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Abstract

We show that an equilibrium always exists in the Rothschild-Stiglitz insurance market model with adverse selection and an arbitrary number of risk types, when insurance contracts include policy dividend rules. The Miyazaki-Wilson-Spence state-contingent allocation is an equilibrium allocation (defined as a set of type-dependent lotteries sustained at a symmetric equilibrium of a market game), and it is the only one when out-of-equilibrium beliefs satisfy a robustness criterion. It is shown that stock insurers and mutuals may coexist, with stock insurers offering insurance coverage at actuarial price and mutuals cross-subsidizing risks.

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1   Introduction

The fact that no equilibrium may exist in the Rothschild-Stiglitz (1976) model of insurance markets under adverse selection has been at the origin of an abundant literature in economic theory. In one way or another, most articles in this area have moved away from the basic premise of the Rothschild-Stiglitz approach. This approach consisted of modelling the strategic interactions between insurers who simultaneously offer contracts under hidden information about the risk types of insurance seekers.

An important avenue of research that followed the seminal contribution of Rothschild and Stiglitz (1976) has its origin in the article by Wilson (1977). It focuses attention on competitive mechanisms when insurers interact in a dynamic way. This includes the "anticipatory equilibrium" of Miyazaki (1977), Wilson (1977) and Spence (1978), the "reactive equilibrium" of Riley (1979), and the variations on the equilibrium concept introduced by Hellwig (1987) and Engers and Fernandez (1987), and in more recent papers surveyed by Mimra and Wambach (2014), in particular Mimra and Wambach (2011), and Netzer and Scheuer (2014). Another line of research, illustrated by the works of Dubey and Geanakoplos (2002) and Bisin and Gottardi (2006) among others, departs from the strategic dimension and considers atomistic insurance markets under adverse selection in line with the approach by Prescott and Townsend (1984). Unlike these two strands of research,[1] our purpose is to reexamine the equilibrium issue

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[1]The fact that there may be no equilibrium in the Rothschild-Stiglitz model is related to the discontinuity of insurers' payoff functions, since small changes in their contract offers may lead all individuals of a given type to switch to other insurers, with a possible jump in the insurers' expected profits. Dasgupta and Maskin (1986a,b) have established existence theorems for mixed strategy equilibria in a class of games where payoff functions have discontinuity points, and, as shown by Rosenthal and Weiss (1984) in the case of the Spence model of education choices, such a mixed strategy equilibrium exists in the Rothschild-Stiglitz insurance market model. However, assuming that firms play mixed strategies at the contract offer stage has not been considered as reasonable in the subsequent literature on markets with adverse selection. In addition, as shown by Rosenthal

in a perspective that remains framed within the initial Rothschild-Stiglitz approach. This requires a few preliminary explanations.

Rothschild and Stiglitz (1976) considered a simple setting in which each insurer is constrained to offering a single contract, with a free entry equilibrium concept, but they emphasized that such an equilibrium could be viewed as a Nash equilibrium of a game in which insurers interact by offering contracts simultaneously. They also noted that a next step was to test a less restrictive definition of insurers' strategies. In particular, they observed that allowing insurers to offer menus of contracts would make the condition under which an equilibrium exists even more restrictive. When commenting on the approach by Wilson (1977), they noted that "the peculiar provision of many insurance contracts, that the effective premium is not determined until the end of the period (when the individual obtains what is called a dividend), is perhaps a reflection of the uncertainty associated with who will purchase the policy, which in turn is associated with the uncertainty about what contracts other insurance firms will offer". In other words, many insurance contracts, mostly those offered by mutuals, have a participating dimension which should not be ignored when we seek to understand how competition works in the real word.[2]

Our objective in the present paper is to move forward in that direction. In a first approach (Picard, 2014), we have studied how allowing insurers to offer either participating or non-participating contracts, or in other words to act as mutuals or as stock insurers,[3] affects the conclusion about the existence of an equilibrium if all other as-

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and Weiss (1984), at a mixed-strategy equilibrium, a potential entrant could make positive profit. This reinforces the fundamental conclusion of Rothschild and Stiglitz, that is, that an entry-deterring equilibrium may not exist.

[2]Mutuals differ according to the role of the premium charged at the start of each policy period. *Advance premium mutuals* set premium rates at a level that is expected to be sufficient to pay the expected losses and expenses while providing a margin for contingencies, and policyholders usually receive dividends. In contrast, *assessment mutuals* collect an initial premium that is sufficient only to pay typical losses and expenses and levy supplementary premiums whenever unusual losses occur.

[3]This mapping between the nature of contracts (participating or non-participating) and the cor-

sumptions of the Rothschild-Stiglitz model are unchanged. An equilibrium (within the meaning of Rothschild and Stiglitz) always exists in such a setting, and the socalled Miyazaki-Wilson-Spence (MWS) allocation is a state contingent equilibrium allocation. Furthermore, mutuals offering participating contracts is the corporate form that emerges in markets where cross-subsidization provides a Pareto-improvement over the Rothschild-Stiglitz separating pair of contracts, a case where no equilibrium exists in the standard Rothschild-Stiglitz model. However, these conclusions were reached under quite restrictive assumptions: we postulated that there were only two risk types (high risk and low risk), as in the initial Rothschild-Stiglitz model, and we restricted attention to linear policy dividend rules that allow insurers to distribute a fixed proportion of their aggregate underwriting profit to policyholders. Furthermore, we did not present conditions under which a unique equilibrium allocation exists. The objective of the present paper is to reexamine these issues in a setting with an arbitrary number of risk types and a more general definition of admissible policy dividend rules, and also to obtain conditions under which there is a unique equilibrium allocation.

It turns out that, beyond the extended validity of our conclusions, considering an arbitrary number of risk types provide an endogenous structure of corporate forms in the insurance industry: mutuals emerge for risk type subgroups that require cross-subsidization, while stock insurers and mutuals may provide coverage to subgroups without cross-subsidization. We will thus explain why the coexistence of mutuals and stock insurers is a natural outcome of competitive interactions in insurance markets,

porate form (mutuals or stocks) is of course an oversimplification of the insurance market. Firstly, insurers may offer participating and non-participating contracts simultaneously. In particular, most life insurance contracts include profit participation clauses, even in the case of stock insurers. Furthermore, whatever the corporate structure, the participation of policyholders in profit may take other forms than policy dividends: in particular, it may be in the form of discounts when contracts are renewed, which is a strategy available to stock insurers and mutuals. In addition, the superiority of one corporate form over another may also reflect other factors, including agency costs and governance problems.

a conclusion that fits with the facts observed in many countries.[4] Finally, we will also examine the issue of equilibrium uniqueness, and we will highlight a robustness criterion under which there is a unique equilibrium. However, considering an arbitrary number of types and non-linear policy dividend rules and extending the approach to conditions under which a unique equilibrium exists requires a more formal approach than the geometry-based reasoning that is sufficient for more simple cases, such as the seminal article of Rothschild and Stiglitz (1976).

The rest of this article is organized as follows. Section 2 presents our setting, which is an insurance market under adverse selection with an arbitrary number of risk types, where insurance contracts may include policy dividend rules. Section 3 is the core of the paper: it analyzes the market equilibrium by defining a market game and an equilibrium of this game, as well as the MWS allocation in the manner of Spence (1978). We show that this allocation is sustained by a symmetric equilibrium of the market game and, more specifically, that it may be sustained by participating contracts for subgroups with cross-subsidization and non-participating contracts in the other cases. Finally, we show that the MWS allocation is the only equilibrium allocation under a robustness criterion derived from evolutionary stability criterions in games with a continuum of players. Section 4 concludes. Proofs are in the Appendix.

2 The setting

We consider a large population represented by a continuum of individuals, with mass 1, facing idiosyncratic risks of having an accident.[5] All individuals are risk averse:

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[4]The mutual market share is over 40% in Japan, France and Germany. It is almost 50% in the Netherlands and it is over 60% in Austria. In the US, it reached 36.3% in 2013. These aggregate figures mask important disparities between the life and non-life lines of business.

[5]The word "accident" is taken in its generic meaning: it refers to any kind of insurable loss, such as health care expenditures or fire damages.

they maximize the expected utility of wealth $u(W)$, where W denotes wealth and the (twice continuously differentiable) utility function u is such that $u' > 0$ and $u'' < 0$. If no insurance policy is taken out, we have $W = W_N$ in the no-accident state and $W = W_A$ in the accident state; $A = W_N - W_A$ is the loss from an accident. Individuals differ according to their probability of accident π, and they have private information on their own accident probability. There are n types of individuals, with $\pi = \pi_i$ for type i with $0 < \pi_n < \pi_{n-1} < ... < \pi_1 < 1$. Hence, the larger the index i, the lower the probability of an accident. λ_i is the fraction of type i individuals among the whole population with $\sum_{i=1}^{n} \lambda_i = 1$.

Insurance contracts are offered by m insurers ($m \geq 2$) indexed by $j = 1, ..., m$ who may offer participating or non-participating contracts. In other words, insurers are entrepreneurs who may be stock insurers or mutual insurers. Stock insurers pool risks between policyholders through non-participating insurance contracts, and they transfer underwriting profit to risk-neutral shareholders. Mutual insurers have no shareholders: they share risks between their members only through participating contracts. Insurers offer contracts in order to maximize their residual expected profit (i.e. the expected corporate earnings after policy dividends have been distributed).[6]

We assume that each individual can take out only one contract. An insurance contract is written as (k, x), where k is the insurance premium and x is the net payout

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[6]Thus, the insurance corporate form is a consequence of the kind of insurance contracts offered at the equilibrium of the insurance market. It is not given *ex ante*. The underwriting activity as well as all other aspects of the insurance business (e.g. claims handling) are supposed to be costless. Insurers earn fixed fees in a competitive market. The mere fact that they may transfer risks to risk-neutral investors leads them to maximize the expected residual profit. If an insurer could increase its residual expected profit by offering other insurance policies, then it could contract with risk neutral investors and secure higher fixed fees. Note that the residual profit of a mutual is zero if profits are distributed as policy dividends or losses are absorbed through supplementary premiums. In that case, if the mutual insurer could make positive residual profit, then he would benefit from becoming a stock insurer.

in case of an accident. Hence, $x+k$ is the indemnity. Participating insurance contracts also specify how policy dividends are paid or supplementary premiums are levied. We will restrict attention to deterministic policies in which dividend rules define the (non-random) policy dividend D as a function of average profits and of the number of policyholders, for each contract offered by the insurer (see below for details).[7] The expected utility of a type i policyholder is then written as:

$$Eu = (1 - \pi_i)u(W_N - k + D) + \pi_i u(W_A + x + D).$$

Our objective is to characterize a subgame perfect Nash equilibrium of a two stage game called "the market game", where insurers can offer participating or non-participating contracts. At stage 1, insurers offer menus of contracts, and at stage 2 individuals respond by choosing the contracts they prefer among the offers made by the insurers.

It is of utmost importance to note that the choices of individuals depend on the intrinsic characteristics of the contracts that have been offered at stage 1, but also on expected policy dividends. Expected policy dividends should coincide with true dividends (for contracts that are actually chosen by some individuals), that are themselves dependent on the distribution of risk types among policyholders for each contract. Thus, at stage 2, the participating nature of contracts induces a form of interdependence between individuals' strategies that is absent in the standard model with only non-participating contracts.

At stage 1, the strategy of insurer j is defined by a menu of n contracts, one for each type of individual, written as $C^j = (C_1^j, C_2^j, ..., C_n^j, D^j(.))$, where $C_h^j = (k_h^j, x_h^j)$ specifies the premium k_h^j and the net indemnity x_h^j. $D^j(.)$ is a policy dividend rule, i.e., a way to distribute the net profits made on C^j. We write $D^j(.) = (D_1^j(.), ..., D_n^j(.))$,

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[7]D will be non random because the law of large numbers allows us to evaluate the average profit by the expected profit made on a policyholder who is randomly drawn among the customers. $D < 0$ corresponds to a supplementary call.

where $D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j)$ denotes the policy dividend paid to each individual who has chosen contract C_h^j when N_i^j is the fraction of individuals in the whole population who have chosen C_i^j with underwriting profit (the difference between premium and indemnity) per policyholder P_i^j, with $\sum_{j=1}^m \sum_{i=1}^n N_i^j = 1$.[8] C^j is non-participating if $D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j) \equiv 0$ for all h, and otherwise it is said to be participating. In particular, C^j is fully participating if underwriting profits are entirely distributed as policy dividends, that is, if[9]

$$\sum_{h=1}^n N_h^j D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j) \equiv \sum_{h=1}^n N_h^j P_h^j.$$

We will assume that $D_h^j(N_1^j, P_1^j, ..., N_n^j, P_n^j)$ is non-decreasing with respect to $P_1^j, ..., P_n^j$ and homogeneous of degree zero with respect to $(N_1^j, ..., N_n^j)$. We can write the policy dividend as

$$D_h^j = D_h^j(\theta_1^j, P_1^j, ..., \theta_n^j, P_n^j),$$

where

$$\theta_h^j \equiv \frac{N_h^j}{\sum_{i=1}^n N_i^j}$$

is the fraction of insurer j's customers who have chosen C_h^j, with $\sum_{h=1}^n \theta_h^j = 1$.

The homogeneity assumption is made for the sake of mathematical simplicity, but also because it fits with the standard policy dividend rules we may think of. For instance, if insurer j shares a fraction $\gamma^j \in [0,1]$ of its underwriting profit evenly among all its policyholders, then we have

$$D_h^j = \gamma^j \sum_{i=1}^n \theta_i^j P_i^j \quad \text{for all } h = 1, ..., n.$$

If insurer j distributes a fraction $\gamma^j \in [0,1]$ of the underwriting profit made on C_h^j to the policyholders who have chosen this contract, then

$$D_h^j = \gamma^j P_h^j \quad \text{for all } h = 1, ..., n.$$

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[8] $D_h^j < 0$ corresponds to a supplementary premium levied on C_h^j.

[9] C^j may be fully participating with $D_h^j \equiv 0$ for some h. In other words, a fully participating menu may include non-participating policies.

If insurer j distributes a fraction $\gamma^j \in [0, 1]$ of its underwriting profit to the policyholders, with different rights to dividend according to the contract, then we may postulate that there exist coefficients $\eta_h^j \geq 0$, with $\eta_1^j = 1$ such that $D_h^j = \eta_h^j D_1^j$, which gives

$$D_h^j = \frac{\eta_h^j}{\sum\limits_{h=1}^{n} \theta_i^j \eta_i^j} \gamma^j \sum_{h=1}^{n} \theta_h^j P_h^j \quad \text{for all } h = 1, ..., n.$$

Thus, although the homogeneity assumption reduces the generality of our analysis, it nevertheless encompasses a broad variety of cases that we observe in practice.

3 Market equilibrium

Let $C \equiv (C^1, C^2, ..., C^m)$ be the profile of contract menus offered by insurers at stage 1 of the market game, with $C^j = (C_1^j, C_2^j, ..., C_n^j, D^j(.))$. At stage 2, the strategy of a type i individual[10] specifies for all j and all h the probability $\sigma_{ih}^j(C)$ to choose C_h^j as a function of C. The contract choice strategy of type i individuals is thus defined by $\sigma_i(C) \equiv \{\sigma_{ih}^j(C) \in [0, 1] \text{ for } j = 1, ..., m \text{ and } h = 1, ..., n \text{ with} \sum_{j=1}^{m} \sum_{h=1}^{n} \sigma_{ih}^j(C) = 1\}$ for all C. Let $\sigma(.) \equiv (\sigma_1(.), \sigma_2(.), ..., \sigma_n(.))$ be a profile of individuals' strategies.

When an insurance contract $C_h^j = (k_h^j, x_h^j)$ is taken out by a type i individual, with (non-random) policy dividend D_h^j, the policyholder's expected utility and the expected underwriting profit are respectively written as:

$$U_i(C_h^j, D_h^j) \equiv (1 - \pi_i)u(W_N - k_h^j + D_h^j) + \pi_i u(W_A + x_h^j + D_h^j),$$

$$\Pi_i(C_h^j) \equiv (1 - \pi_i)k_h^j - \pi_i x_h^j.$$

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[10]Hence, for the sake of notational simplicity, it is assumed that all individuals of the same type choose the same mixed strategy. In a more general setting, different individuals of the same type could choose different mixed strategies. This extension would not affect our conclusions insofar as the policy dividends paid by an insurer only depend on the distribution of customers among its contracts and by the proportion of each type for each contract, and not on the identity of the individuals who purchase a given contract. See the proof of Lemma 4 in the Appendix.

When type i individuals choose C_h^j with probability σ_{ih}^j, we may write θ_h^j and P_h^j as functions of individual choices and contracts:

$$\theta_h^j(\sigma) \equiv \frac{\sum_{i=1}^{n} \lambda_i \sigma_{ih}^j}{\sum_{i=1}^{n} \sum_{k=1}^{n} \lambda_i \sigma_{ik}^j} \quad \text{if} \quad \sum_{i=1}^{n} \sum_{k=1}^{n} \lambda_i \sigma_{ik}^j > 0,$$

$$P_h^j(C_h^j, \sigma) = \frac{\sum_{i=1}^{n} \lambda_i \sigma_{ih}^j \Pi_i(C_h^j)}{\sum_{i=1}^{n} \lambda_i \sigma_{ih}^j} \quad \text{if} \quad \sum_{i=1}^{n} \lambda_i \sigma_{ih}^j > 0,$$

where $\sigma = (\sigma_1, ..., \sigma_n)$ with $\sigma_i = (..., \sigma_{ih}^j, ...)$.

We are now in a position to define a market equilibrium more formally.

3.1 Definition of a market equilibrium

Definition 1 *A profile of strategies* $\widetilde{\sigma}(.), \widetilde{C} \equiv (\widetilde{C}^1, ..., \widetilde{C}^m)$*, where* $\widetilde{C}^j = (\widetilde{C}_1^j, ..., \widetilde{C}_n^j, \widetilde{D}^j(.))$*, is a subgame perfect Nash equilibrium of the market game (in short a market equilibrium) if*

$$\sum_{j=1}^{m} \sum_{h=1}^{n} \widetilde{\sigma}_{ih}^j(C) U_i(C_h^j, \overline{D}_h^j(C)) = \max\{U_i(C_h^j, \overline{D}_h^j(C)); j = 1, ..., m, h = 1, ..., n\}$$

for all $i = 1, ..., n$ *and all* $C,$ $\quad\quad\quad\quad$ (1)

$$\overline{\Pi}^j(\widetilde{C}) \geq \overline{\Pi}^j(C^j, \widetilde{C}^{-j}) \text{ for all } C^j \text{ and all } j = 1, ..., m \quad\quad\quad\quad (2)$$

where $C \equiv (C^1, ..., C^m), C^j = (C_1^j, ..., C_n^j, D^j(.)), \widetilde{C}^{-j} = (\widetilde{C}^1, ..., \widetilde{C}^{j-1}, \widetilde{C}^{j+1}, ..., \widetilde{C}^m)$ *and*

$$\overline{D}_h^j(C) \equiv D_h^j(\overline{\theta}_1^j(C), \overline{P}_1^j(C), ..., \overline{\theta}_n^j(C), \overline{P}_n^j(C)), \quad\quad\quad\quad (3)$$

$$\overline{\Pi}^j(C) \equiv \sum_{i=1}^{n} \sum_{h=1}^{n} \lambda_i \widetilde{\sigma}_{ih}^j(C)[\Pi_i(C_h^j) - \overline{D}_h^j(C)], \quad\quad\quad\quad (4)$$

with

$$\overline{\theta}_h^j(C) = \theta_h^j(\widetilde{\sigma}(C)) \ \ for \ all \ h \ \ if \ \sum_{i=1}^{n}\sum_{k=1}^{n} \lambda_i \widetilde{\sigma}_{ik}^j(C) > 0,$$

$$\overline{P}_h^j(C) = P_h^j(C_h^j, \widetilde{\sigma}(C)) \ \ for \ all \ h \ \ if \ \sum_{i=1}^{n} \lambda_i \widetilde{\sigma}_{ih}^j(C) > 0,$$

$$\overline{\theta}_h^j(C) \geq 0 \ and \ \overline{P}_h^j(C) \in [\Pi_1(C_h^j), \Pi_n(C_h^j)] \ for \ all \ h, \ with \ \sum_{h=1}^{n} \overline{\theta}_h^j(C) = 1,$$

$$if \ \sum_{i=1}^{n}\sum_{k=1}^{n} \lambda_i \widetilde{\sigma}_{ik}^j(C) = 0.$$

The notations in Definition 1 are as follows. Consider a profile of contracts $C = (C^1, ..., C^m)$ where $C^j = (C_1^j, C_2^j, ..., C_n^j, D^j(.))$ is the menu offered by insurer j. Then $\overline{\theta}_h^j(C)$ is the proportion of insurer j's policyholders who choose C_h^j when C is offered, with $\overline{P}_h^j(C)$ the corresponding profit per policyholder. When insurer j attracts policyholders, then $\overline{\theta}_h^j(C)$ and $\overline{P}_h^j(C)$ are derived from individuals' contract choice strategy $\widetilde{\sigma}(C)$. Otherwise, $\overline{\theta}_h^j(C)$ and $\overline{P}_h^j(C)$ are out-of-equilibrium beliefs that fullfill the coherency conditions stated in Definition 1. Then $\overline{D}_h^j(C)$ and $\overline{\Pi}^j(C)$ defined by (3) and (4) denote the policy dividend for C_h^j and the residual profit of insurer j, respectively. They depend on the set of contracts C offered in the market and on the profile of individuals' contract choice strategy $\widetilde{\sigma}(.)$. In particular, $\overline{D}_h^j(C) = \widetilde{D}_h^j(\overline{\theta}_1^j(C), \overline{P}_1^j(C), ..., \overline{\theta}_n^j(C), \overline{P}_n^j(C))$ if $C^j = \widetilde{C}^j$.

Keeping these notations in mind, (1) and (2) correspond to the standard definition of a subgame perfect Nash equilibrium. From (1), choosing C_h^j with probability $\widetilde{\sigma}_{ih}^j(C)$ is an optimal contract choice for type i individuals, given expected policy dividends.[11] (2) means that $\widetilde{C}^j$ is an optimal offer by insurer j (i.e., an offer that maximizes residual

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[11]Since there is a continuum of individuals in the population, when a type i individual chooses her mixed strategy $\sigma_i(C)$, she considers that expected underwriting profit $\overline{P}_h^j(C)$ and expected policy dividends $\overline{D}_h^j(C)$ are independent from her own choices. This is implicit in equation (1): type i individuals choose their insurance contract for given expectations on policy dividends, because they believe they are infinitesimal in the group of insureds who choose the same contract. If C_h^j is chosen by nobody, or more generally if insurer j does not attract any customer, then individuals estimate

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profit, that is, the difference between underwriting profit and policy dividend) when $\widetilde{C}^{-j}$ is offered by the other insurers, given the contract choice strategy of individuals.

Let C^* denote the menu of contracts at a *symmetric equilibrium of the market game* (defined as an equilibrium where all active insurers, i.e., all insurers with customers, offer the same menu and individuals are evenly shared between insurers), with $\widetilde{C}^j = C^* \equiv (C_1^*, C_2^*, ..., C_n^*, D^*(.))$ for each active insurer j and $C_i^* = (k_i^*, x_i^*)$ for all $i = 1, ..., n$ and $D^*(.) \equiv (D_1^*(.), ..., D_n^*(.))$. If individuals do not randomize between contracts, $C_i^* = (k_i^*, x_i^*)$ denotes the contract chosen by type i individuals.

A symmetric equilibrium of the market game sustains an *equilibrium allocation* $\{(W_i^{1*}, W_i^{2*}), i = 1, ..., n\}$, where (W_i^{1*}, W_i^{2*}) is the lottery on final wealth induced by the equilibrium strategies for type i individuals (meaning that their final wealth is W_i^{1*} with probability $1 - \pi_i$ and W_i^{2*} with probability π_i), with $W_i^{1*} = W_N - k_i^* + D_i^*$ and $W_i^{2*} = W_A + x_i^* + D_i^*$, where $D_i^* \equiv D_i^*(\lambda_1, \Pi_1^*, ..., \lambda_n, \Pi_n^*)$ with $\Pi_i^* \equiv \Pi_i(C_i^*)$.

Our main objective in what follows is to establish the existence and uniqueness of such an equilibrium allocation. To do that, we first characterize a candidate equilibrium allocation by following the Spence (1978) approach to the Miyazaki-Wilson equilibrium with an arbitrary number of types (we will call it the MWS allocation), and next we show that this allocation is sustained by a profile of strategies which is a symmetric equilibrium of the market game.

3.2 The MWS allocation

When a type i individual takes out a contract $C_i = (k_i, x_i)$ and receives policy dividend D_i, then she is facing lottery $(W_i^1, W_i^2) = (W_N - k_i + D_i, W_A + x_i + D_i)$, and the insurer's

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$\overline{P}_h^j(C)$ and $\overline{D}_h^j(C)$ by considering themselves as members of a deviant group with infinitesimal mass who would choose contracts offered by insurer j, and their out-of-equilibrium beliefs correspond to the composition of this hypothetical deviant group.

expected residual profit (in short, its profit) is

$$\Pi_i(C_i) - D_i = W_N - (1 - \pi_i)W_i^1 - \pi_i(W_i^2 + A). \tag{5}$$

This allows us to characterize candidate equilibrium allocations as follows. Let us define a sequence of expected utility levels u_i^* by $u_1^* = u(W_N - \pi_1 A)$, and for $2 \leq i \leq n$:

$$u_i^* = \max(1 - \pi_i)u(W_i^1) + \pi_i u(W_i^2)$$

with respect to $W_h^1, W_h^2, h = 1, ..., i$, subject to

$$(1 - \pi_h)u(W_h^1) + \pi_h u(W_h^2) \geq u_h^* \text{ for } h < i, \tag{6}$$

$$(1 - \pi_h)u(W_h^1) + \pi_h u(W_h^2) \geq (1 - \pi_h)u(W_{h+1}^1) + \pi_h u(W_{h+1}^2) \text{ for } h < i, \tag{7}$$

$$\sum_{h=1}^{i} \lambda_h[W_N - (1 - \pi_h)W_h^1 - \pi_h(W_h^2 + A)] = 0. \tag{8}$$

Let $\mathbb{P}_i$ denote the problem which defines u_i^*, with $i = 2, ..., n$. The objective function in $\mathbb{P}_i$ is the expected utility of type i individuals by restricting attention to individuals with types 1 to i. Constraints (6) ensure that higher risk individuals (i.e. $h < i$) get expected utility no less than u_h^*. (7) are incentive compatibility constraints: type h individuals (with $h < i$) are deterred from choosing the policy targeted at the adjacent less risky type $h+1$. (8) is the break-even constraint over the set of risk types $h \leq i$. For $n = 2$, the optimal solution to $\mathbb{P}_2$ is the Miyazaki-Wilson equilibrium allocation. Let $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ be the optimal solution to $\mathbb{P}_n$. It is characterized in Lemmas 1 and 2, which are adapted from Spence (1978), and, as usual in the literature, we may call it the MWS allocation.

Lemma 1 *There exist* $T \in \mathbb{N}, 0 \leq T \leq n - 1$, *and* $\ell_t \in \{0, ..., n\}$, $t = 0, ..., T + 1$ *with* $\ell_0 = 0 \leq \ell_1 \leq \ell_2 ... \leq \ell_T < \ell_{T+1} = n$ *such that for all* $t = 0, ..., T$

$$\sum_{i=\ell_t+1}^{h} \lambda_i[W_N - (1 - \pi_i)\widehat{W}_i^1 - \pi_i(\widehat{W}_i^2 + A)] < 0 \quad \text{for all } h = \ell_t + 1, ..., \ell_{t+1} - 1, \tag{9}$$

$$\sum_{i=\ell_t+1}^{\ell_{t+1}} \lambda_i[W_N - (1 - \pi_i)\widehat{W}_i^1 - \pi_i(\widehat{W}_i^2 + A)] = 0. \tag{10}$$

Furthermore, we have

$$(1 - \pi_i)u(\widehat{W}_i^1) + \pi_i u(\widehat{W}_i^2) \quad = \quad u_i^* \ \textit{if } i \in \{\ell_1, \ell_2, ..., n\}, \tag{11}$$

$$(1 - \pi_i)u(\widehat{W}_i^1) + \pi_i u(\widehat{W}_i^2) \quad > \quad u_i^* \quad \textit{otherwise.} \tag{12}$$

In $\mathbb{P}_n$, for each risk type i lower than n, the optimal lottery $(\widehat{W}_i^1, \widehat{W}_i^2)$ trades off the increase in insurance cost against the relaxation of the adjacent incentive constraint. In addition, the minimal expected utility level u_i^* has to be reached. Lemma 1 states that this trade-off results in pooling risk types in $T + 1$ subgroups indexed by t. Subgroup t includes risk types $i = \ell_t + 1, ..., \ell_{t+1}$ with $\ell_0 = 0$ and $\ell_{T+1} = n$. From (12), within each subgroup t, all types i except the highest (i.e. $i = \ell_t + 1, ..., \ell_{t+1} - 1$) get more than their reservation utility u_i^*, and from (9) there is negative profit over this subset of individuals. They are cross-subsidized by the highest risk type (i.e., by type ℓ_{t+1}). From (11) and (10), type ℓ_t just reaches its reservation utility $u_{\ell_t}^*$, for $t = 1, ..., T + 1$, with zero profit over the whole subgroup t. In what follows, I will denote the set of risk types in subgroups with cross-subsidization, i.e.

$$i \quad \in \quad I \subset \{1, ..., n\} \ \text{if} \ \ell_t < i \leq \ell_{t+1}$$

$$\text{for } t \quad \in \quad \{0, ..., T\} \text{ such that } \ell_{t+1} - \ell_t \geq 2.$$

When $n = 2$, we know from Crocker and Snow (1985)[12] that there exists $\lambda^* \in (0, 1)$ such that $I = \{1, 2\}$ if $\lambda_1 < \lambda^*$ and $I = \emptyset$ if $\lambda_1 \geq \lambda^*$. When $n > 2$, the population is distributed among subgroups. A case with $n = 5, T = 2, \ell_1 = 3$ and $\ell_2 = 4$ is illustrated in Figure 1. There are three subgroups in this example: type $i = 3$ cross-subsidizes types 1 and 2, while the contracts offered to types 4 and 5 make zero profit. We thus have $I = \{1, 2, 3\}$ and $\widehat{u}_h > u_h^*$ for $h = 1, 2$ and $\widehat{u}_h = u_h^*$ for $h = 3, 4$ and 5, where $\widehat{u}_h$ is the type h expected utility at the optimal solution to $\mathbb{P}_n$.[13]

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[12]See also Picard (2014).

[13]The structure of cross-subsidization subgroups follows from the interaction of the π_i and λ_i in a complex way, which makes a more precise characterization difficult. For given π_i, intuition suggests

Figure 1

Lemma 2 *There does not exist any incentive compatible allocation* $\{(W_i^1, W_i^2), i = 1, ..., n\}$ *such that*

$$(1 - \pi_{\ell_t})u(W_{\ell_t}^1) + \pi_{\ell_t}u(W_{\ell_t}^2) \geq u_{\ell_t}^* \text{ for all } t = 1, ..., T + 1 \tag{13}$$

and

$$\sum_{i=1}^{n} \lambda_i[W_N - (1 - \pi_i)W_i^1 - \pi_i(W_i^2 + A)] > 0. \tag{14}$$

Lemma 2 states that no insurer can make positive profit by attracting all individuals and offering more than $u_{\ell_t}^*$ to threshold types ℓ_t. Suppose that there exists a profitable allocation close to $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ that provides more than $u_{\ell_t}^*$ to types ℓ_t. Such an allocation would provide an expected utility larger than u_h^* for all h (this is just a consequence of the second part of Lemma 1), which would contradict the definition of u_n^*. The proof of Lemma 2 extends this argument to allocations that are not close to $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$. The main consequence of Lemma 2 is that it is impossible to make positive profit in a deviation from $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ if threshold types ℓ_t are guaranteed to get at least $u_{\ell_t}^*$.

3.3 Existence of an equilibrium

Proposition 1 $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ *is an equilibrium allocation. It is sustained by a symmetric equilibrium of the market game where each insurer j offers $C^j = C^* \equiv (\widehat{C}_1, ..., \widehat{C}_n, D^*(.))$, type i individuals choose $\widehat{C}_i \equiv (\widehat{k}_i, \widehat{x}_i) = (W_N - \widehat{W}_i^1, \widehat{W}_i^2 - W_A)$ and*

that the case described in Figure 1 emerges from a situation where λ_1/λ_3 and λ_2/λ_3 are relatively small so that cross-subsidizing risk types 1 and 2 allows a higher expected utility u_3^* for type 3 to be reached, while λ_3/λ_4 and λ_4/λ_5 are relatively large so that it would be too costly to cross-subsidize risk types 3 and 4.

$D^*(.) = (D_1^*(.), ..., D_n^*(.))$ *is any policy dividend rule such that*

$$\sum_{i \in I} N_i D_i^*(N_1, P_1, ..., N_n, P_n) \equiv \sum_{i \in I} N_i P_i, \tag{15}$$

$$D_i^*(\lambda_1, \Pi_1(\widehat{C}_1), ..., \lambda_n, \Pi_n(\widehat{C}_n)) = 0 \quad \text{for all } i = 1, ..., n, \tag{16}$$

$$D_{\ell_t}^*(N_1, P_1, ..., N_n, P_n) \equiv 0 \text{ for all } t = 1, ..., T+1. \tag{17}$$

At the symmetric equilibrium of the market game described in Proposition 1, each insurer offers $C^* = (\widehat{C}_1, ..., \widehat{C}_n, D^*(.))$, and type i individuals choose $\widehat{C}_i$. The conditions on $D^*(.)$ are sufficient for C^* to be an equilibrium contract offer. (15) means that profits are fully distributed among the individuals who choose a contract with cross-subsidization at equilibrium, and from (16) no policy dividend is paid on the equilibrium path. From (17), threshold types ℓ_t are excluded from the sharing of profits.

To intuitively understand how Proposition 1 is deduced from Lemma 2, consider an allocation induced by $C^{j_0} \neq C^*$ offered by a deviant insurer j_0. This corresponds to a compound lottery generated by individuals' mixed strategies over C^{j_0} and C^*. The aggregate residual profit of this allocation is larger or equal to the profit made on C^{j_0} alone, because non-deviant insurers $j \neq j_0$ offer a menu of contracts with full distribution of profits or payment of losses on $\{\widehat{C}_i, i \in I\}$ and non-negative profits on $\{\widehat{C}_i, i \notin I\}$. Furthermore, Condition (17) assures that all threshold types ℓ_t get at least $u_{\ell_t}^*$. Lemma 2 shows that this allocation cannot be profitable, hence deviant insurer j_0 does not make positive profit[14].

Remark 1 *Note that equilibrium premiums are not uniquely defined, since insurers may compensate higher premiums through higher dividends. More precisely, the equilibrium allocation $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ can also be sustained by an equilibrium of the market game where insurers offer contracts $\widehat{C}_i' \equiv (\widehat{k}_i', \widehat{x}_i')$ where $\widehat{k}_i' = \widehat{k}_i + \delta$ and $\widehat{x}_i' = \widehat{x}_i -$*

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[14]More precisely, Proposition 1 follows from a straightforward extension of Lemma 2 to allocations with randomization between contracts. See Lemma 3 in the Appendix.

δ, with policy dividend rule $D_i^{**}(N_1, P_1, ..., N_n, P_n) \equiv D_i^*(N_1, P_1 - \delta, ..., N_n, P_n - \delta) + \delta$, with $\delta > 0$. In that case, dividends include a fixed part δ paid to all policyholders and a variable part that does not concern threshold types. Hence, the fundamental meaning of Condition (17) is not the fact that threshold types do not receive policy dividends, since they may actually receive such dividends according to the level of premiums: Condition (17) assures us that threshold types cannot be penalized when deviant insurers offer new contracts.[15]

Although no policy dividend (or dividend δ) is paid on the equilibrium path, there may be variations in policy dividends when a deviant insurer j^0 offers a menu C^{j_0} that differs from $C^* = (\widehat{C}_1, ..., \widehat{C}_n)$. Such a deviation may affect the distribution of types among individuals who still choose a contract in C^*, with possible variations in profits or losses of insurers $j \neq j_0$, and thus policy dividends or supplementary premiums. Variations in policy dividends can then act as an implicit threat that dissuades deviant insurers from undertaking competitive attacks. For the sake of illustration, assume

$$D_i^*(N_1, P_1, ..., N_n, P_n) = \frac{\widehat{k}_i - \widehat{k}_{\ell_{t+1}}}{\sum\limits_{h=\ell_t+1}^{\ell_{t+1}} N_h(\widehat{k}_h - \widehat{k}_{\ell_{t+1}})} \sum\limits_{h=\ell_t+1}^{\ell_{t+1}} N_h P_h \qquad (18)$$

for all $i \in \{\ell_t + 1, ..., \ell_{t+1}\} \subset I$. Here, $D^*(.)$ involves the sharing of profit within each subgroup t with cross-subsidization. The total profit made within subgroup t is $\sum_{h=\ell_t+1}^{\ell_{t+1}} N_h P_h$. It is distributed to policyholders within the same subgroup. Furthermore, according to the policy dividend rule, the larger the premium, the larger

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[15]Condition (17) seems necessary to get an equilibrium existence result when $n > 2$. For the sake of illustration, assume $n = 3$ and consider a case where $\widehat{C}_1$ is in deficit and $\widehat{C}_2$ and $\widehat{C}_3$ are profitable when respectively chosen by types $1, 2$ and 3 (a case where $I = \{1, 2, 3\}$ and $T = 0$). Assume also that underwriting profit or losses are uniformly shared between policyholders, including type 3. In that case, if λ_2 is small enough, there exists a profitable non-participating contract C_2' closed to $\widehat{C}_2$ which would attract type 2 individuals if offered in deviation from equilibrium, while types 1 and 3 would keep choosing $\widehat{C}_1$ and $\widehat{C}_3$ and pay (small) supplementary premiums.

the policy dividend in absolute value. There is no right to receive a policy dividend for the individuals who pay the smallest premium (i.e. for type ℓ_{t+1}), while rights are larger for types i who pay larger premiums, which reflects the practice of mutuals that pay larger dividends to policyholders who have paid larger premiums. We have $\sum_{h=\ell_t+1}^{\ell_{t+1}} \lambda_h \Pi(\widehat{C}_h) = 0$ for all t from (10), and thus this policy dividend rule satisfies conditions (15)-(17). If a deviant insurer j_0 attracts some individuals who cross-subsidize other risk types within subgroup t, then after the deviation we will have $\sum_{h=\ell_t+1}^{\ell_{t+1}} N_h P_h < 0$ for non-deviant insurers $j \neq j_0$, and consequently the welfare of these other individuals will deteriorate if they keep choosing the same contract because they will have to pay supplementary premiums. It may then be impossible for insurer j_0 to not also attract them, which will make its offer non-profitable. The proof of Proposition 1 shows that this is indeed the case.[16]

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[16] It might be objected that, in practice, a deviant insurer could limit its offer to a small number of individuals by rationing demand, which would lessen the effect of its action on non-deviant insurers. In this way, if a deviant insurer restricts its offer to a small group of size ε, then its deviation only entails a small effect on the profit of non-deviant insurers: the lower ε, the smaller the shift in the lotteries offered by non-deviant insurers, which would open the door to profitable deviations attracting type ℓ_t individuals when $I \neq \emptyset$. A complete analysis of the market equilibrium with quantity rationing is beyond the scope of the present paper and would require a thorough analysis. However, at this stage, we may observe that insurers could use discontinuous policy dividend rules to prevent deviant competitors from attracting a small group of their policyholders. For example, participating contracts may stipulate that no policy dividend will be distributed unless the insurer's profit reaches a predetermined target level. Equilibrium strategies may consist of offering $\widehat{C}'_i \equiv (\widehat{k}'_i, \widehat{x}'_i)$, as defined in Remark 1 if $i \in I$, and committing to pay positive dividend δ if the profit is at least δ and nothing otherwise. Any deviation that attracts ε type ℓ_t individuals would cancel the payment of policy dividends by non-deviant insurers. Consequently, there exists a continuation equilibrium where the deviant does not make profit. Regarding competition with quantity rationing in the insurance market, see Inderst and Wambach (2001).

More generally, we may choose $D^*(.)$ such that

$$\sum_{\substack{i=\ell_t+1 \\ i \in I}}^{\ell_{t+1}} N_i D_i^*(N_1, P_1, ..., N_n, P_n) \equiv \sum_{\substack{i=\ell_t+1 \\ i \in I}}^{\ell_{t+1}} N_i P_i,$$

for all subgroup t with cross-subsidization, which shows that the equilibrium allocation is also sustained by equilibrium strategies where each insurer sells insurance to a given subgroup of individuals (gathering risk types $i = \ell_t+1, ..., \ell_{t+1}$ in I) or to a combination of these subgroups. Insurers who sell insurance to subgroups with only one risk type (i.e. to types $i \notin I$) or to a combination of these subgroups do not cross-subsidize risks. They offer non-participating policies, and we may consider them as stock insurers. Insurers who sell insurance policies to individuals who belong to subgroups with cross-subsidization (i.e. to types $i \in I$) offer fully participating policies: they act as mutuals do. In the example illustrated in Figure 1, mutuals would offer participating contracts to subgroup $t = 1$ (that includes types 1, 2 and 3) and stock insurers would offer non-participating contracts to subgroups $t = 2$ and 3. Hence, the model explains why stock insurers and mutuals may coexist: mutuals offer insurance contracts that are robust to competitive attacks when there is cross-subsidization, while stock insurers offer insurance contracts at actuarial price. The following corollary recaps our results more compactly.

Corollary 1 *The MWS allocation is also sustained by a market equilibrium where mutual insurers offer participating contracts to subgroups of individuals with types $i \in I$ and stock insurers offer non-participating contracts to types $i \notin I$.*

3.4 Uniqueness of equilibrium

Participating contracts induce an interdependence between the individuals' contract choices. Consequently: multiple continuation equilibria[17] may exist after menus of

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[17]Contract choice strategies $\widetilde{\sigma}(C) = (\widetilde{\sigma}_1(C), \widetilde{\sigma}_2(C), ..., \widetilde{\sigma}_n(C))$ define a continuation equilibrium associated with the contract offer C when they satisfy (1), with $\overline{D}_h^j(C)$ given by (3).

contracts have been offered at stage 1. Typically, type i individuals may decide to choose a participating contract offered by insurer j if they anticipate that less risky types i' (i.e., $i' > i$) are going to do the same, but they may make another choice for other expectations. This creates leeway in the characterization of a continuation equilibrium after a deviation at stage 1, and it opens the door to multiple equilibrium issues in the market game itself. In particular, contracts may not be chosen by anyone because of pessimistic expectations about the contracts offered by inactive insurers: insurance seekers may anticipate that the insurers who offer these contracts are going to attract only high-risk individuals, with negative underwriting profit. These pessimistic expectations (i.e., out-of-equilibrium beliefs) may annihilate profitable deviations, although such deviations would exist under more optimistic expectations. An equilibrium sustained by arbitrarily pessimistic beliefs is not very convincing if choosing contracts offered by a deviant insurer were beneficial to some policyholders. Definition 2 introduces a robustness criterion, that eliminates such equilibria.

Definition 2 *A market equilibrium $\widetilde{\sigma}(.), \widetilde{C}$ is based on robust beliefs if there does not exist a deviation C^{j_0} where insurer j_0 does not attract any customer, i.e.,*

$$\sum_{i=1}^{n} \sum_{h=1}^{n} \lambda_i \widetilde{\sigma}_{ih}^{j_0}(C^{j_0}, \widetilde{C}^{-j_0}) = 0,$$

and a risk type i_0 such that:

(i) Type i_0 individuals would be better off if they choose a contract $C_{i_0}^{j_0}$ in C^{j_0} in a deviation from their equilibrium strategy, and if they belong to an infinitely small subset of type i_0 individuals who are the only ones to do so, i.e.,

$$U_{i_0}(C_{i_0}^{j_0}, \overline{\overline{D}}_{i_0}^{j_0}) > \max\{U_{i_0}(\widetilde{C}_h^j, \overline{D}_h^j(C)); j \neq j_0, h = 1, ..., n\},$$

where $\overline{\overline{D}}_{i_0}^{j_0}$ is the policy dividend received by type i_0 individuals when they are the only ones to choose a contract in C^{j_0},[18]

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[18] Since $D_h^{j_0}(N_1^{j_0}, P_1^{j_0}, ..., N_n^{j_0}, P_n^{j_0})$ is homogeneous of degree zero with respect to $(N_1^{j_0}, ..., N_n^{j_0})$, $\overline{\overline{D}}_{i_0}^{j_0}$ does not depend on the mass of the subset ot type i_0 individual who choose $C_{i_0}^{j_0}$.

(ii) insurer j_0 attracts type i_0 policyholders, and possibly other individuals, in at least one other continuation equilibrium following the deviation from $\widetilde{C}^{j_0}$ to C^{j_0} and makes positive profit at all such continuation equilibria.

A robust equilibrium allocation is sustained by a symmetric market equilibrium based on robut beliefs.

Proposition 2 *The MSW allocation is the only robust equilibrium allocation.*

Presumably, individuals may make error in the real world, and this is the logic of the robustness criterion used to eliminate equilibria based on arbitrarily pessimistic beliefs. In Definition $2 - (i)$, if a subgroup of type i_0 individuals with positive measure do such an error (i.e., they choose $C_{i_0}^{j_0}$), then they would observe that this departure from their equilibrium contracts is in fact favorable to them. Definition $2-(ii)$ adds the condition that this improvement would be confirmed at all continuation equilibria where insurer j_0 attracts policyholders, and that such continuation equilibria exist and are profitable to insurer j_0. Definition 2 says that an equilibrium is based on robust beliefs if such deviations do not exist and Proposition 2 states that the MWS allocation is the only equilibrium allocation when beliefs are required to be robust.

Remark 2 *Definition 2 is inspired by robustness criterions in games with a continuum of players (non-atomic games). In an evolutionary game setting with a large group of identical players, a (mixed or pure) strategy of a given player is said to be neutrally stable (NSS) if there does not exist another strategy that would be strongly prefered by this player if this alternative strategy were played by a small enough fraction of similar individuals[19]. Definition $2-(i)$ adapts the NSS criterion to any subgame that follows*

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[19]The NSS criterion was introduced by Maynard Smith (1982). In the terminology of evolutionary games, the alternative strategy is played by a small group of "mutants" who appears in a large population of individuals who are programmed to play the same incumbent strategy. Following the biological intuition, we may assume that evolutionary forces select against the mutant strategy if and only if its postentry payoff (or fitness) is not larger than that of the incumbent strategy. Thus, a

a deviation by some insurer j_0. Definition 2 −(ii) weakens this equilibrium selection criterion by requiring that alternative strategies also provide a higher expected utility to the deviant individuals at another equilibrium (thus, not only when they are played in deviation from equilibrium by a small subgroup of individuals) and that insurer j_0 makes positive profit in such continuation equilibria.

4   Concluding comments

Thus, the MWS allocation is always an equilibrium allocation in the Rothschild-Stiglitz model when insurers can issue participating or non-participating policies. It is the only equilibrium allocation when out-of-equilibrium beliefs satisfy a robustness criterion. This equilibrium allocation is characterized by a classification of individuals into subgroups as done by Spence (1978), with cross-subsidization within each subgroup that includes several risk types. Participating policies act as an implicit threat which prevents deviant insurers from attracting low-risk individuals only. If a deviant insurer attracts individuals who cross-subsidize other risk types within a given subgroup, then these other individuals will have to pay supplementary premiums or receive lower dividends if they keep choosing the same contract from their non-deviant insurer. Consequently, it will be impossible for the deviant insurer to not also attract them, which will make its offer non-profitable.

This mechanism is similar to the logic of the MWS equilibrium. In both cases, a deviant insurer is deterred from attracting low risk individuals because it is expected that ultimately its offer would also attract higher risks, which would make it unprofitable. However, in the MWS equilibrium, insurers are protected from these competitive at-

neutrally stable strategy cannot be destabilized by deviations of a small group of mutants. NSS is a weakening of the evolutionary stability criterion (ESS) introduced by Maynard Smith and Price (1973) and Maynard Smith (1974). On the connections between evolutionary stability criteria and other robustness criteria of Nash equilibria, see Weibull (1995).

tacks because they can react by withdrawing contracts that become unprofitable. This assumption may be considered as unsatisfactory because it means that insurers are not committed to actually offer the announced contracts. It can also be legitimately argued that this description of the dynamic relationship between insurers is arbitrary. Other timings are possible, as shown by Riley (1979), Hellwig (1987) and others. Mimra and Wambach (2014) list papers that have departed from the original game structure of Rosthschild and Stiglitz (1976), and we have to admit that no particular timing has an obvious superiority over the others. Moving away from the Rothschild-Stiglitz game structure may be like opening a Pandora's box, since there always exist new ways to describe the dynamic competitive interaction between firms.

We have taken a different route. Our analysis has not stepped away from the instantaneous strategic interaction between insurers that characterizes the Rothschild-Stiglitz model, and we have explored the consequences of deleting an exogenous restriction on the content of insurance policies.[20] As observed by Rothschild and Stiglitz (1976) themselves, extending their model in order to include "the peculiar provision of many insurance contracts", firstly by considering menus, and secondly by allowing insurers to pay policy dividends, is a natural way to reconcile the empirical observation and the theoretical definition of a market equilibrium, and this is what we have done in this paper. Of course, we may consider that the glass is half empty rather than half full, and that even more general contracts, e.g., with quantity rationing, should be considered. This is another research avenue worth exploring. However, the case where firms commit to honour the offers made to clients, without restricting these offers to a subset of consumers, seems to be a natural starting point for the analysis

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[20]To be honest, it must be acknowledged that there are two possible game theory interpretation of the Rothschild-Stiglitz framework. In the most usual one, insurers face a continuum of individuals of various possible risk types, and they know the fraction of each type, but not any given individual's type. This is the interpretation we have come up with in this paper. In another one, insurers compete for a single potential insured individual whose type is privately observed, and insurers have a common prior over this type. Only the first interpretation is compatible with our analysis.

of competitive markets.

The main outcome of this modelling, apart from the existence and uniqueness of an equilibrium, is the fact that it leads to an endogenous definition of corporate forms, where mutuals and stock insurers may coexist, with specific functions: mutuals may provide coverage to risk groups that require cross-subsidization, while at the same time being protected against competitive attacks that would target their least risky policyholders. Subgroups without cross-subsidization do not require such endogenous protection, and they purchase non-participating or participating contracts. If, for some other reasons, stock insurers benefit from competitive advantages, for instance because they can transfer systemic risks to stockholders, then we may reach a complete market structuring that trades off the ability of mutuals to implement efficient cross-subsidization and the superiority of stock insurers in the face of macroeconomic risks. The diversity of market structures that we may observe in practice suggests that the balance is not always on the same side.

Appendix

Proof of Lemma 1

If $\sum_{h=1}^{i} \lambda_h [W_N - (1 - \pi_h)\widehat{W}_h^1 - \pi_h(\widehat{W}_h^2 + A)] > 0$ for $i \in \{1, ..., n\}$, then it would be possible to provide a higher expected utility than u_h^* for all $h = 1, ..., i$, while breaking even over the subset of individuals $h = 1, ..., i$, which would contradict the definition of u_i^*.[21] We thus have $\sum_{h=1}^{i} \lambda_h [W_N - (1 - \pi_h)\widehat{W}_h^1 - \pi_h(\widehat{W}_h^2 + A)] \leq 0$ for all $i \in \{1, ..., n\}$, which yields the first part of the Lemma.

We have $(1 - \pi_i)u(\widehat{W}_i^1) + \pi_i u(\widehat{W}_i^2) \geq u_i^*$ for all i from the definition of $\mathbb{P}_n$. If $i \in \{\ell_1, \ell_2, ..., n\}$, we have $\sum_{h=1}^{i} \lambda_h [W_N - (1 - \pi_h)\widehat{W}_h^1 - \pi_h(\widehat{W}_h^2 + A)] = 0$ from the first part of the Lemma, and we deduce $(1 - \pi_i)u(\widehat{W}_i^1) + \pi_i u(\widehat{W}_i^2) = u_i^*$, for otherwise

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[21] More explicitly, let ε be a positive real number and let $\{(W_h^1(\tau), W_h^2(\tau)), h = 1, ..., i\}$ that satifies (7) for all $\tau > 0$ with $W_h^1(0) = \widehat{W}_h^1, W_h^2(0) = \widehat{W}_h^2$ for all $h = 1, ..., i$, and $dW_h^1/d\tau = dW_h^2/d\tau > \varepsilon$ for all τ and all $h = 1, ...i$. There exists $\widehat{\tau} > 0$ such that $\{(W_h^1(\widehat{\tau}), W_h^2(\widehat{\tau})), h = 1, ..., i\}$ satifies (8), with $U_i(W_h^1(\widehat{\tau}), W_h^2(\widehat{\tau})) > u_h^*$ for all $h = 1, ..., i$, which contradicts the definition of u_i^*.

we would have a contradiction with the definition of u_i^*. Conversely, suppose we have $(1 - \pi_i)u(\widehat{W}_i^1) + \pi_i u(\widehat{W}_i^2) = u_i^*$ and $i \notin \{\ell_1, \ell_2, ..., n\}$. We would then have $\sum_{h=1}^i \lambda_h[W_N - (1 - \pi_h)\widehat{W}_h^1 - \pi_h(\widehat{W}_h^2 + A)] < 0$. Hence the allocation $\{(\widehat{W}_h^1, \widehat{W}_h^2), h = 1, ..., i\}$ is in deficit. Let $\{(W_h^{1\prime}, W_h^{2\prime}), h = 1, ..., i\}$ be the optimal solution to $\mathbb{P}_i$. Replacing $\{(\widehat{W}_h^1, \widehat{W}_h^2), h = 1, ..., i\}$ with $\{(W_h^{1\prime}, W_h^{2\prime}), h = 1, ..., i\}$ allows us to improve the optimal solution to $\mathbb{P}_n$, since the same type i expected utility u_i^* can be reached while breaking even on the set $h = 1, ..., i$, which provides additional resources that could be used to raise $(1 - \pi_n)u(W_n^1) + \pi_n u(W_n^2)$ over $(1 - \pi_n)u(\widehat{W}_n^1) + \pi_n u(\widehat{W}_n^2)$. We thus obtain a contradiction with the fact that $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$ is the optimal solution to $\mathbb{P}_n$.

Proof of Lemma 2

We first restrict attention to incentive compatible allocations $\{(W_i^1, W_i^2), i = 1, ..., n\}$ located in a neighbourhood of $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$. Suppose that such an allocation satisfies (13)-(14). Lemma 1 shows that

$$(1 - \pi_i)u(W_i^1) + \pi_i u(W_i^2) \geq u_i^* \text{ for all } i = 1, ..., n,$$

if (W_i^1, W_i^2) is close enough to $(\widehat{W}_i^1, \widehat{W}_i^2)$. Hence $\{(W_i^1, W_i^2), i = 1, ..., n\}$ satisfies the constraints of $\mathbb{P}_n$ with positive profits and expected utility larger or equal to u_n^* for type n, hence a contradiction.

We now prove that there does not exist any incentive compatible allocation $\{(W_i^1, W_i^2), i = 1, ..., n\}$ that satisfies (13)-(14), even if we do not restrict attention to allocations close to $\{(\widehat{W}_i^1, \widehat{W}_i^2), i = 1, ..., n\}$. Let us define $z_i^s \equiv u(W_i^s)$ and $\widehat{z}_i^s \equiv u(\widehat{W}_i^s)$ for $i = 1, ..., n$ and $s = 1, 2$. With this change of variable, the Lemma states that there does not exist

$\{(z_i^1, z_i^2), i = 1, ..., n\}$ such that

$$(1 - \pi_{\ell_t})z_{\ell_t}^1 + \pi_{\ell_t}z_{\ell_t}^2 \geq u_{\ell_t}^* \text{ for all } t = 1, ..., T + 1 \ , \tag{19}$$

$$(1 - \pi_i)z_i^1 + \pi_i z_i^2 \geq (1 - \pi_i)z_{i+1}^1 + \pi_i z_{i+1}^2 \text{ for } i = 1, ..., n - 1 \ , \tag{20}$$

$$\sum_{i=1}^{n} \lambda_i \{(1 - \pi_i)[W_N - u^{-1}(z_i^1)] - \pi_i[u^{-1}(z_i^2) - W_A]\}$$

$$> \sum_{i=1}^{n} \lambda_i \{(1 - \pi_i)[W_N - u^{-1}(\hat{z}_i^1)] - \pi_i[u^{-1}(\hat{z}_i^2) - W_A]\}. \tag{21}$$

The set of $\{(z_i^1, z_i^2), i = 1, ..., n\}$ that satisfies the conditions (19)-(21) is convex. Hence if there is any allocation $\{(z_i^1, z_i^2), i = 1, ..., n\}$ that satisfies these conditions, there is an allocation in any neighbourhood of $\{(\hat{z}_i^1, \hat{z}_i^2), i = 1, ..., n\}$ that satisfies them, which contradicts our previous result.

Remark 3 *Lemmas 1 and 2 easily extend to allocations where individuals of a given type may randomize between contracts that are equivalent for themselves. An allocation is then a type-dependent randomization over a set of lotteries. Formally, an allocation is defined by a set of lotteries $\{(W_s^1, W_s^2), s = 1, ..., S\}$ and individuals' choices $\sigma \equiv (\sigma_1, \sigma_2, ..., \sigma_n)$ with $\sigma_i = (\sigma_{i1}, ..., \sigma_{iS})$, where σ_{is} is the probability that a type i individual chooses (W_s^1, W_s^2), with $\sum_{s=1}^{S} \sigma_{is} = 1$. In other words, type i individuals get a compound lottery generated by their mixed strategy σ_i over available lotteries $\{(W_s^1, W_s^2), s = 1, ..., S\}$. An allocation is incentive compatible if*

$$\sum_{s=1}^{S} \sigma_{is}[(1 - \pi_i)u(W_s^1) + \pi_i u(W_s^2)] = \max\{(1 - \pi_i)u(W_s^1) + \pi_i u(W_s^2), s = 1, ..., S\},$$

for all $i = 1, ..., n$. In words, an allocation is incentive compatible when individuals only choose their best contract with positive probability. The definition of Problem $\mathbb{P}_i$ for $i = 1, ..., n$ can be extended straightforwardly to this more general setting, with an unchanged definition of u_i^*. In particular, individuals choose only one (non compound) lottery at the optimal solution to $\mathbb{P}_i$, and the MWS lotteries are still an optimal solution

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to $\mathbb{P}_n$. Lemma 1 is thus still valid. Lemma 3 extends Lemma 2 to the case where individuals may randomize between contracts.

Lemma 3 *There does not exist any incentive compatible allocation with randomization* $\{(W_s^1, W_s^2), s = 1, ..., S; \sigma \equiv (\sigma_1, \sigma_2, ..., \sigma_n)\}$ *such that*

$$\sum_{s=1}^{S} \sigma_{\ell_t, s}[(1 - \pi_{\ell_t})u(W_s^1) + \pi_{\ell_t}u(W_s^2)] \geq u_{\ell_t}^* \quad \text{for all } t = 1, ..., T + 1 \qquad (22)$$

and

$$\sum_{h=1}^{n} \lambda_h \{\sum_{s=1}^{S} \sigma_{hs}[W_N - (1 - \pi_h)W_s^1 - \pi_h(W_s^2 + A)]\} > 0. \qquad (23)$$

Proof of Lemma 3

For a given incentive compatible allocation with randomization $\{(W_s^1, W_s^2), s = 1, ..., S; \sigma \equiv (\sigma_1, \sigma_2, ..., \sigma_n)\}$, let $(\overline{W}_h^1, \overline{W}_h^2) = (W_{s(h)}^1, W_{s(h)}^2)$ be one of the the most profitable lotteries which are chosen by type h individuals with positive probability, i.e., $s(h)$ is such that $\sigma_{h, s(h)} > 0$ and

$$(1 - \pi_h)W_{s(h)}^1 + \pi_h W_{s(h)}^2 \leq (1 - \pi_h)W_{s'}^1 + \pi_h W_{s'}^2$$

for all s' such that $\sigma_{h, s'} > 0$. If (22) and (23) hold for the initial allocation with randomization, then (13) and (14) also hold for the non-randomized incentive compatible allocation $\{(\overline{W}_h^1, \overline{W}_h^2), h = 1, ..., n\}$, which contradicts Lemma 2.

Lemma 4 *For any contract offer* $C = (C^1, ..., C^m)$ *made at stage 1, there exists at least one continuation equilibrium* $\sigma(C) = (\sigma_1(C), \sigma_2(C), ..., \sigma_n(C))$ *at stage 2.*

Proof of Lemma 4

Let $C = (C^1, ..., C^m)$ with $C^j = (C_1^j, ..., C_n^j, D^j(.))$ be a contract offer. Consider a discretization of the stage 2 subgame that follows C, with N individuals. Individuals are indexed by $t = 1, ..., N$ and S_i^N is the set of type i individuals, with $\sum_{i=1}^{N} |S_i^N| = N$. In this discretized game, a *pure strategy* of individual t is the choice of a contract in

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C. Let us denote $s_{th}^j = 1$ if individual t chooses C_h^j and $s_{th}^j = 0$ otherwise. The expected utility of a type i who chooses C_h^j is $U_i(C_h^j, X_h^j)$, where $X_h^j = D_h^j(\theta_1^j, P_1^j, ..., \theta_n^j, P_n^j)$, with

$$\theta_h^j = \frac{\sum_{t=1}^N s_{th}^j}{\sum_{t=1}^N \sum_{k=1}^n s_{tk}^j} \quad \text{if} \quad \sum_{t=1}^N \sum_{k=1}^n s_{tk}^j > 0,$$

$$P_h^j = \frac{\sum_{i=1}^n \sum_{t \in S_i^N} s_{th}^j \Pi_i(C_h^j)}{\sum_{t=1}^N s_{th}^j} \quad \text{if} \quad \sum_{t=1}^N s_{th}^j > 0,$$

This discretized subgame is a finite strategic-form game. Consider an $\varepsilon-$perturbation of this game, with $\varepsilon > 0$, where all individuals may play mixed strategy and are required to choose each contract C_h^j with probability larger or equal to ε. This perturbated game is characterized by N and ε and it has a mixed strategy equilibrium, where all type i individuals choose C_h^j with probability $\sigma_{ih}^{j*N}(\varepsilon) \geq \varepsilon$.[22] Let $\sigma_i^{*N}(\varepsilon) = (\sigma_{ih}^{j*N}(\varepsilon))$. Thus, if $t \in S_i^N$, we have

$$E\left[U_i(C_h^j, X_{ht}^{j*N}(\varepsilon) \,|\, \sigma^{*N}(\varepsilon)\right] = \max\left\{E\left[U_i(C_k^j, X_{kt}^{j*N}(\varepsilon) \,|\, \sigma^{*N}(\varepsilon)\right] \text{ for all } j, k\right\}$$
$$\text{if } \sigma_{ih}^{j*N}(\varepsilon) > \varepsilon, \tag{24}$$

where expected value $E\left[.\,|\,\sigma^{*N}(\varepsilon)\right]$ is conditional on the equilibrium mixed strategies played by all individuals except t, and where $X_{ht}^{j*N}(\varepsilon)$ is the equilibrium random policy dividend when all individuals except t play the equilibrium type-dependent mixed strategy $\sigma^{*N}(\varepsilon) = (\sigma_1^{*N}(\varepsilon), ..., \sigma_n^{*N}(\varepsilon))$ and individual t chooses C_h^j.

Consider a sequence of such discretized subgames indexed by $N \in \mathbb{N}$, where ε depends on N, with $\varepsilon \equiv \varepsilon^N > 0$, such that $\left|S_i^N\right|/N \to \lambda_i$ for all i and $\varepsilon^N \to 0$ when $N \to \infty$. The sequence $\{\sigma^{*N} = (..., \sigma_i^{j*N}(\varepsilon^N), ...)\}_{N \in \mathbb{N}}$ is in a compact set, and thus it includes a converging subsequence: $\sigma^{*N} \to \sigma^* = (..., \sigma_{ih}^{j*}, ...)$ with $\sum_{j=1}^m \sum_{h=1}^n \sigma_{ih}^{j*} = 1$ for all i, when $N \to \infty, N \in \mathbb{N}' \subset \mathbb{N}$. Let $\theta_k^{j*N}, P_k^{j*N}$ be the equilibrium proportion of insurer j's policyholders who choose C_k^j and the corresponding equilibrium profit per

---

[22] The payoff functions are such that there is always an equilibrium of the discretized game where individuals of the same type play the same mixed strategy.

policyholder, respectively. The weak law of large numbers yields

$$\theta_h^{j*N} \xrightarrow{P} \frac{\displaystyle\sum_{i=1}^{n} \lambda_i \sigma_{ih}^{j*N}(\varepsilon^N)}{\displaystyle\sum_{i=1}^{n}\sum_{k=1}^{n} \lambda_i \sigma_{ik}^{j*N}(\varepsilon^N)} \equiv \overline{\theta}_h^{j*N},$$

$$P_h^{j*N} \xrightarrow{P} \frac{\displaystyle\sum_{i=1}^{n} \lambda_i \sigma_{ih}^{j*N}(\varepsilon^N)\Pi_i(C_h^j)}{\displaystyle\sum_{i=1}^{n} \lambda_i \sigma_{ih}^{j*N}(\varepsilon^N)} \equiv \overline{P}_h^{j*N},$$

when $N \to \infty$. We have

$$\overline{\theta}_h^{j*N} \to \frac{\displaystyle\sum_{i=1}^{n} \lambda_i \sigma_{ih}^{j*}}{\displaystyle\sum_{i=1}^{n}\sum_{k=1}^{n} \lambda_i \sigma_{ik}^{j*}} \equiv \overline{\theta}_h^{j*} \text{ if } \sum_{i=1}^{n}\sum_{k=1}^{n} \lambda_i \sigma_{ik}^{j*} > 0,$$

$$P_h^{j*N} \to \frac{\displaystyle\sum_{i=1}^{n} \lambda_i \sigma_{ih}^{j*}\Pi_i(C_h^j)}{\displaystyle\sum_{i=1}^{n} \lambda_i \sigma_{ih}^{j*}} \equiv \overline{P}_h^{j*} \text{ if } \sum_{i=1}^{n} \lambda_i \sigma_{ih}^{j*} > 0,$$

when $N \to \infty, N \in \mathbb{N}'$. If $\sum_{i=1}^{n}\sum_{k=1}^{n} \lambda_i \sigma_{ik}^{j*} = 0$, then we have $\overline{\theta}_h^{j*N} \to \overline{\theta}_h^{j*} \geq 0$ and $P_h^{j*N} \to \overline{P}_h^{j*}$, with $\sum_{h=1}^{n} \overline{\theta}_h^{j*} = 1$ and $\overline{P}_h^{j*} \in [\Pi_1(C_h^j), \Pi_n(C_h^j)]$ for all h, when $N \to \infty, N \in \mathbb{N}'$.

We have $\left| X_{ht}^{j*N}(\varepsilon^N) - D_h^j(\theta_1^{j*N}, P_1^{j*N}, ..., \theta_n^{j*N}, P_n^{j*N}) \right| \longrightarrow 0$ for all t when $N \longrightarrow \infty$. Hence, $X_{ht}^{j*N}(\varepsilon^N) \xrightarrow{P} \overline{D}_h^{j*} \equiv D_h^j(\overline{\theta}_1^{j*}, \overline{P}_1^{j*}, ..., \overline{\theta}_n^{j*}, \overline{P}_n^{j*})$ for all t when $N \to \infty, N \in \mathbb{N}'$. Taking the limit of (24), when $N \to \infty, N \in \mathbb{N}'$, then gives

$$U_i(C_h^j, \overline{D}_h^{j*}) = \max\{U_i(C_k^j, \overline{D}_k^{j*}) \text{ for all } j, k\} \text{ if } \sigma_{ih}^{j*} > 0.$$

Using $\sum_{j=1}^{m}\sum_{h=1}^{n} \sigma_{ih}^{j*} = 1$ then yields

$$\sum_{j=1}^{m}\sum_{h=1}^{n} \sigma_{ih}^{j*} U_i(C_h^j, \overline{D}_h^{j*}) = \max\{U_i(C_h^j, \overline{D}_h^{j*}) \text{ for all } j, h\},$$

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which shows that σ^* is an equilibrium of the stage 2 subgame when insurers offer C at stage 1 and policy dividends are $\overline{D}_h^{j*}$.

Proof of Proposition 1

Assume that each insurer offers $\widehat{C} = (\widehat{C}_1, \widehat{C}_2, ..., \widehat{C}_n, D^*(.))$ such that (15)-(17) hold. Then $\widehat{C}_i$ is an optimal choice of type i individuals if no policy dividend is paid on any contract. (16) shows that this is actually the case when all individuals are evenly shared among insurers.

Suppose some insurer j_0 deviates from $\widehat{C}$ to another menu $C^{j_0} = \{C_1^{j_0}, C_2^{j_0}, ..., C_n^{j_0}, D^{j_0}(.)\}$ with $C_i^{j_0} = (k_i^{j_0}, x_i^{j_0})$. Let $\widetilde{\sigma}(C^{j_0}, \widehat{C}^{-j_0})$ be a continuation equilibrium following the deviation, i.e., equilibrium contract choices by individuals in the subgame where C^{j_0} and $\widehat{C}$ are simultaneously offered, respectively by insurer j_0 and by all the other insurers $j \neq j_0$. Lemma 4 shows that such a continuation equilibrium exists. Let us restrict the definition of this subgame by imposing $\widetilde{\sigma}_{i-1,i}^j = 0$ for all $i \notin I, j \neq j_0$. From (17), type $i - 1$ individuals weakly prefer $\widehat{C}_{i-1}$ to $\widehat{C}_i$ if $i \notin I$, so that any equilibrium of the restricted game is also an equilibrium of the original game. Let $\overline{P}_h^j$ be the profit per policyholder made by insurer $j \neq j_0$ on contract $\widehat{C}_h$ and $\overline{\theta}_h^j$ be the proportion of insurer j' s customers who choose $\widehat{C}_h$, after the deviation by insurer j_0. Consider a continuation equilibrium where individuals of a given type are evenly shared between insurers $j \neq j_0$, i.e., where $\widetilde{\sigma}_{ih}^j(C^{j_0}, \widehat{C}^{-j_0}) = \widetilde{\sigma}_{ih}^{j'}(C^{j_0}, \widehat{C}^{-j_0})$ for all h if $j \neq j', j, j' \neq j_0$ [23]. We may then use more compact notations $\widetilde{\sigma}_{ih}^0 \equiv \widetilde{\sigma}_{ih}^{j_0}(C^{j_0}, \widehat{C}^{-j_0})$ and $\widetilde{\sigma}_{ih}^1 \equiv \widetilde{\sigma}_{ih}^j(C^{j_0}, \widehat{C}^{-j_0}), \overline{P}_h^1 = \overline{P}_h^j, \overline{N}_h^1 = \overline{N}_h^j$ for all $j \neq j_0$. Let also $\overline{P}_h^0$ and $\overline{\theta}_h^0$ be, respectively, the average profit made on $C_h^{j_0}$ and the proportion of the customers of insurer j_0 who choose $C_h^{j_0}$.

After the deviation by insurer j_0, type i individuals get the following lottery on

---

[23] Such a continuation equilibrium exists because it is a Nash equilibrium of an equivalent game with only two insurers that respectively offer $\widehat{C}^{-j_0}$ and C^{j_0}. Note that this equivalence is possible because $D_h^j(.)$ is homogeneous of degree 1 with respect to $(N_1^j, ..., N_n^j)$.

final wealth:

$$
\begin{aligned}
(W_{0h}^1, W_{0h}^2) &\equiv (W_N - k_h^{j_0} + \overline{D}_h^0, W_A + x_h^{j_0} + \overline{D}_h^0) \text{ with probability } \widetilde{\sigma}_{ih}^0, \\
(W_{1h}^1, W_{1h}^2) &\equiv (\widehat{W}_h^1 + \overline{D}_h^1, \widehat{W}_h^2 + \overline{D}_h^1) \text{ with probability } \widetilde{\sigma}_{ih}^1 (n-1),
\end{aligned}
$$

where

$$
\begin{aligned}
\overline{D}_h^0 &= D_h^{j_0}(\overline{\theta}_1^0, \overline{P}_1^0, ..., \overline{\theta}_n^0, \overline{P}_n^0), \\
\overline{D}_h^1 &= D_h^*(\overline{\theta}_1^1, \overline{P}_1^1, ..., \overline{\theta}_n^1, \overline{P}_n^1),
\end{aligned}
$$

for $h = 1, ..., n$, with $\sum_{h=1}^n [\widetilde{\sigma}_{ih}^0 + \widetilde{\sigma}_{ih}^1(n-1)] = 1$. Let us denote this lottery by $\mathcal{L}$. Let Δ denote the residual profit made by insurer j_0. We have

$$
\Delta = \sum_{i=1}^n \lambda_i \{ \sum_{h=1}^n \widetilde{\sigma}_{ih}^0 [W_N - (1 - \pi_i) W_{0h}^1 - \pi_i (W_{0h}^2 + A)] \}. \tag{25}
$$

We know from (15) that $D^*(.)$ involves the full distribution of profits made by non-deviant insurers on the set of contracts $\{\widehat{C}_i, i \in I\}$. Furthermore, we have $\widetilde{\sigma}_{hi}^1 = 0$ if $h < i - 1$ when $i \notin I$, because types h strongly prefer $\widehat{C}_{i-1}$ to $\widehat{C}_i$ for all $h < i - 1$.[24] Thus we have $\widetilde{\sigma}_{hi}^1 = 0$ if $h \leq i$ when $i \notin I$, and consequently the profit made on $\widehat{C}_i$ by non-deviant insurers is non-negative when $i \notin I$. We deduce that non-deviant insurers j make non-negative residual profit. We thus have

$$
\sum_{i=1}^n \lambda_i \{ \sum_{h=1}^n \widetilde{\sigma}_{ih}^1 [W_N - (1 - \pi_i) W_{1h}^1 - \pi_i (W_{1h}^2 + A)] \} \geq 0. \tag{26}
$$

(25) and (26) then yield

$$
\begin{aligned}
\Delta \leq &\sum_{i=1}^n \lambda_i \{ \sum_{h=1}^n \widetilde{\sigma}_{ih}^0 [W_N - (1 - \pi_i) W_{0h}^1 - \pi_i (W_{0h}^2 + A)] \\
&+ (n-1) \sum_{h=1}^n \widetilde{\sigma}_{ih}^1 [W_N - (1 - \pi_i) W_{1h}^1 - \pi_i (W_{1h}^2 + A)] \}. \tag{27}
\end{aligned}
$$

---

[24] Note that we here use $D_i^* \equiv 0$ and $D_{i-1}^* \equiv 0$ when $i \notin I$, which follows from (17).

Furthermore, we have

$$\sum_{h=1}^{n} \widetilde{\sigma}_{\ell_t,h}^0[(1-\pi_{\ell_t})u(W_{0h}^1) + \pi_{\ell_\theta}u(W_{0h}^2)]$$

$$+(n-1)\sum_{h=1}^{n} \widetilde{\sigma}_{\ell_t,h}^1[(1-\pi_{\ell_t})u(W_{1h}^1) + \pi_{\ell_t}u(W_{1h}^2)$$

$$\geq u_{\ell_t}^* \text{ for all } t = 1, ..., T+1 \tag{28}$$

because $(W_{1\ell_t}^1, W_{1\ell_t}^2) = (\widehat{W}_{1\ell_t}^1, \widehat{W}_{1\ell_t}^2)$ since $\overline{D}_{\ell_t}^1 = 0$ from (17), and $(1-\pi_{\ell_t})u(\widehat{W}_{1\ell_t}^1) + \pi_{\ell_t}u(\widehat{W}_{1\ell_t}^2) = u_{\ell_t}^*$, and $\{\widetilde{\sigma}_{\ell_t,h}^0, \widetilde{\sigma}_{\ell_t,h}^1, h = 1, ..., n\}$ is an optimal contract choice strategy of type ℓ_t individuals. The right-hand side of (27) is the expected profit associated with $\mathcal{L}$. Lemma 3 applied to lottery $\mathcal{L}$ then gives $\Delta \leq 0$. Hence the deviation is non-profitable, which completes the proof.

Proof of Proposition 2

In the proof of Proposition 1, it has been shown that the MWS allocation is sustained by a market equilibrium where stage 1 deviations are non-profitable at all continuation equilibrium. Hence this equilibrium allocation is robust.

Let $\{(\widetilde{W}_i^1, \widetilde{W}_i^2), i = 1, ..., n\}$ be an equilibrium allocation that differs from the MWS allocation, with expected utility $\widetilde{u}_i$ for type i. This allocation satisfies incentive compatibility constraints (7) for all $h = 1, ..., n-1$, and it is sustained by a symmetric Nash equilibrium of the market game with m_a active insurers $(m_a \leq m)$ where each active insurer offers $\widetilde{C} = (\widetilde{C}_1, \widetilde{C}_2, ..., \widetilde{C}_n, \widetilde{D}(.))$, with $\widetilde{D}(.) = (\widetilde{D}_1(.), \widetilde{D}_2(.), ..., \widetilde{D}_n(.))$. At such an equilibrium, insurers make non-negative residual profit, for otherwise they would deviate to a "zero contract". Hence $\{(\widetilde{W}_i^1, \widetilde{W}_i^2), i = 1, ..., n\}$ satisfies (8) for $i = n$, rewritten as a weak inequality (with sign $\leq$). Since $\{(\widetilde{W}_i^1, \widetilde{W}_i^2), i = 1, ..., n\}$ satisfies (7) and (8) for $i = n$ and it is not an optimal solution to $\mathbb{P}_n$, we deduce that there is i_0 in $\{1, ..., n\}$ such that $\widetilde{u}_i \geq u_i^*$ if $i < i_0$ and $\widetilde{u}_{i_0} < u_{i_0}^*$. Thus, there exists an allocation $\{(W_i^1, W_i^2), i = 1, ..., i_0\}$ in the neighbourhood of the optimal solution to $\mathbb{P}_{i_0}$, with expected utility u_i for type i, that satisfies (6) and (7) as strong inequalities and (8) rewritten as a strong inequality (with sign $<$) for $i = i_0$. Let

$k_i = W_N - W_i^1$ and $x_i = W_i^2 - W_A$ for $i \leq i_0$. Let j_0 be some insurer that belongs to the set of inactive insurers if $m_a = 1$ and that may be active or inactive if $m_a > 1$. Suppose insurer j_0 deviates from $\widetilde{C}$ to $C^{j_0} = \{C_1^{j_0}, C_2^{j_0}, ..., C_n^{j_0}, D^{j_0}(.)\}$ with $D^{j_0}(.) = (D_1^{j_0}(.), D_2^{j_0}(.), ..., D_n^{j_0}(.))$, where $C_i^{j_0} = (k_i, x_i)$ if $i \leq i_0$, $C_i^{j_0} = (0,0)$ if $i > i_0$ and

$$D_i^{j_0}(N_1^{j_0}, P_1^{j_0}, ..., N_n^{j_0}, P_n^{j_0}) = \begin{cases} 0 & \text{if } \sum_{h=1}^{i_0} N_h^{j_0} P_h^{j_0} > 0 \\ -K & \text{if } \sum_{h=1}^{i_0} N_h^{j_0} P_h^{j_0} \leq 0 \end{cases} \quad \text{if } i \leq i_0,$$

$$D_i^{j_0}(N_1^{j_0}, P_1^{j_0}, ..., N_n^{j_0}, P_n^{j_0}) \equiv 0 \text{ if } i > i_0,$$

with $K > 0$. For K large enough, insurer j_0 makes positive profit at any continuation equilibrium after the deviation to C^{j_0} where it attracts some individuals. This is the case when all type i_0 individuals choose $C_{i_0}^{j_0}$ and reach expected utility u_{i_0} (with $u_{i_0} \geq u_{i_0}^* > \widetilde{u}_{i_0}$) and possibly other individuals choose a contract in C^{j_0}. Thus, any market equilibrium where insurer j_0 does not attract some individuals after deviating from $\widetilde{C}$ to C^{j_0} is not based on robust beliefs. We deduce that $\{(\widetilde{W}_i^1, \widetilde{W}_i^2), i = 1, ..., n\}$ is not a robust equilibrium allocation.

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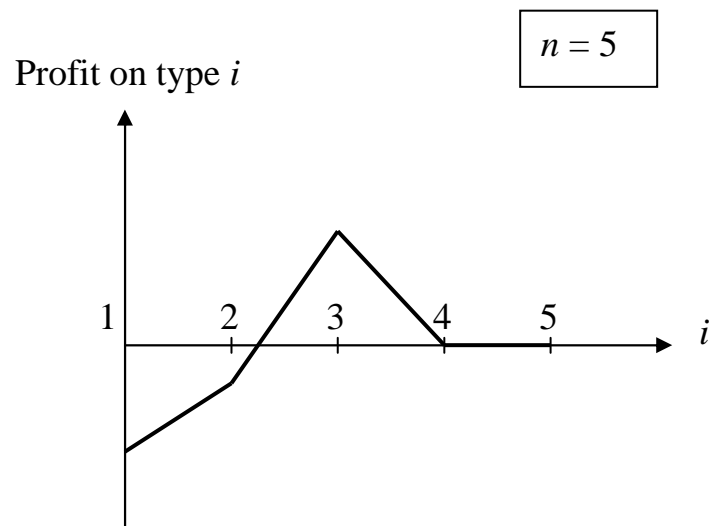
Figure 1