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Abstract. This paper presents an obstruction-free solution to the (n, k)-set agreement problem in an asynchronous anonymous read/write system using solely $(n - k + 1)$ registers. We then extend this algorithm into *(i)* a space-optimal solution for the repeated version of (n, k)-set agreement, and *(ii)* an x-obstruction-free solution using $(n - k + x)$ atomic registers (with $1 \leq x \leq k < n$).

1 Context & motivation

Due to failures, concurrent processes have to deal not only with finite asynchrony, i.e., finite but arbitrary process speed, but also with infinite asynchrony. In this context, mutex-based synchronization is useless, and pioneering works in *fault-tolerant* distributed computing, such as [7], have instead promoted the design of concurrent algorithms.

A first challenge: multi-writer registers. When processes communicate with *Single-Writer Multi-Reader* (SWMR) atomic registers, a concurrent algorithm usually associates each process with a register. In the case where processes communicate with *Multi-Writer Multi-Reader* (MWMR) atomic registers, as any process can write any register, the previous association is no longer granted for free. To still benefit from existing SWMR registers-based solutions, a classical reduction consists in emulating SWMR registers on top of MWMR registers. In a system of n processes, n MWMR atomic registers are needed when the simulation is non-blocking [4]. Hence, if the underlying system provides less than n MWMR registers, the simulation approach is irrelevant and novel techniques must be found.

A second challenge: anonymity. Some algorithms based on MWMR registers require processes to write control values that include their identities. On the contrary, in an *anonymous* system, processes have no identity, the same code, and the same initialization of their local variables. Hence, they are in a strong sense identical. In such a context, the core question that interests us is the following: “Is it possible to solve a given problem with MWMR registers and anonymous processes, and if the answer is “yes”, how many registers do we need ?”

Consensus and k-set agreement. This paper focuses on the k-set agreement problem in a system of n processes. This problem introduced in [3], and denoted (n, k)-set agreement in the following, is a generalization of consensus, which corresponds to the case where $k = 1$. Assuming that each participating process proposes a value, every non-faulty process must decide a value (termination), which was proposed by some process (validity), and at most k different values are decided (agreement).

Impossibility results and the case of obstruction-freedom. When k or more processes may fail, there is no deterministic wait-free read/write solution to (n, k)-set agreement [2]. To sidestep this impossibility result, we consider a progress property weaker than wait-freedom, namely *obstruction-freedom*. This property states for (n, k)-set agreement that a process decides a value only if it executes solo during a "long enough period" without interruption. The notion of x-obstruction-freedom [8] generalizes this idea to any group of at most x processes.

2 Contributions of the paper

This paper details a *genuine obstruction-free* algorithm solving the (n, k)-set agreement problem in an *asynchronous anonymous read/write* system where any number of processes may crash. Our algorithm makes use of $(n - k + 1)$ MWMR registers, i.e., exactly n registers for consensus. In anonymous systems, (n, k)-set agreement requires $\Omega(\sqrt{\frac{n}{k} - 2})$ MWMR registers [6]. On another hand, the best obstruction-free (n, k)-set agreement algorithm known so far requires $2(n - k) + 1$ registers [5]. Hence, our algorithm provides a gain of $(n - k)$ MWMR registers.

In the *repeated* version of the (n, k)-set agreement problem, processes participate in a sequence of (n, k)-set agreement instances. It was recently proved [6] that $(n-k+1)$ atomic registers are necessary to solve repeated (n, k)-set agreement. This paper shows that a simple modification of our base construction solves *repeated* (n, k)-set agreement without additional atomic registers, being consequently optimal.

Our base algorithm, its extension to solve repeated (n, k)-set agreement, as well as an x-obstruction-free variation that uses $n - k + x$ MWMR registers are all detailed in our companion technical report [1].

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