Maximum likelihood frequency estimation in smart grid applications

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***Abstract*—This paper focuses on the estimation of the fundamental frequency in balanced three-phase power systems. Specifically, we propose a Maximum Likelihood Estimator (MLE) that exploits the multidimensional nature of electrical signals. For perfectly sinusoidal signals, we show that the MLE can be expressed according to the periodogram of the instantaneous positive component. For harmonic signals, we demonstrate that the MLE can be approximated by a cumulated periodogram of the zero, positive and negative sequence components. As compared to single-phase estimators, statistical analysis and simulation results prove that the proposed estimator decreases the Mean Square Error by a factor of three, whatever the Signal to Noise Ratio (SNR) or data length. Furthermore, simulations with experimental data show that the proposed technique outperforms classical spectral estimators such as MUSIC.**

***Index Terms*—Frequency Estimation, Power System, Smart Grid, Maximum Likelihood Estimator, Cramér-Rao Bounds**

I. Introduction

The electrical network has undergone profound changes during the last decades. These changes are mainly due to two factors: the integration of renewable energy sources and the emergence of new consumer usages. Despite its ecological benefit, this mutation greatly complicates the management of the electricity grid. On one hand, renewable energy sources are intermittent and diffuse that make the energy generation hardly controllable and predictable. On the other, new usages such as the charging of Electric Vehicles are large energy consumers and make difficult to maintain the balance between energy production and consumption. To solve this problem, a promising solution relies on the use of Information and Communications Technology. The future "Smart Grid" is expected to be able to monitor and optimize its structure to maintain the balance between energy production and consumption.

In Smart-Grid, the electrical signal is measured and analyzed at substations with Phasor Measurement Units (PMUs) to monitor the grid state. A PMU is a device that is able to estimate several signal parameters from the current or voltage signals, such as the phasor or fundamental frequency. Among these parameters, the fundamental frequency plays a key role. Indeed, deviation from the nominal value can indicate a mismatch between production and consumption [1]. Furthermore, a large deviation can cause damage to equipment and should be detected at an early stage. For these reasons, frequency estimation is the backbone of smart grid monitoring.

Many frequency estimation techniques have been proposed [2], [3] and applied to electrical signals. These include the Discrete Fourier Transform [4], PLL [5], high resolution approaches [6], [7], Least square [8], and adaptive filtering [9]. Nevertheless, these techniques are general and do not take into account the particularities of electrical signals. From a statistical point of view, these particularities offer opportunities to improve the frequency estimation. Following this idea, several authors have developed parametric estimators specifically designed for electrical signals. A Least square estimators exploiting the harmonic structure of electrical signal has been described in [10], [11]. Another interesting feature of electrical signals is their multidimensional nature. Indeed, electrical signals are composed of three "phases" that can be used jointly for frequency estimation. Several techniques that exploit this property are available in the literature [12]–[20]. Most of them are based on Power electronic analysis tools such as the Clarke or Fortescue Transform. Despite their simplicity, these techniques have several limitations. First, these techniques assume perfect sinusoidal signals and their performance degrade drastically in the presence of harmonic components. Then, since these techniques focus on instantaneous frequency estimation, they may produce suboptimal estimators if the fundamental frequency is near constant over a period of time.

Despite this rich literature, there is no technique that jointly exploits the multidimensional and harmonic structure for frequency estimation. Furthermore, it is not clear how these particularities improve the performance of the frequency estimator. In this study, we address the fundamental frequency estimation problem for three-phase systems. Specifically, under the assumption of a balanced system, we propose a parametric frequency estimator that jointly exploits the multidimensional and harmonic structure of electrical signals. The contribution of this study is twofold: first, we derive the Maximum Likelihood Estimator (MLE) of the fundamental frequency, then we provide a statistical analysis aiming at measuring the benefit of using the multidimensional and harmonic structure. The remainder of the paper is organized as follows. Section II presents the signal model and the assumptions. Section III describes the MLE frequency estimator and Section IV provides an analysis of the Cramér-Rao Bounds. Finally, Section V reports on the performance of the proposed estimator with synthetic and experimental signals.

II. SIGNAL MODEL

In a three-phase system, voltage or current signals are composed of three components. Under balanced conditions, these components are phase-rotated from each other by $\frac{2\pi}{3}$. In the ideal case, each component is a sinusoidal signal with (normalised) angular frequency $\omega_0 \in [0, \pi[$. In practice, electrical signals may be corrupted by harmonic content [21]. These frequency components are introduced by large non-linear loads [21], [22]. Using these properties, the n^{th} sample on the m^{th} phase ($m = 0, 1, 2$) can be expressed by [23]

$$x_m[n] = \sum_{l=1}^{L} a_l \cos\left(l\omega_0 n + \varphi_l - \frac{2ml\pi}{3}\right) + b_m[n]. \quad (1)$$

where a_l and φ_l correspond to the amplitude and initial phase of the l^{th} harmonic component ($l = 1, 2, \cdots L$) and $b_m[n]$ is the additive noise. Note that this signal model corresponds to the one described in the IEEE Standard 1459 for three-Phase nonsinusoidal balanced systems (see [23, Section 3.2.3]). In this paper, the problem of interest is to estimate ω_0 from N observations of the three-phase signal. Using matrix notation, the three-phase signal can be expressed as

$$\mathbf{x} = \mathbf{G}(\omega_0)\mathbf{s} + \mathbf{b} \quad (2)$$

where

- $\mathbf{x}$ and $\mathbf{b}$ are $3N \times 1$ column vector which are defined respectively as
$$\mathbf{x} \triangleq [x_0[0], \cdots, x_0[N-1], \cdots, x_2[0], \cdots, x_2[N-1]]^T$$
$$\mathbf{b} \triangleq [b_0[0], \cdots, b_0[N-1], \cdots, b_2[0], \cdots, b_2[N-1]]^T$$
where $(.)^T$ denotes the matrix transpose.
- $\mathbf{s}$ is a $2L \times 1$ vector containing the amplitude and initial phase. This vector is defined as $\mathbf{s} \triangleq [a_1\cos(\varphi_1), a_1\sin(\varphi_1), \cdots, a_L\cos(\varphi_L), a_L\sin(\varphi_L)]^T$.
- $\mathbf{G}(\omega_0)$ is a $3N \times 2L$ matrix that is defined as
$$\mathbf{G}(\omega_0) \triangleq \begin{bmatrix} \mathbf{G}_0(1,\omega_0) & \mathbf{G}_0(2,\omega_0) & \cdots & \mathbf{G}_0(L,\omega_0) \\ \mathbf{G}_1(1,\omega_0) & \mathbf{G}_1(2,\omega_0) & \cdots & \mathbf{G}_1(L,\omega_0) \\ \mathbf{G}_2(1,\omega_0) & \mathbf{G}_2(2,\omega_0) & \cdots & \mathbf{G}_2(L,\omega_0) \end{bmatrix}$$
where $\mathbf{G}_m(l,\omega_0) \triangleq [\Re e[\mathbf{a}_m^T(l,\omega_0)], -\Im m[\mathbf{a}_m^T(l,\omega_0)]]$ is a $N \times 2$ matrix and $\mathbf{a}_m(l,\omega_0)$ is defined as
$$\mathbf{a}_m(l,\omega_0) \triangleq e^{-j\frac{2\pi lm}{3}} \times \left[1, e^{2l\omega_0}, \cdots, e^{l\omega_0(N-1)}\right]$$

From a statistical point of view, the goal of this study is to estimate ω_0 from $\mathbf{x}$ when $\mathbf{s}$ and $\mathbf{b}$ are unknown. To achieve this goal, we make the following assumptions:

- AS1) The number of components L is known.
- AS2) The signal on each phase is corrupted by an additive white gaussian noise with zero mean and covariance σ^2, i.e. $\mathbf{b} = \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I}_{3N})$, where $\mathbf{I}_{3N}$ corresponds to the identity matrix of size $3N \times 3N$.
- AS3) The number of samples, N, is greater than $(2L + 1)/3$.

Note that if L is unknown, it can be estimated with information criteria techniques [24]. Concerning AS2), this assumption is motivated by the Central Limit Theorem.

III. MAXIMUM LIKELIHOOD ESTIMATION

In this section, we derive the Maximum Likelihood Estimator (MLE) of the fundamental frequency. Under assumption AS2), the MLE corresponds to the least squares estimator. The estimate of ω_0 and $\mathbf{s}$ are therefore obtained by minimizing

$$\|\mathbf{x} - \mathbf{G}(\omega)\mathbf{s}\|^2 \quad (3)$$

with respect to ω and $\mathbf{s}$, where $\|.\|^2$ corresponds to the vector norm. This nonlinear problem is a separable least squares problem. Consequently, it can be shown that the estimate of ω_0 is obtained by maximizing [25, Section 8.9]

$$\mathcal{C}(\omega) = \mathbf{x}^T\mathbf{G}(\omega)\left(\mathbf{G}^T(\omega)\mathbf{G}(\omega)\right)^{-1}\mathbf{G}^T(\omega)\mathbf{x} \quad (4)$$

In the general case, the maximization of (4) must be performed by a grid-search algorithm or iterative techniques. Another issue is the computation of the inverse of $\mathbf{G}^T(\omega)\mathbf{G}(\omega)$ in (4). Using the expression of $\mathbf{G}(\omega)$, this matrix can be decomposed as

$$\mathbf{G}^T(\omega)\mathbf{G}(\omega) = \begin{bmatrix} \mathbf{P}_{1,1}(\omega) & \mathbf{P}_{1,2}(\omega) & \cdots & \mathbf{P}_{1,L}(\omega) \\ \mathbf{P}_{2,1}(\omega) & \mathbf{P}_{2,2}(\omega) & \cdots & \mathbf{P}_{2,L}(\omega) \\ \vdots & & \ddots & \vdots \\ \mathbf{P}_{L,1}(\omega) & \mathbf{P}_{L,2}(\omega) & \cdots & \mathbf{P}_{L,L}(\omega) \end{bmatrix} \quad (5)$$

where

$$\mathbf{P}_{u,v}(\omega) \triangleq \sum_{m=0}^{2} \mathbf{G}_m^T(u,\omega)\mathbf{G}_m(v,\omega) \quad (6)$$

Although $\mathbf{P}_{u,v}(\omega)$ has a simple form, the inverse of $\mathbf{G}^T(\omega)\mathbf{G}(\omega)$ is difficult, if not impossible, to obtain analytically. However, in some particular cases, the structure of $\mathbf{P}_{u,v}(\omega)$ can be simplified and the inverse of $\mathbf{G}^T(\omega)\mathbf{G}(\omega)$ derived analytically.

A. Single sinusoidal component ($L = 1$)

In the ideal case, the electrical signal contains only a sinusoidal component. Setting $L = 1$ in (5), we obtain

$$\mathbf{G}^T(\omega)\mathbf{G}(\omega) = \mathbf{P}_{1,1}(\omega) = \frac{3N}{2}\mathbf{I}_2. \quad (7)$$

Using this result, the cost function in (4) can be expressed as

$$\mathcal{C}(\omega) = \frac{2}{3N}\left\|\mathbf{G}^T(\omega)\mathbf{x}\right\|^2 \quad (8)$$

This expression can be simplified using the definition of $\mathbf{G}(\omega)$ and $\mathbf{x}$. By introducing the complex number $j^2 = -1$, we obtain

$$\mathcal{C}(\omega) = \frac{2}{3N}\left|\sum_{n=0}^{N-1}\sum_{m=0}^{2} x_m[n]e^{-j(\omega n - \frac{2m\pi}{3})}\right|^2 \quad (9)$$

where $|.|$ corresponds to the complex modulus.

Theorem 1: When $L = 1$, the MLE of the fundamental frequency is obtained by maximizing

$$P_{y_1}(\omega) = \frac{1}{N}\left|\sum_{n=0}^{N-1} y_1[n]e^{-j\omega n}\right|^2 \quad (10)$$

where $y_1[n] \triangleq \sum_{m=0}^{2} x_m[n] e^{\frac{2jm\pi}{3}}$ is called the (instantaneous) positive sequence component.

Theorem 1 shows that the MLE of ω is given by the highest peak of the periodogram of the *positive sequence component*. We can note that the three-phase MLE is closely related to the MLE proposed by Rife [26]. Indeed, the main difference between these two estimators is the analysed signal. Specifically, the estimator proposed by Rife is based on the analytic signal whereas our estimator uses the positive sequence component.

B. Estimation for $N \gg 1$

For $N \gg 1$, it can be shown that

$$\mathbf{P}_{u,v}(\omega) \approx \frac{3N\delta(u-v)}{2}\mathbf{I}_2 \tag{11}$$

where $\delta(u)$ corresponds to the Kronecker delta. Using (11) in (5), we obtain

$$\mathbf{G}^T(\omega)\mathbf{G}(\omega) \approx \frac{3N}{2}\mathbf{I}_{2L} \tag{12}$$

Using this approximation in (4) and the definition of $\mathbf{G}(\omega)$, the cost function can be simplified as

$$\mathcal{C}(\omega) \approx \frac{2}{3N}\sum_{l=1}^{L}\left|\sum_{n=0}^{N-1} y_l[n]e^{-jl\omega n}\right|^2 \tag{13}$$

where $y_l[n] \triangleq \left[1 \ e^{2j\pi l/3} \ e^{4j\pi l/3}\right] x[n]$. The components $y_l[n]$ have a particular structure since $y_{l+3}[n] = y_l[n]$. To simplify the cost function, we make use of the Fortescue transform, which is defined as

$$\begin{bmatrix} y_0[n] \\ y_1[n] \\ y_2[n] \end{bmatrix} \triangleq \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{2j\pi/3} & e^{4j\pi/3} \\ 1 & e^{-2j\pi/3} & e^{-4j\pi/3} \end{bmatrix} \begin{bmatrix} x_0[n] \\ x_1[n] \\ x_2[n] \end{bmatrix} \tag{14}$$

where $y_0[n]$, $y_1[n]$ and $y_2[n]$ are called the zero, positive and negative sequence components, respectively. This transform has been introduced by Fortescue in 1918 to simplify the analysis of unbalanced polyphase systems [27], [28]. Using (14) in (13), we obtain the following theorem.

Theorem 2: For $N \gg 1$, the MLE of the fundamental frequency is obtained by maximizing

$$S(\omega) = \sum_{l=1}^{L}\left(\frac{1}{N}\left|\sum_{n=0}^{N-1} y_{l\mathrm{mod}(3)}[n]e^{-jl\omega n}\right|^2\right) \tag{15}$$

where mod(.) is the modulo operator.

Theorem 2 shows that the cost function corresponds to a *cumulated* periodogram involving the periodogram of $y_0[n]$, $y_1[n]$ and $y_2[n]$. It is interesting to note that the Fortescue transform separates the harmonic content of the three-phase signals. Indeed, for a particular harmonic order, only one sequence component contributes to the cost function. Specifically, information about the harmonics of order $l = 3q$, $l = 3q+1$ and $l = 3q+2$ ($q \in \mathbb{N}$) are respectively carried by the zero, positive and negative sequence components.

IV. Cramer Rao Bound

A natural criterion to assess the performance of an estimator is the mean square error (MSE). The MSE of $\widehat{\omega}$ is defined as $MSE[\widehat{\omega}] = \text{bias}^2(\widehat{\omega}) + \text{var}(\widehat{\omega})$, where $\text{bias}(\widehat{\omega})$ and $\text{var}(\widehat{\omega})$ correspond to the bias and variance of the estimator. The variance of any unbiased estimator of the fundamental frequency, $\widehat{\omega}$, is bounded by [25]

$$\text{var}(\widehat{\omega}) \geq CRB(\widehat{\omega}) \tag{16}$$

where $CRB(\widehat{\omega})$ corresponds to the Cramer Rao Bound of $\widehat{\omega}$. Let us define $\theta \triangleq [a_1, \varphi_1, \cdots, a_L, \varphi_L, \omega]^T$ the $2L+1$ vector containing the unknown parameters. The CRB of ω is given by

$$CRB(\widehat{\omega}) = \left[\mathbf{F}(\theta)^{-1}\right]_{2L+1,2L+1} \tag{17}$$

where $\mathbf{F}(\theta)^{-1}$ is the inverse of the Fisher information matrix and $[.]_{ij}$ corresponds to the element located at the i^{th} row and the j^{th} column. As $\mathbf{b}[n] \sim \mathcal{N}(0, \sigma^2\mathbf{I}_{3N})$, the ij^{th} element of the Fisher Information Matrix is given by [25]

$$[\mathbf{F}(\theta)]_{ij} \triangleq \frac{1}{\sigma^2}\left(\frac{\partial \mathbf{G}(\omega)\mathbf{s}}{\partial \theta_i}\right)^T\left(\frac{\partial \mathbf{G}(\omega)\mathbf{s}}{\partial \theta_i}\right) \tag{18}$$

Using the definitions of $\mathbf{G}(\omega)$ and $\mathbf{s}$, we obtain

$$\frac{\partial \mathbf{G}(\omega)\mathbf{s}}{\partial a_l} = \begin{bmatrix} \mathbf{G}_0(l,\omega) \\ \mathbf{G}_1(l,\omega) \\ \mathbf{G}_2(l,\omega) \end{bmatrix}\begin{bmatrix} \cos(\varphi_l) \\ \sin(\varphi_l) \end{bmatrix} \tag{19}$$

$$\frac{\partial \mathbf{G}(\omega)\mathbf{s}}{\partial \varphi_l} = \begin{bmatrix} \mathbf{G}_0(l,\omega) \\ \mathbf{G}_1(l,\omega) \\ \mathbf{G}_2(l,\omega) \end{bmatrix}\begin{bmatrix} -a_l\sin(\varphi_l) \\ a_l\cos(\varphi_l) \end{bmatrix} \tag{20}$$

$$\frac{\partial \mathbf{G}(\omega)\mathbf{s}}{\partial \omega} = \begin{bmatrix} \mathbf{H}_0(1,\omega) & \cdots & \mathbf{H}_0(L,\omega) \\ \mathbf{H}_1(1,\omega) & \cdots & \mathbf{H}_1(L,\omega) \\ \mathbf{H}_2(1,\omega) & \cdots & \mathbf{H}_2(L,\omega) \end{bmatrix}\mathbf{s} \tag{21}$$

where $\mathbf{H}_m(u,\omega) \triangleq -u\left[\ \Im m[\mathbf{D}\mathbf{a}_m^T(u,\omega)]\ \Re e[\mathbf{D}\mathbf{a}_m^T(u,\omega)]\ \right]$ and $\mathbf{D} = \text{diag}\left([0, 1, 2, \cdots, (N-1)]\right)$. In the general case, the inverse of the Fisher Information Matrix is difficult to obtain analytically. However, we show in the next subsections that analytical forms can be derived when $L = 1$ and $N \to \infty$. For the sake of comparison, the Signal to Noise Ratio (SNR) is defined in the next subsections as

$$\rho \triangleq \frac{1}{2\sigma^2}\sum_{l=1}^{L} a_l^2. \tag{22}$$

A. Closed form expression for $L = 1$

For $L = 1$, the Fisher Information Matrix can be expressed as

$$\mathbf{F}(\theta) = \frac{3}{\sigma^2}\begin{bmatrix} \frac{N}{2} & 0 & 0 \\ 0 & \frac{a_1^2 N}{2} & \frac{a_1^2}{2}\sum_{n=0}^{N-1} n \\ 0 & \frac{a_1^2}{2}\sum_{n=0}^{N-1} n & \frac{a_1^2}{2}\sum_{n=0}^{N-1} n^2 \end{bmatrix} \tag{23}$$

The CRB of ω is derived from the inverse of $\mathbf{F}(\theta)$. After some manipulations, the CRB for sinusoidal signals, denoted $CRB_1(\widehat{\omega})$, reduces to

$$CRB_1(\widehat{\omega}) = \frac{4}{N(N^2-1)\rho}. \tag{24}$$

Equation (24) shows that the CRB depends on the sample length, N, and Signal to Noise Ratio, ρ. Specifically, the CRB decreases as ρ or N increases. Furthermore it shows that the CRB is highly sensitive to N since this bound decreases as $1/(N^3)$. It is interesting to compare this CRB with that obtained for single-phase systems [25, Example 3.14]. First, we note that the CRB expression in (24) is exact for all N while the expression provided in [25] holds asymptotically. Next, we observe that the CRB of ω for three-phase systems is three times smaller than that obtained for single-phase systems. This result can be generalized to M-phase system and demonstrates the benefit of using the multidimensional nature of electrical signals for frequency estimation.

B. Asymptotic expression for $N \gg 1$

For $N \gg 1$, the Fisher Information matrix can be decomposed as follows

$$\mathbf{F}(\theta) = \frac{1}{\sigma^2}\begin{bmatrix} \mathbf{K}_{11}(\omega) & 0 & \cdots & 0 & \mathbf{f}_1(\omega) \\ 0 & \mathbf{K}_{22}(\omega) & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \mathbf{K}_{LL}(\omega) & \mathbf{f}_L(\omega) \\ \mathbf{f}_1^T(\omega) & \mathbf{f}_2^T(\omega) & \cdots & \mathbf{f}_L^T(\omega) & t(\omega) \end{bmatrix} \tag{25}$$

where

$$\mathbf{K}_{uu}(\omega) = \frac{3N}{2}\begin{bmatrix} 1 & 0 \\ 0 & a_u^2 \end{bmatrix} \tag{26a}$$

$$\mathbf{f}_u(\omega) = \frac{3ua_u^2N^2}{4}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{26b}$$

$$t(\omega) = \frac{N^3}{2}\sum_{l=1}^{L} l^2 a_l^2 \tag{26c}$$

Using the result on the inverse of a partitioned matrix, the asymptotic CRB, denoted $CRB_\infty[\widehat{\omega}]$, can be expressed as

$$CRB_\infty(\widehat{\omega}) = \frac{4\beta}{N^3\rho} \tag{27}$$

where $\beta \leq 1$ is defined as

$$\beta \triangleq \frac{\sum_{l=1}^{L} a_l^2}{\sum_{l=1}^{L} l^2 a_l^2}. \tag{28}$$

From (27), we observe that the CRB depends on the signal length, SNR and parameter β. Comparing the asymptotic bound in (27) with that derived in [29], we observe that the CRB for three-phase systems is three times smaller than that obtained for single-phase systems. Comparing the asymptotic bound with that obtained for sinusoidal signals, we observe that $CRB_\infty(\widehat{\omega}) \approx \beta CRB_1(\widehat{\omega})$. As $\beta \leq 1$, it follows that the CRB for harmonic signals is $1/\beta$ times smaller than that obtained for sinusoidal signals. This statement demonstrates the utility of using the harmonic content for frequency estimation.

V. Simulation results

In this section, we evaluate the performance of the proposed frequency estimators. Specifically, we focus on three estimators, namely the exact MLE in (4), the sinusoidal MLE in (10), and the approximated MLE in (15). For sake of brevity, we refer to these technique as MLE, MLE_1, and MLE_∞, respectively. These estimators are compared with the single-phase MLE [10] and with the Cramer Rao Bounds reported in Section IV. For each estimator, the maximisation step is performed using the Nelder-Mead simplex algorithm [30]. This algorithm is initialized at the nominal fundamental frequency, i.e. $\widehat{w} = 2\pi \times 50/F_s$ rad/s, and the termination tolerance is set to 10^{-8}. The frequency estimators are tested using signals modeled by (1) under different signal length, N. The performance are evaluated in terms of Mean Square Error (MSE). The MSE is estimated using $K = 1000$ Monte Carlo Trials by

$$MSE \approx \frac{1}{K}\sum_{k=0}^{K-1}(\omega_0 - \widehat{\omega})^2 \tag{29}$$

where ω_0 is the (true) fundamental frequency and $\widehat{\omega}$ corresponds to the frequency estimate. In each simulation, the sampling frequency and the fundamental frequency are set to $F_s = 1$kHz and $\omega_0 = 2\pi \times (51.5)/F_s$ rad/s, respectively.

A. Estimation for $L = 1$

In this subsection, we evaluate the performance for sinusoidal signals. In each simulation, parameters are fixed to $L = 1$, $a_1 = 1$, $\varphi = 0.3$ and $SNR = 10$dB. Figure 1 presents the performance of the three-phase and single-phase MLE. We note that the three-phase MLE achieves the CRB. Furthermore, we observe that the three-phase estimator outperforms the single-phase estimator, whatever the number of samples. Specifically, the former decreases the MSE by a factor of three, which is consistent with the conclusions of subsection IV-A.

B. Estimation for $L \geq 1$

In this subsection, we present the performance of MLE, MLE_1 and MLE_∞ for harmonic signals with $L = 4$. Simulation parameters are given in Table I. Figure 2 displays the MSE versus the data-length at $SNR = 10$ dB. We observe that the exact and asymptotic CRBs have similar values whatever the signal length. Furthermore, among the considered estimators, we note that the exact MLE is the only one that achieves the CRB at high SNRs. As compared to single-phase and MLE_1 estimators, the exact MLE significantly decreases the MSE. Concerning the approximated MLE, MLE_∞, we see that the performances highly depend on the signal length N. Indeed, the performances tend to those of the exact MLE for $N \rightarrow \infty$. In addition, we observe that the MSEs are broadly similar for $N \approx 2k\pi/\omega$ ($k \in \mathbb{N}$). This behaviour comes from the fact that the equality in (11) is attained for these particular values of N.

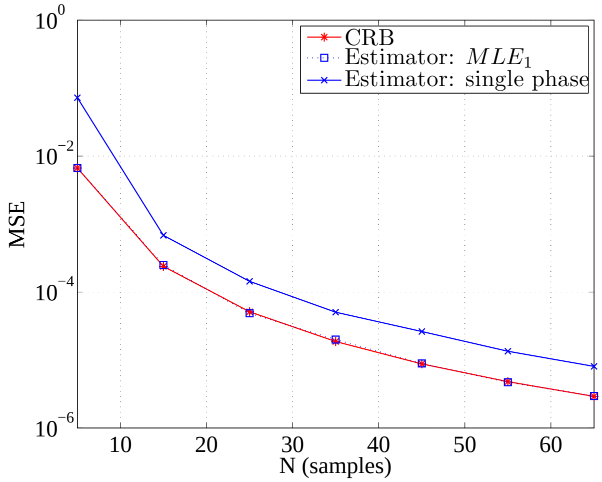


Fig. 1: MSE versus signal length for sinusoidal signals ($L = 1$, $SNR = 10$dB).

TABLE I: Simulation parameters ($L = 4$).

Parameter	a_1	a_2	a_3	a_4	φ_1	φ_2	φ_3	φ_4
Value	1	0.1	0.105	0.366	0.052	0.1	0.4	0.5

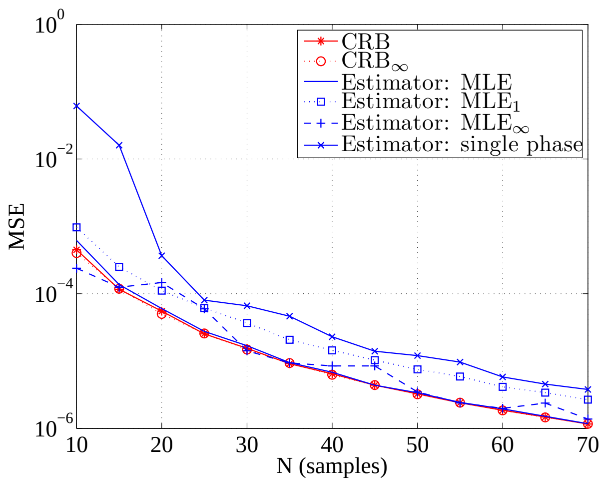
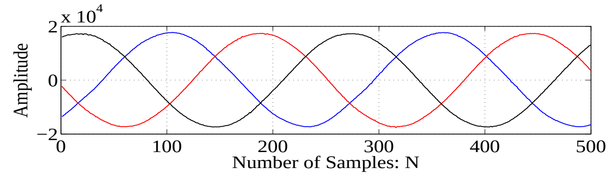


Fig. 2: MSE versus signal length, N, for harmonic signals ($L = 4$, $\beta = 0.5$, $SNR = 10$dB).

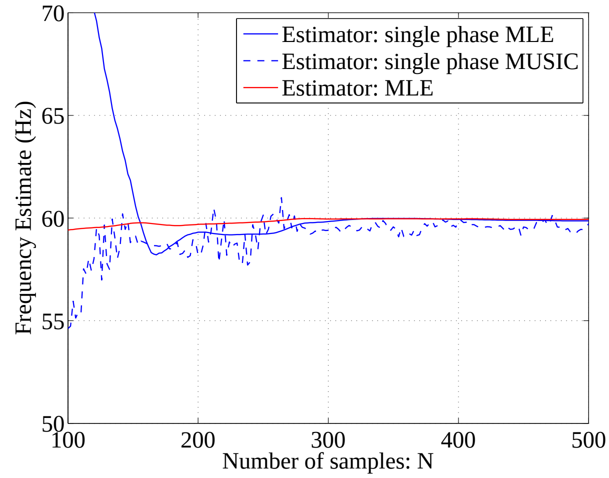
C. Experimental signals

In this subsection, the frequency estimator MLE is applied to experimental data. Experimental signals come from the DOE/EPRI National Database of Power System Events. The sampling frequency is equal to $F_e = 15360$ Hz. The analysed signals are presented in Figures 3 and 4. These signals correspond to the events 3127 and 3163. The fundamental frequency is estimated using the exact three-phase MLE estimator with $L = 3$. Frequency estimates obtained with the single phase MLE ($L = 3$) and root MUSIC are also shown for comparison.

Figure 3 displays the estimated fundamental frequencies versus N for the event 3127. We observe that the three estimators converge to the same value, $\widehat{f_0} = 59.9$Hz, when $N > 500$. However, we see that the exact MLE converges more rapidly than the single-phase MLE and root MUSIC. In particular, the exact MLE is the only estimator that provides accurate results even for a small number of samples. Specifically, this estimator converges roughly after 275 samples ($\approx$ 1 cycle).



(a) Experimental Signal.



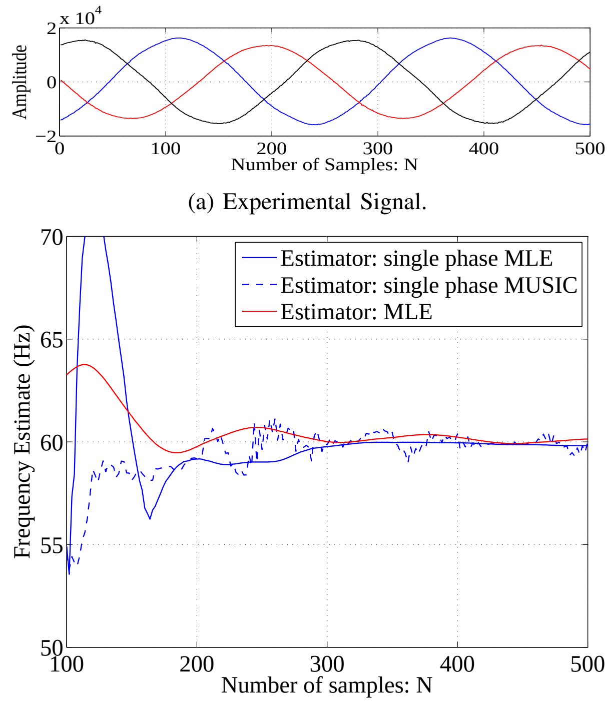
(b) Frequency estimation versus the number of samples N.

Fig. 3: Event 3127: Estimation of the fundamental frequency.

As shown in Figure 4, the event 3163 introduces a moderate amount of three-phase unbalance in the voltage signal. Although the proposed estimator is designed for balanced systems, we note that the single-phase and proposed estimator lead to the same frequency estimate for $N > 500$ samples ($\approx$ 2 cycles). Nevertheless, the three-phase MLE exhibits extra oscillations due to the unbalanced conditions. We can note that these oscillations significantly increase the estimator's settling time. Under unbalanced conditions, it is clear that better estimates could be obtained by treating the unbalance parameters as nuisance parameters.

VI. Conclusions

In this paper, we have focused on the Maximum Likelihood estimation of the fundamental frequency in three-phase balanced systems. A new estimator that jointly uses the multidimensional and harmonic structure of electrical signals has been proposed. For sinusoidal signals, this estimator is obtained by maximizing the periodogram of the positive sequence component. For harmonic signals, the estimator is obtained by maximizing a cumulated periodogram of the zero, positive and negative components. The analysis of the Cramer Rao Bound has clearly demonstrated the benefit of using



(a) Experimental Signal.

(b) Frequency estimation versus the number of samples N.

Fig. 4: Event 3163: Estimation of the fundamental frequency.

the multidimensional and harmonic structure for frequency estimation. Indeed, the use of the multidimensional nature of electrical signals allows to decreases the CRB by of a factor of 3, and the use of the harmonic structure by a factor of $1/\beta$.

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