic field (36 Tesla)

Challenges:

- Modeling: multi-physics non-linear models, complex geometries, generality
- Account for uncertainties: uncertainty quantification, sensitivity analysis
- Optimization: shape of magnets, robustness of design

Strategy: Reduced basis method

Design of high field magnet

- Hydraulics - 0D
- Mechanics - 3D Linear
- Thermics - 3D Non Linear
- Joules losses
- Electromagnetism - 2D axi