Effects of the local resonance on the dynamic behavior of periodic frame structures

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ABSTRACT: This work investigates the dynamic behaviour of structures obtained by repeating a unit cell made up of interconnected beams or plates forming an unbraced frame. Such structures can represent an idealization of numerous reticulated systems, for example the microstructure of foams, plants, bones, the sandwich panels, stiffened plates and truss beams used in aerospace and marine structures, buildings. As beams are much stiffer in tension-compression than in bending, the propagation of compressional waves with wavelengths much greater than the cell size and the bending modes of the elements can occur in the same frequency range. Thus frame structures can behave as metamaterials and exhibit unusual dynamic properties.

Since the condition of scale separation is respected for the compressional waves, the homogenization method of periodic discrete media is used to rigorously derive the macroscopic behaviour at the leading order. The main advantages of the method are the analytical formulation and the possibility to study the behaviour of the elements at the local scale. This provides a clear understanding of the mechanisms governing the dynamics of the material.

In the presence of the local resonance, the form of the equations is unchanged but some macroscopic parameters depend on the frequency. In particular, this applies to the mass leading to a generalization of the Newtonian mechanics. As a result, the longitudinal modes of periodic frame beams and the transfer function have unusual properties. The same macroscopic modal shape is associated with several resonant frequencies (but the deformation of the elements at the local scale is different). These atypical behaviours are first established theoretically and then, they are confirmed by numerical simulations.

KEY WORDS: Local resonance; Reticulated material; Meta-material; Homogenization.

1 INTRODUCTION

Locally resonant metamaterials are a class of composite materials with a high rigidity contrast between the constituents. The propagation of waves in the stiff component with a wavelength much greater than the heterogeneity size can then induce the resonance of the soft component. This phenomenon which differs from diffraction leads to unusual effective properties investigated in the pioneering work [1] (see also [2]) and observed experimentally in [3] and [4]. In particular, the effective density is different from the real density and depends on the frequency. The description of such composites at the macroscopic scale is a generalization of the Newtonian mechanics [5]. In [6] it was shown that reticulated materials with only one constituent can also behave as locally resonant metamaterials. Reticulated materials are made up of interconnected beams or plates. Examples include the microstructure of foams, plants, bones, the sandwich panels, stiffened plates and truss beams used in aerospace and marine structures. Since beams and plates are much stiffer in tension-compression than in bending, the propagation of compressional waves and the bending modes of the elements can occur in the same frequency range.

In this paper, we investigate the consequences of the local resonance in bending on the dynamic behaviour of periodic frame structures which can represent idealised buildings. Instead of considering wave propagation as in [6], emphasis is put on the modification of the features of the longitudinal modes. For the first modes of a structure with a sufficiently large number of frames, deformations occur on a lengthscale much greater than the size of a frame. Therefore we can use the homogenization method of periodic discrete media (HPDM method) to obtain a macroscopic description. This method, elaborated by Caillerie [7] has been extended to situations with local resonance [6]. Its main advantage is that the macroscopic behaviour is derived rigorously from the properties of the basic frame. Moreover the analytic formulation enables to understand the role of each parameter. This method has already given interesting results on the transverse dynamics of frame structures [8].

The principles of the HPDM method and the studied structures are described in Section 2. Section 3 presents the two possible macroscopic behaviours: without and with local resonance. In Section 4, the consequences of the local resonance on the modal properties and the transfer function of the structure are analysed. The results are confirmed by finite element simulations.

2 STUDIED STRUCTURES AND OVERVIEW OF THE HPDM METHOD

The studied structures are constituted by a pile of a large number $N$ of identical unbraced frames called cells and made of a floor supported by two walls (see Figure 1). The walls and the floors are beams or plates which behave as Euler-Bernoulli beams in out-of-plane motion. The parameters of floors ($i = f$) and walls ($i = w$) are: length $l_i$, thickness $a_i$, cross-section area $A_i$, second moment of area $I_i = a_i^4 h/12$ in direction $e_i$, density $\rho_i$, elastic modulus $E_i$, Poisson ratio $\nu_i$. As the connections are assumed perfectly rigid, the motions of
each endpoint connected to the same node are identical and define the discrete nodal kinematic variables of the system.

![Diagram](image)

Figure 1. Left: class of studied structures, right: longitudinal kinematics.

The analysis of such a periodic lattice of interconnected beams with the homogenization method of periodic discrete media (HPDM) is performed in two steps [9]: first, the discretization of the balance of the structure under harmonic vibrations; second, the homogenization, leading to a continuous model elaborated from the discrete description. As the studied structures are symmetric, a change of variables is added to uncouple the transverse and longitudinal kinematics. An outline of this method and its adaptation to situations with local resonance is given hereafter.

The discretization consists in integrating the dynamic balance (in harmonic regime) of the beams, the unknown displacements and rotations at their extremities being taken as boundary conditions. The forces applied by an element on its extremities are then expressed as functions of the nodal kinematic variables. The balance of each element being satisfied, it remains to express the balance of the forces applied to the nodes. Thus, the balance of the whole structure is rigorously reduced to the balance of the nodes.

The kinematics is characterized at any level \( n \) by the motions of the two nodes in the plane \((e_1, e_2)\), i.e., the displacements in the two directions and the rotation \((u_n, u_{n+1}, \theta)\). These six variables can be replaced by (see Figure 1):

- Three variables associated to the rigid body motion of the level \( n \): the mean displacements, \( U(n) \) along \( e_1 \) and \( V(n) \) along \( e_2 \), and the global rotation \( \alpha(n) \) (differential vertical nodal motion divided by \( \ell(n) \)).

- Three variables corresponding to its deformation: the mean and differential rotations of the nodes, \( \theta(n) \) and \( \Phi(n) \), and the transverse dilatation \( \Delta(n) \).

As mentioned before, the transverse and longitudinal kinematics, respectively governed by \((U, \alpha, \theta)\) and \((V, \Phi, \Delta)\), are uncoupled. This paper focuses on the longitudinal vibrations. The study of the transverse vibrations can be found in [10,8].

Then the principles of homogenization are used to derive the differential equation describing the behaviour of the equivalent beam. The key assumption is that the cell size in the direction of periodicity \( \ell(n) \) is small compared to the characteristic size \( L \) of the vibrations of the structure. Thus, the scale ratio is small: \( \varepsilon = \ell(n)/L \ll 1 \). The condition of scale separation implies that the method is limited to the first modes of vibration which have wavelengths that are much larger than the cell size. The existence of a macroscopic scale is expressed by means of macroscopic space variable \( x \). The unknowns are continuous functions of \( x \) coinciding with the discrete variables at any level, e.g. \( V(x) = V(x_n) = F(\text{level } n) \). These quantities, assumed to converge when \( \varepsilon \) approaches zero, are expanded in powers of \( \varepsilon \): \( V(x) = V(x) + \varepsilon V(x) + \varepsilon^2 V(x) + \ldots \). Similarly, all other unknowns, including the modal frequency, are expanded in powers of \( \varepsilon \). As \( \ell(n) \approx L \) is a small increment with respect to \( x \), the variations of the variables between neighbouring nodes are expressed using Taylor’s series; this in turn introduces the macroscopic derivatives.

To account properly for the local physics, the geometrical and mechanical characteristics of the elements are scaled according to the powers of \( \varepsilon \). As for the modal frequency, scaling is imposed by the balance of elastic and inertia forces at the macroscopic level. This scaling ensures that each mechanical effect appears at the same order whatever the \( \varepsilon \) value is. Therefore, the same physics is kept when \( \varepsilon \) approaches zero, i.e. for the homogenized model. Finally, the expansions in \( \varepsilon \) powers are introduced in the nodal balances. Those relations, valid for any small \( \varepsilon \), lead for each \( \varepsilon \)-order to balance equations which describe the macroscopic behaviour.

In general the scale separation requires wavelengths of the compression and bending vibrations generated in each local element to be much longer than the element length at the modal frequency of the global system. In that case the nodal forces can be expanded in Taylor’s series with respect to \( \varepsilon \). This situation corresponds to a quasi-static state at the local scale. Nevertheless, at higher frequencies, it may occur that only the compression wavelength is much longer than the length of the elements while local resonance in bending appears. The expansion of the shear force and the bending moment is no longer possible. However, the homogenization remains possible through the expansion of the compression forces and leads to atypical descriptions with inner dynamics. Above this frequency range, the local resonance in both compression and bending makes impossible the homogenization process.

3 LONGITUDINAL VIBRATIONS

3.1 Quasi-static state at the local scale

We first illustrate the classical homogenization by considering a structure as in Figure 1 with thick enough elements to have a quasi-static state at the local scale. The walls and the floors are made of the same material. Their geometrical characteristics and the order of magnitude of the circular frequencies \( \omega \) corresponding to the longitudinal vibrations are given below.

\[
\frac{a_w}{\ell} = O(\sqrt{\varepsilon}) ; \quad \frac{a_f}{\ell} = O(\varepsilon) ; \quad \ell_w = O(1) ; \quad \omega = O\left(\frac{1}{L \sqrt{\rho_w}}\right)
\]

(1)

In that case, the leading order equations obtained by homogenization are:
\[
\Lambda \omega_b^2 V^0(x) + 2E_w A_w V^0/ (x) = 0 \quad (2a)
\]
\[
(K_f + 3K_w) \Phi^0(x) - \Lambda f E_f \omega_b^2 V^0(x) = 0 \quad (2b)
\]
\[
12E_f A_f V(x) - 3K_w \Phi^0(x) = 0 \quad (2c)
\]
with
\[
\Lambda = \Lambda_w + \Lambda_f ; \quad \Lambda_w = 2 \rho_w A_w ; \quad \Lambda_f = \rho_f A_f E_f / \ell_w
\]
\[
K_w = 24 E_w I_w / \ell_w^3 ; \quad K_f = 12 E_f I_f / \ell_w \ell_f
\]

Equation (2a) expresses the balance of vertical forces while Equation (2b) and (2c) come from the balance of differential moments and of differential horizontal forces. The macroscopic behaviour is given by Equation (2a) which corresponds to the classical description of a beam in tension-compression with the compression modulus of the two walls \( 2 E_w A_w \) and the linear density \( \Lambda \). Once the mean vertical displacement \( V \) is known, the "hidden" variables \( \Phi \) and \( \Lambda \) are determined thanks to Equations (2b) and (2c) which describe the inner equilibrium of the cell. Note that \( \Phi \) and \( \Lambda \) depend on the rigidity of the elements in bending \( K_w \) and \( K_f \).

A systematic study in which the properties of the elements vary shows that the longitudinal vibrations with a quasi-static state at the local scale are always described by Equation (2a) [10].

### 3.2 Local resonance

To investigate the effects of the bending resonance of the floors, we now consider a structure with floors thinner than walls.

\[
a_w / \ell_w = O(\sqrt{\varepsilon}) ; \quad a_f / \ell_w = O(\varepsilon) ; \quad \ell_w / \ell_f = O(1) ;
\]
\[
\omega = O \left( \frac{1}{L \sqrt{\varepsilon}} \right)
\]

As the order of magnitude of \( a_w / a_1 \) is not a whole power of \( \varepsilon \), the unknowns should now be expanded in powers of \( \varepsilon^{1/2} \).

Moreover, because of the local resonance, the shear and bending moment in the floors cannot be expanded in Taylor's series. Thus the homogenization provides the following equations for the first two significant orders of the balance of vertical forces. The main difference with Section 3.1 is the multiplication of \( A_f \) by a function \( f \) depending on the frequency.

\[
\Lambda_w \omega_b^2 V^0(x) + 2E_w A_w V^0/ (x) = 0 \quad (4a)
\]
\[
\Lambda_w \omega_b^2 V^{1/2}(x) + 2E_w A_w V^{1/2} (x) = 0 \quad (4b)
\]

Because of the thickness contrast between the walls and the floors, \( A_f \) is negligible compared to \( A_w \). This is the reason why we obtain a degenerate equation at the leading order. At most of the frequencies \( f(\omega_0) = O(1) \) and Equation (4a) is sufficient for the description of the macroscopic behaviour. However, we will see that \( f(\omega_0) \) can become infinite. In that case, the inertial term related to the floors is no longer negligible. To build a macroscopic description valid for the whole frequency range of the longitudinal vibrations we take \( f(x) = f^0(x) + \varepsilon^{1/2} f^1(x) \), \( f_0 = \omega_0 + \varepsilon^{1/2} \omega_1 \) and we sum the Equations (4).

\[
\Lambda(\omega) \omega^2 V(x) + 2E_w A_w V^0/ (x) = 0 \quad (5a)
\]
\[
\omega = \omega_0 + \varepsilon^{1/2} \omega_1 \quad (5b)
\]

Equation (5a) looks like Equation (2a) but differs fundamentally by the effective mass \( \Lambda(\omega) \) which depends on the frequency. The HPDM method provides an analytical expression of the function \( f \). Its variations according to the frequency are plotted in Figure 2. It shows that \( f(\omega) \to 1 \) when \( \omega \to 0 \) as expected and that \( \int f(\omega) d\omega \to +\infty \) when \( \omega \to \omega_f \), where \( \omega_f \) is the circular frequencies of the odd normal modes of the floors with two fixed ends in bending. At most of the circular frequencies higher than \( \omega_f \), we have \( f(\omega) < 1 \), which means that the structure seems lighter thanks to the local resonance. Note also that \( f(\omega) \) can be negative.

Figure 2. Variations of the function \( f(\omega) \) according to the nondimensional frequency \( \omega / \omega_f \). As the resonance frequencies of the Euler-Bernoulli beams are proportional to the sequence of the squares of the odd integers, the modes of the floors correspond to the following abscissas: \( \omega_f / \omega_f = 1, \omega_f / \omega_f = 5^2 / 3^2 = 2.78, \omega_f / \omega_f = 7^2 / 3^2 = 5.44 \), etc.

The effective mass differs so much from the real mass because the points of the cell are in relative motion. According to the definition of the macroscopic variables, \( V(x) \) describes the mean vertical displacement of the nodes. At low frequencies, the whole cell undergoes the same translational motion. Consequently, the sum of inertia forces acting on the whole frame equals the real mass of the frame multiplied by the acceleration of the nodes. When bending resonance occurs, the motion of the points of the floors can strongly differ from the one of the nodes and some points can even be in antiphase. In these conditions, the sum of inertia forces acting on the basic frame is modified.
This analysis of the physical origin of the effective mass is confirmed by the consistency between the deformation of the floors and the variations of the function $f$. As $\omega$ approaches $\omega_f$, the deflection is getting larger and larger because of the resonance. It is in-phase with the nodes when $\omega$ is below $\omega_f$ and in antiphase when $\omega$ is above $\omega_f$. At the frequency of the second bending mode $\omega_{f2}$, the in-phase motion of the two walls does not cause the resonance of the floors. Nevertheless, the motion is not uniform, which induces an effective mass smaller than the real mass.

The previous study focuses on the macroscopic description but the inner equilibrium of the cell is also affected by the local resonance. These equations contain inertial terms and they depend on the rigidity in bending of the elements. For the floors, the effective mass and the effective rigidity become infinite for the frequencies of the odd bending modes. If the walls are in resonance, their effective mass becomes infinite for the frequencies of the odd bending modes but the rigidity becomes infinite for the frequencies of the even bending modes. The modes of the whole cell can also be excited. In the neighbourhood of all these frequencies, the structure is likely to behave atypically.

4 SOME CONSEQUENCES OF THE LOCAL RESONANCE

This section presents the consequences of the local resonance and the variations of the effective mass on the dynamic behaviour of a given structure by considering two problems: the modal analysis and the response to an imposed harmonic motion at the bottom. In the latter case, damping is introduced. The studied structure has been specially designed to highlight the effects of the local resonance. It is a frame structure as in Figure 1 with $N = 15$ levels. The walls and the floors have the same length $L_w = L_f = 3 \, \text{m}$ and the thickness to length ratios correspond to the orders of magnitude of Section 3.2. In [10], it was shown that, for the first macroscopic mode of a structure, the scale ratio can be estimated by $\varepsilon = \pi/(2N)$. The walls are made of concrete but the density of the floors is increased in order to have $A_f = A_w$ and to increase the influence of their resonance. The chosen characteristics are summarized in Equation (6).

\[
\begin{align*}
L_w &= L_f = 3 \, \text{m}, \quad h_w = h_f = 1 \, \text{m} \\
a_w &= 0.971 \, \text{m}, \quad a_f = 0.314 \, \text{m} \\
E_w &= E_f = 30000 \, \text{MPa}, \quad v_w = v_f = 0.2, \\
\rho_w &= 2300 \, \text{kg/m}^3, \quad \rho_f = 14225 \, \text{kg/m}^3
\end{align*}
\]  

(6)

The resonance frequencies of the floors are $f_{f1} = 52.08 \, \text{Hz}$, $f_{f2} = 143.56 \, \text{Hz}$, $f_{f3} = 281.44 \, \text{Hz}$.

4.1 Modal analysis

For a structure fixed at the bottom and free at the top, the solution of Equation (5a) is written below.

\[
V(x) = B \sin(\alpha x) \quad \text{with} \quad \alpha = \frac{\Lambda(\omega) \omega^2}{2E_w A_w} \left(\frac{2H}{\pi}\right)^2 = (2k-1)^2
\]  

Thus the resonance frequencies are given by Equation (8).

\[
\alpha_k H = \frac{(2k - 1)\pi}{2} \Rightarrow \frac{\Lambda(\omega) \omega^2}{2E_w A_w} \left(\frac{2H}{\pi}\right)^2 = (2k - 1)^2
\]  

In the absence of local resonance, the effective mass is constant and there is only one solution $\omega_k$ for each value of $k$. For the considered structure, this approach leads to the frequencies given in the second column of Table 1. As these frequencies are in the same range as the resonance frequencies of the floors, we cannot neglect the variations of the effective mass. Then Figure 3 shows that it has two important consequences. First, the values of the resonance frequencies of the structure are modified. Second, there are several solutions $\omega_k$ for each value of $k$. This means that the structure has the same macroscopic modal shape for several frequencies. However, at the local scale, the deformation of the floors is different. Note also that, because of the great variations of the effective mass in the neighbourhood of the odd natural frequencies of the floors, there is a solution close to these frequencies for each value of $k$. Consequently there is a large number of modes in a small frequency range. Just after the resonance of the floors, the effective mass is negative and there is a frequency range with no longitudinal modes. Usually the condition of scale separation limits homogenization to low frequencies. Since several modes of the studied structure can have the same wavelength, some high frequency modes can be correctly described by homogenization. Here, the scale ratio $\varepsilon$ does not monotonically increase with the frequency and there is an alternation of frequencies at which homogenization applies and frequencies at which homogenization does not apply.

\[\Lambda(\omega) \omega^2 \left(\frac{2H}{\pi}\right)^2\]

Figure 3. The thin red line is obtained by replacing the effective mass by the real mass. The thick blue line takes into account the resonance of the floors. The horizontal dashed lines indicate the first values of $(2k \cdot 1)^2$. The resonance frequencies of the structure are the abcissas of the intersections of the continuous curves with the dashed lines.

All these results are confirmed numerically with the finite element code CESAR-LCPC. The modes are determined for two structures. For the first one, the elements behave as Euler-Bernoulli beams as in the HPDM method. For the second one, the elements behave as Timoshenko beams which
is more realistic for the chosen thickness. The resonance frequencies calculated with the finite element method and Equation (8) are given in Table 1. There is an excellent agreement between the two approaches. The use of Timoshenko beams slightly modifies the frequencies but the modal shapes are identical. The modal shapes are presented in Figure 4, and they are compared with Equation (7) in Figure 5.

### Table 1. Comparison of the resonance frequencies (Hz) of the structure estimated thanks to the homogenized models and with the finite element code CESAR-LCPC.

<table>
<thead>
<tr>
<th>k</th>
<th>Resonance of the floors neglected (Equation (2a))</th>
<th>Resonance of the floors taken into account (Equation (8))</th>
<th>CESAR-LCPC with Euler-Bernoulli beams</th>
<th>CESAR-LCPC with Timoshenko beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.19</td>
<td>14.00 - 65.14 - 297.0</td>
<td>13.76 - 64.83</td>
<td>13.73 - 61.35</td>
</tr>
<tr>
<td>2</td>
<td>42.56</td>
<td>36.73 - 74.35 - 297.5</td>
<td>35.99 - 74.22</td>
<td>35.12 - 71.71</td>
</tr>
<tr>
<td>3</td>
<td>70.94</td>
<td>46.42 - 97.68 - 298.5</td>
<td>45.46 - 97.37</td>
<td>43.32 - 96.04</td>
</tr>
<tr>
<td>4</td>
<td>99.31</td>
<td>49.36 - 127.8 - 300.4</td>
<td>48.28 - 127.0</td>
<td>45.66 - 125.5</td>
</tr>
<tr>
<td>5</td>
<td>127.69</td>
<td>50.49 - 158.9 - 303.4</td>
<td>49.76 - 157.5</td>
<td>46.93 - 154.2</td>
</tr>
</tbody>
</table>

Figure 4. Modal shapes and resonance frequencies (Hz) calculated with the finite element code CESAR-LCPC.

![Figure 4](image-url)

Figure 5. Comparison of the vertical displacements estimated with the homogenized model (Equation (7)) and the finite element code CESAR-LCPC.

![Figure 5](image-url)
4.2 Harmonic motion at the bottom

A harmonic vertical motion of amplitude $V_0$ is now imposed at the bottom of the structure. We are interested in the evolution with the frequency of the amplitude of the displacement at the top. The solution of the homogenized model (Equation (5a)) depends on the sign of the effective mass as indicated in Equations (9).

$$\Lambda(\omega) \geq 0$$

$$\Rightarrow V(x) = V_0 \left[ \cos(\alpha x) + \tan(\alpha H) \sin(\alpha x) \right]$$  \hspace{1cm} (9a)

$$\Rightarrow H(\omega) = \frac{V(H)}{V_0} = \frac{1}{\cos(\alpha H)}$$

$$\Lambda(\omega) < 0$$

$$\Rightarrow V(x) = V_0 \left[ \cosh(\alpha x) - \tanh(\alpha H) \sinh(\alpha x) \right]$$  \hspace{1cm} (9b)

$$\Rightarrow H(\omega) = \frac{1}{\cosh(\alpha H)}$$

with $\alpha^2 = \frac{|\Lambda(\omega)| \omega^2}{2E_w A_w}$

The variations of the transfer function $H(\omega)$ according to the frequency are plotted in Figure 6. As expected, there are peaks at the natural frequencies of the structure determined in Section 4.1. Just before the first resonance of the floors at 52 Hz, the peaks are very numerous because of the great variations of the effective mass. However, a lot of them does not correspond to real modes of the structure. The homogenization process replaces the structure by an equivalent beam which has an infinite number of degrees of freedom and therefore a infinite number of longitudinal modes. On the contrary, the studied structure has only 15 possible macroscopic modal shapes. Just after the resonance of the floors, the effective mass is negative and we observe a bandgap, that is to say a frequency range with no motion at the top of the structure.

All these results are confirmed numerically. The transfer function computed with the finite element method is almost identical (Figure 6). As explained above, there are less peaks before the resonance of the floors and the bandgap begins at slightly lower frequencies. An example of the deformation of the structure inside the bandgap is also presented in Figure 6. The first floors experience large deformations because of the resonance but the energy is not transferred to the upper storeys. From the fourth floor, there is no more vibration.

The introduction of damping does not modify fundamentally the results. We use a complex elastic modulus $E_f = E_w = E e^{i\eta}$ in the homogenized model (Equations (9)). The modulus of the transfer function is plotted in Figure 7 for $\eta = 2.10^{-2}$. The main difference with Figure 6 is that the frequency bandgap is larger. As a result, the peaks of the transfer function before the resonance no longer exist. Finite element simulations are still in good agreement with the transfer function of the homogenized model (Figure 7).
5 CONCLUSION

Because of the rigidity contrast between tension-compression and bending in beams and plates, the longitudinal modes of a reticulated structure can appear in the same frequency range as the bending modes of the elements. The extension of the HPDM method to these situations with local resonance shows that the real mass should be replaced by an effective mass which depends on the frequency. When the frequency approaches the odd natural frequencies of the resonating elements in bending, the effective mass becomes infinite and it changes its sign. This phenomenon has two main consequences. First, several normal modes of the structure associated to different natural frequencies can have the same macroscopic modal shapes. Second, when the effective mass is negative, the vibrations do not propagate inside the structure and we have frequency bandgaps.

These theoretical results are confirmed by finite element simulations. The studied structure has been specially designed to highlight the effects of the local resonance. In particular, mass has been added to the floors. The increase of the length of the floors would have been another possibility. If the proportion of the mass affected by the resonance is smaller, the variations of the effective mass far from the resonance frequencies can be negligible. However, at the resonance, the effective mass is still considerable and the vibrations are attenuated.

REFERENCES
