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A REVIEW OF NUMERICAL MODELLING OF WAVE ENERGY CONVERTER ARRAYS

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ABSTRACT

Large-scale commercial exploitation of wave energy is certain to require the deployment of wave energy converters (WECs) in arrays, creating 'WEC farms'. An understanding of the hydrodynamic interactions in such arrays is essential for determining optimum layouts of WECs, as well as calculating the area of ocean that the farms will require. It is equally important to consider the potential impact of wave farms on the local and distal wave climates and coastal processes; a poor understanding of the resulting environmental impact may hamper progress, as it would make planning consents more difficult to obtain. It is therefore clear that an understanding the interactions between WECs within a farm is vital for the continued development of the wave energy industry.

To support WEC farm design, a range of different numerical models have been developed, with both wave phase-resolving and wave phase-averaging models now available. Phase-resolving methods are primarily based on potential flow models and include semi-analytical techniques, boundary element methods and methods involving the mild-slope equations. Phase-averaging methods are all based around spectral wave models, with supra-grid and sub-grid wave farm models available as alternative implementations.

The aims, underlying principles, strengths, weaknesses and obtained results of the main numerical methods currently used for modelling wave energy converter arrays are described in this paper, using a common framework. This allows a qualitative comparative analysis of the different methods to be performed at the end of the paper. This includes consideration of the conditions under which the models may be applied, the output of the models and the relationship between array size and computational effort. Guidance for developers is also presented on the most suitable numerical method to use for given aspects of WEC farm design. For instance, certain models are more suitable for studying near-field effects, whilst others are preferable for investigating far-field effects of the WEC farms. Furthermore, the analysis presented in this paper identifies areas in which the numerical modelling of WEC arrays is relatively weak and thus highlights those in which future developments are required.

1. INTRODUCTION

As development continues in WEC technology there is an increasing interest in investigating how WECs interact with one another and the environment when they are deployed in an array. This understanding is vital to support wave farm design as commercialisation of WEC technologies progresses. In recognition of this the Wave Energy Converter Array Network (WECAN) was formed in 2010 as an international forum for researchers and developers active in the field of WEC arrays to discuss relevant current research and development so as to provide guidance and expert opinion on WEC arrays. This paper is the result of a review of WEC array numerical modelling techniques held at the WECAN meeting in Ghent, Belgium in October 2011.

A significant area of current research and development in WEC arrays is their numerical modelling. The first numerical models of WEC arrays were developed in the late 1970s and were based on potential flow models. Based on the number of publications, this numerical modelling technique remains the most popular method for determining interactions between WECs; however, in the last 5 years a number of alternative numerical modelling techniques have been developed or are being developed that provide alternatives for assessing the WEC interactions as well as the potential environmental impact of a wave farm. These techniques include the use of Boussinesq or mild-slope wave models, the use of spectral wave models and also the use of nonlinear boundary element and CFD models.

It would be wrong to assume that these alternative numerical modelling techniques for WEC arrays may be exact replacements for potential flow models and that there is a single best numerical modelling technique for WEC arrays. Each of the four identified basic types of WEC array numerical modelling techniques has a certain set of characteristics that make it more or less suitable for particular modelling requirements. Even within these basic types there are sub-types which offer further possibilities to optimise the numerical modelling approach for a particular case study.

To assist in the comparative analysis of the WEC array numerical modelling techniques, each of the techniques is described and then assessed using a common set of defining characteristics. To assist analysis, these defining characteristics are separated into three basic types: fundamental modelling characteristics, computational processing characteristics and usability characteristics. The fundamental modelling characteristics include the assumptions inherent in the model, together with the consequential limitations, strengths, weaknesses and issues. The computational processing characteristics include the factors that define the computation effort such as the model complexity, the number of WECs and spatial extent. Finally, the usability characteristics include the required skill of the user, the degree of ease of use and availability of suitable software (including cost and user friendliness).

This paper starts by reviewing all of the current WEC array numerical modelling techniques and is followed by a comparative analysis of the techniques identified. This comparative analysis uses the defining characteristics mentioned above to identify which techniques are most suitable for particular numerical modelling tasks. The tasks considered include: evaluation of localised interactions and impacts, evaluation of WEC array control strategy, estimation of power productivity of a small WEC array (2-10 units), estimation of power productivity of a wave farm (100+ units) and assessment of distal environmental impact. Finally, the state-of-the-art in WEC array numerical modelling is discussed and the requirements for further research and development identified.

2. POTENTIAL FLOW MODELS

2.1 Boundary element methods

To model the wave-structure interaction of a single WEC with the incident wave field, the present state-of-the-art is to use Boundary Element Method (BEM) based numerical codes such as the well known WAMIT, ANSYS Aqwa, Aquaplus amongst others. When it comes to arrays of WECs, these numerical tools are theoretically able to deal with any number of devices without restrictions except the ones related to the use of linear potential theory.

As a brief summary, potential flow methods are based upon the following assumptions:

- The fluid is inviscid.
- The flow is irrotational. Therefore, there exists a velocity potential $\phi(M,t)$ from which the velocity can be derived everywhere in the fluid domain: $\vec{V}\left(M,t\right)=\vec{\nabla}\phi\left(M,t\right)$.
- The flow is incompressible. Adopting mass conservation, this assumption leads to Laplace’s equation: $\Delta\phi\left(M,t\right)=0$ everywhere in the fluid domain.

Formulating a set of boundary conditions that satisfy the Laplace equation results in a nonlinear Boundary Value Problem which remains challenging to solve (nonlinear potential flow formulations are discussed below). As a result, the problem is usually simplified further by being linearised.

The two assumptions for linearisation are:

- The ratios of wave height to wavelength (i.e. wave steepness) and wave height to water depth must both be much smaller than 1.
- The motions of the body are small and around a fixed mean position: the ratio of the typical amplitude of motion to the typical dimension of the body is much smaller than 1.

The next step is to transform the volumic problem into a surfacic problem by making use of Green’s second identity. Then, by using an adequate Green’s function, the problem can be discretised and solved numerically, usually in the frequency-domain (which is possible as a consequence of the linearisation). Figure 1, taken from Borgarino *et al.* [1], shows an example of such a calculation for an array composed of two clusters of 8 floating OWSCs (Oscillating Wave Surge Converters). The figure shows the normalised perturbation of the incident significant wave height.

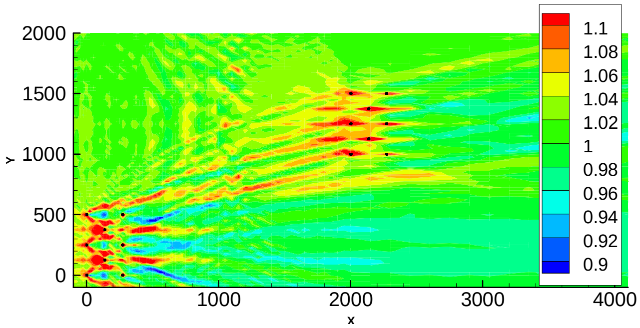

Figure 1: Ratio of the significant height of the total wave field around an array of two clusters of 8 floating OWSCs with respect to the significant height of the incident wave. Sea state is Hs=1 m, Tp = 8 seconds. Picture is taken from [1]

Usually, assumptions related to linear potential theory apply well for application to WECs in small to moderate sea states; comparison of numerical predictions with experiments or more complex CFD calculations agree well (see for example Durand *et al.* [2] or Thilleul *et al.* [3]). However, for larger sea states, significant discrepancies can be observed as the linearity assumptions are violated and other effects such as vortex shedding become important.

One attractive aspect of BEM based numerical codes is that they are fast when compared to CFD techniques. However, with these codes, the computational resources increase rapidly with the number of WECs in the array, as numerical complexity is proportional to the square of the number of unknowns. For this reason, only recently did it become possible to consider small arrays (typically 5-10 devices) with these numerical models [4-9]. For arrays larger than 10 devices, CPU time and memory requirements become a real issue, even for higher-order panel methods which are available in codes like WAMIT. However, it should be noted that there is ongoing research into how to overcome this issue, using the Fast Multipole Algorithm [10].

Another limitation of the BEM approach is that constant water depth is usually assumed over the whole array. This might be very inaccurate in the case of large arrays of WECs, for which bottom induced effects might be significant (such as shoaling or refraction, for example). Theoretically, one could take into account the bathymetry with BEM by meshing the sea bottom, but this would correspond to an enormous number of additional unknowns. In this case, computational resources would, again, be an issue.

2.2 Semi-analytical techniques

Much of the existing research on the subject of hydrodynamic interactions in arrays of wave energy converters has employed semi-analytical representations of a potential flow solution. These involve analytical expressions that either approximate or converge to an 'exact' solution in the limit of an infinite series. There are several main classes of such an approach which are described here: the 'point absorber', 'plane wave', 'multiple scattering' and 'direct matrix' methods. All of these techniques are based upon linear wave theory and are therefore subject to the relevant assumptions of Section 2.1.

The principal assumption in the point absorber method [11, 12] is that wave energy absorbers are sufficiently small compared to their separation and to the incident wavelength that the far-field radiation pattern from each one is not significantly affected by the presence of the other absorbers. Hence, the diffraction of radiated and diffracted waves from each device by others in the array is neglected. Typically further assumptions, that the devices are all identical bodies of revolution, oscillating vertically and optimally controlled, are also made. The resulting solution for the quantity of maximum power absorption by the array may be written in terms of the inverse of a square matrix of order N (the number of absorbers in the array).

The plane wave method [13] requires that the spacing between axisymmetric absorbers is large compared to the incident wavelength. This allows the diverging wave coming from one device to be approximated as a plane wave upon reaching others in the array. Unlike the point absorber approximation, the diffraction of all waves by each element in the array is accounted for here, albeit using an approximation. A version of the solution that includes the effects of evanescent waves [14], requires the inversion of a square matrix of order $N(N\text{-}1)$.

Multiple body radiation and diffraction was dealt with by Mavrakos [15] as a succession of distinct scattering events. In principle, this multiple scattering method is capable of providing the exact linear wave theory solution by means of an infinite summation over horizontal angular and vertical 'eigenfunction modes' as well as over successive 'orders of interaction'. In practice all three constants must be truncated to a finite number (for example, to integers M, P and S respectively). Although matrix inversion is not required in the main array calculation, the number and order of matrix multiplication operations increase with N as well as M, P and S (which in turn depend on the physical scenario under consideration).

Kagemoto and Yue [16] provided a 'direct matrix' method to address the same problem as the multiple scattering technique, solving for the amplitudes of all scattered waves simultaneously without the need for iteration. This interaction procedure may be combined with a single body solution, such as the numerical scheme used in [16] or another eigenfunction expansion (see [17]). In common with the multiple scattering approach, the accuracy of this technique is dependent on the number of horizontal angular and vertical eigenfunction modes required. The solution of the hydrodynamic problem reduces to the inversion of a square matrix of order MNP.

In the latter three techniques, there are additional mild assumptions on the geometrical arrangement of the array required. Due to the use of Graf's addition theorem, the vertical projections of interacting bodies onto a horizontal plane must not overlap and a circumscribed vertical cylinder around each body centred on its imaginary origin must not contain the origin of any other body.

Power absorption values are available from all methods (in the form of an upper bound for the point absorber technique). The methods that derive velocity potentials (plane wave, multiple scattering and direct matrix) also give rise to hydrodynamic forces on body surfaces (from which motions may be calculated) whilst additional theory is needed to obtain these quantities using the point absorber approximation. The potential (from the multiple scattering and direct matrix method) may also be used to visualise the free surface elevation in the vicinity of the array.

All of the above methods are computationally efficient (compared to, say, BEMs) for small to medium sized arrays and some are capable of approaching the ‘exact’ linear wave theory solution for the special cases to which they apply. This makes them most suited for use in optimisation routines and preliminary array studies. In addition to the limitations of stated assumptions, the accuracy of the plane wave method may be reduced in the low frequency range [4] and so the more accurate direct matrix method or more efficient point absorber method are now more commonly used.

All of the methods described in this section have typically been applied to heaving point absorber device types in the past, although the multiple scattering and direct matrix methods are capable of being adapted for use with other geometries. The algorithms for the methods described here are freely available in the literature and have been implemented at several academic institutions over the years. However, currently there are no commercial software tools using these techniques.

2.3 Time-domain formulation

This subsection focuses on the linear potential flow problems mentioned in Section 2.1, *i.e.* it assumes small wave steepness (linear waves) and small body motions. However, by moving from the frequency-domain into the time-domain, it is possible to model transient phenomena and to include nonlinear external forces such as nonlinear viscous damping, mooring and power take-off forces.

For an array of N freely floating rigid body WECs, the frequency-domain equations of motion are [18]:

$$[-\omega^2(\mathbf{M}+\mathbf{A}(\omega))+i\omega\mathbf{B}(\omega)+\mathbf{C}]\mathbf{X}(\omega)=\mathbf{F}(\omega), \quad (1)$$

where ω is the angular wave frequency and $\mathbf{M}$, $\mathbf{A}(\omega)$, $\mathbf{B}(\omega)$ and $\mathbf{C}$ are all $6N\times6N$ matrices. The matrices $\mathbf{M}$ and $\mathbf{C}$ are block diagonal matrices and represent the global mass matrix and matrix of hydrostatic and gravitational restoring coefficients, respectively. The 6×6 blocks on each diagonal represent the corresponding body matrices. The matrices $\mathbf{A}(\omega)$ and $\mathbf{B}(\omega)$ are full matrices and are the added mass and added damping matrices respectively. The $6N\times1$ matrices (column vectors) $\mathbf{X}(\omega)$ and $\mathbf{F}(\omega)$ are the Fourier transforms of the body motions $\mathbf{x}(t)$ and wave excitation forces $\mathbf{f}(t)$, respectively.

Taking the inverse Fourier transform of equation (1) yields:

$$(\mathbf{M}+\mathbf{A}(\infty))\ddot{\mathbf{x}}(t)+\int_0^t \mathbf{k}(t-\tau)\dot{\mathbf{x}}(\tau)\,\mathrm{d}\tau+\mathbf{C}\mathbf{x}(t)=\mathbf{f}(t), \quad (2)$$

where $\mathbf{A}(\infty)$ is the added mass matrix at infinite frequency and $\mathbf{k}(t)$ is the matrix of radiation impulse response functions. These functions account for the effects that persist in the free-surface after body motion has occurred. The matrix $\mathbf{k}(t)$ is the inverse Fourier transform of the radiation impedance matrix $\mathbf{K}(\omega) = \mathbf{B}(\omega) + i\omega(\mathbf{A}(\omega) - \mathbf{A}(\infty))$ and $\mathbf{k}(t)$ can be obtained from the added damping via:

$$\mathbf{k}(t)=\tfrac{2}{\pi}\int_0^\infty \mathbf{B}(\omega)\cos\omega t\,\mathrm{d}\omega. \quad (3)$$

Equation (2) represents the linear time-domain equations of motion and it is due to Cummins [19]. Very few papers have actually applied it to arrays, however it has been extensively used for single devices and most of the issues are common to both. For example, there is not much difference between the treatment of a 6×6 $\mathbf{k}(t)$ matrix for a single body and a $6N\times6N$ $\mathbf{k}(t)$ matrix for an array.

Normally, the hydrodynamic data $\mathbf{A}(\omega)$, $\mathbf{B}(\omega)$, $\mathbf{C}$ and $\mathbf{F}(\omega)$ are obtained from hydrodynamic codes like WAMIT or Aquaplus, with $\mathbf{k}(t)$ then derived from equation (3). However, $\mathbf{k}(t)$ can also be computed directly from programs like Achil3D.

The main problem with equation (2) is the convolution term. This term is not well suited for studying and designing WEC dynamics because it is computationally demanding to compute directly. Fortunately, because the convolution is linear, it is possible to replace it by other linear time-invariant systems like transfer functions or a state-space system. System identification is used for this approximation, as discussed by Taghipour *et al.* [20], who compares system identification in the time and frequency-domains. System identification in the time-domain involves approximating the radiation impulse response functions $\mathbf{k}(t)$ and system identification in the frequency-domain involves approximating the radiation impedance $\mathbf{K}(\omega)$. An example of system identification in the time-domain is Prony’s method [21]. Prony’s method uses a sum of exponential functions to approximate $\mathbf{k}(t)$ and this works very well for a single device. However, based on our experience, it does not work so well for arrays because of the cross coupled terms for which the maximum of the response does not happen at the initial time. An example of system identification in the frequency-domain can be found in McCabe *et al.* [22].

An interesting extension to the above linear time-domain models is reported in Babarit *et al*. [23] and this could also be applied to arrays. In that paper, the radiation and diffraction forces remain calculated by linear potential theory but the Froude-Krylov force (*i.e.* the sum of incident wave and hydrostatic forces) is computed on the exact wetted surface. This means that important nonlinearities are taken into account and although there is a moderate increase in computational time, the agreement with experimental results is improved.

2.4 Nonlinear potential flow models

Like all potential flow models, nonlinear potential flow models rely on the assumptions of incompressible, irrotational and inviscid flow. Most commonly, these models are implemented using a Boundary Element Method (BEM). In contrast to the linear BEM frequency-domain models discussed above, the nonlinear counterparts operate in the time-domain.

In early BEM schemes the free surface at each time step was mapped to a closed contour, providing a continuous boundary for the evaluation of the boundary integral equation. More recent nonlinear BEM models are often implemented as Numerical Wave Tanks (NWT) in physical space, where the geometry of the domain is that of the physical environment. In contrast to mapped solutions, physical space offers the benefit of a non-uniform bathymetry, and enables the definition of arbitrary body geometries within the domain.

Within a nonlinear NWT, the following boundary conditions are imposed: (i) a wave generation condition on the input boundary; (ii) a radiation condition on the outflow boundary; (iii) a no-flow condition on the bed and (iv) the dynamic free surface boundary condition and the kinematic free surface boundary condition. Further details concerning nonlinear BEM formulations can be found in Hague and Swan [24] and many others.

The two surface conditions, (iv) above, are computed on the instantaneous position of the free surface. As a result, this formulation retains the full nonlinearity of the underlying hydrodynamics. Likewise, the forces acting on fixed or floating bodies may be computed on the instantaneous position of the fluid-structure intersection, again retaining the full nonlinearity of the problem. In the context of floating bodies this concept is described by Kashiwagi [25].

Nonlinear potential flow formulations have been used extensively for the computation of extreme loads on fixed offshore structures and in the simulation of large vessel motions. Their high computational demand has hindered the simulation of large domains and arrays of wave energy converters. However, with the recent availability of parallel computing (cluster computers) this limitation is expected to vanish in the near future and simulation of small arrays (5-10 devices) is within practical reach. In fact, nonlinear potential flow codes are now being applied to model wave energy converter arrays as part of the PerAWaT project.

An additional application of nonlinear BEM formulations is the provision of nonlinear hydrodynamic data as the input to other models. They could be coupled with Smoothed-Particle Hydrodynamics (SPH), CFD (discussed below) or the nonlinear time-domain simulations discussed above. This coupling may provide the nonlinear fluid particle kinematics in the absence of the structure, or the nonlinear loading on a single fixed or floating structure.

3. BOUSSINESQ / MILD-SLOPE WAVE MODELS

This category includes phase-resolving models which can be subdivided into models based on the linear mild-slope equations and models based on the nonlinear Boussinesq equations. Typical applications of phase-resolving models are at the nearshore/local scale (harbours), using smaller grid cell sizes (down to 1.0 m). The Boussinesq models seem to be accurate predictors of the nearshore hydrodynamic behaviour, such as the propagation of nonlinear waves in deep to shallow water. The complexity of Boussinesq models makes them computationally very demanding when simulating more than a few hours of wave input and can in some circumstances become unstable.

Compared to the Boussinesq models, the models based on the linear mild-slope equations are considered to be fast solvers. The latter models describe the transformation of linear waves when propagating from deep to shallow water. Limitations of these mild-slope models lie in the simplifying assumptions. Nevertheless, they have proven to be an excellent tool when investigating wave penetration in harbours, diffraction issues, wave transformations, etc [26].

3.1 Boussinesq models

Boussinesq models are based on a set of nonlinear partial differential equations known as the Boussinesq Equations. The classic equations basically approximate wave propagation by eliminating the vertical component of velocity but still accounting for the vertical flow structure, assuming an incompressible fluid and irrotational flow. As a result of this depth averaging, the use of the classic equations is limited to water depths less than 0.25 times the deep water wavelength.

Boussinesq models are usually mathematically enhanced versions of the classic Boussinesq equations which include the effects of, deeper water depths; varying bathymetry; frequency dispersion; wave breaking and moving shore line to name a few. One such model is the Boussinesq Wave Editor (MIKE21BW) provided as part of the MIKE 21 suite of software developed by the DHI Water and Environment [27]. This model is based on the enhanced Boussinesq equations formulated by Madsen and Sørensen [28], which calculates the free surface elevation based on flux density, rather than velocity as is the normal method for other models, resulting in improved stability of the simulations. The formulation includes further improvements allowing the theory to be extended into deep water with a max depth limit of 0.5 times the deepwater wavelength. The model accounts for all important wave transformation processes including, shoaling, refraction, diffraction, wave breaking, bottom friction, moving shoreline, partial reflection and transmission, nonlinear wave-wave interaction and frequency and directional spreading. Other phenomena such as surf beats and generation of sub and super harmonics may also be modelled, making it an ideal tool for studies of harbour resonance, seiching etc.

In general, Boussinesq models are not capable of modelling the hydrodynamics of a moving device. However, they may be used to model device characteristics, such as wave transmission reflection and absorption. If radiation characteristics are known, these may be included by use of an internal generation line, although this may become cumbersome when more than 1 WEC is considered and, for this reason, their use warrants caution. Outputs are in the form of surface elevation and flux/velocity components within the model domain. It is also possible to calculate the disturbance coefficient which is the ratio of the significant wave height at a particular point relative to the significant wave height at the input; this is commonly used for port and harbour studies.

Venugopal and Smith [29], carried out an investigation into the change in wave climate around a hypothetical array of 5 individual bottom mounted WECs at the European Marine Energy Centre (EMEC) in the Orkney Islands. This was achieved by utilising the capacity to model partial transmitting and reflecting obstacles in the MIKE21BW modelling tool, with a domain size of 5km by 4.5km. Differing porosity values were used to simulate varying degrees of reflection, absorption and transmission ranging from 0 (i.e. 100% transmission – no WEC in place) to 1 (i.e. 100% reflection – no transmission). The study shows that this method may be used to identify regions of reduced and augmented wave energy in the lee of the array for particular bathymetries, wave conditions and array geometries, although there is currently no experimental nor field data to validate the success of this method. Venugopal and Smith also identified that reductions in wave height vary greatly depending on the values of porosity used; indicating that, if this method is to be used confidently for future array studies, great care should be taken that realistic device characteristics are modelled. This will require calibration of the porosity values to match device specific transmission, reflection and absorption and validation with physical model results.

Aside from the inclusion of nonlinearity and deep water terms, one further advantage to using phase resolving models of the Boussinesq type for modelling of wave farms is the realistic representation of diffraction phenomena. Spectral models and some mild-slope models include only a parameterised representation, which does not accurately represent reality. It remains to be seen whether this will have a significant effect on studies of wave farm interactions or environmental impact.

3.2 Mild-slope models

In general, mild-slope models are based on the linear form of Boussinesq shallow water equations and therefore linear waves are generated, propagating over mildly varying bathymetries. Nevertheless, they calculate velocity potential and surface elevations throughout the numerical domain with a relatively low computational and accuracy cost and with a high stability performance.

Recently, wake effects in the lee of a single and multiple WECs and energy absorption have been studied [30-32] by using the time-dependent mild-slope equation model MILDwave [33] and applying a sponge layer technique, by which the redistribution of wave power both within and behind each farm can be studied in detail. In this phase-resolving model each combination of reflection and transmission characteristics, and consequently absorption characteristics, can be modelled for all individual WECs in a farm [30]. This results in a representation of the wake effects in the lee of a single WEC and in that of a farm of WECs. A WEC is implemented in MILDwave as an array of cells (covering the spatial extent of the WEC) that have been assigned a given degree of absorption using the sponge layer technique. Absorption functions define the absorption coefficient *S* attached to each cell of the WEC in the x-direction and the y-direction. By changing the values of the absorption coefficients or the number of absorbing cells, the degree of reflection and transmission and therefore absorption of the porous structure can be changed [31]. When assuming a constant absorption coefficient *S* for all cells of the WEC, the amount of reflection, transmission and absorption are coupled, as seen in [29]. To avoid this coupling, the shape of the absorption function through the WEC is changed. This way, the degree of absorption (and consequently transmission) of the WEC, given in the power matrix of the WEC, can be tuned for a fixed amount of reflection on the WEC as specified by the developer.

The power absorption of a WEC typically varies with frequency; however it is possible, using MILDwave, to represent the frequency dependent absorption by appropriate definition of the sponge layers. In this way, the wake behind a WEC is studied for each frequency component separately, as the amount of absorption of the WEC in its lee depends on the remaining energy in the considered frequency components. This is also the case for wave direction dependent WECs. The wake is then not only calculated for each frequency component but also for each wave direction.

4. SPECTRAL WAVE MODELS

Another category of model which has been used to simulate WEC arrays is the spectral wave model. Spectral wave models are phase-averaging wave propagation models which predict how the surface wave frequency and directional spectrum will evolve as waves propagate through varying background currents and water depth. While the other models described in this paper solve an equation or set of equations to find the surface elevation of the waves, spectral wave models solve what is essentially an energy conservation equation. In fact the quantity that is solved for is wave action, which is the spectral energy density divided by the intrinsic frequency. Wave action is conserved even in the presence of varying background currents, and thus is the preferred quantity to solve for. Spectral wave models are capable of representing numerous wave transformation processes. These include depth- and current-induced refraction, shoaling, wind forcing, white-capping and bottom friction dissipation, dissipation through bathymetric breaking, and nonlinear quadruplet and triad wave-wave interactions. Because spectral wave models are phase-averaging, they are unable to represent wave diffraction explicitly. However, a phase-decoupled refraction-diffraction representation has been developed that addresses this deficiency reasonably well [34]. There are currently two open source spectral wave models that are readily available: the SWAN model developed by the Delft University of Technology [35], and the TOMAWAC model developed by the Electricité de France [36].

Because spectral wave models solve for the conservation of wave energy, a representation of a WEC array in a spectral wave model must somehow account for the energy absorbed and the energy radiated by the WECs. There are a few existing methods which have been used to represent WEC arrays in a spectral wave model. These can be divided into two categories:

supra-grid scale, in which the whole WEC array is represented over several computational grid points, and sub-grid scale, in which each individual WEC in an array is represented at a single computational grid point.

4.1 Supra-grid models

There are two current examples of supra-grid scale methods. The first uses the built-in obstacle feature in the SWAN model, for which an energy transmission coefficient can be set [37]. The WEC array is represented with a single transmission coefficient and the effect on the coastline is estimated after propagating the waves altered by the array to the shore. However, this method does not allow the energy absorption of the array to depend on frequency. This shortcoming in the method has recently been addressed in another SWAN model study which included the introduction of a frequency dependent transmission coefficient, and the use of an obstacle to represent a single WEC, as opposed to the whole array [38]. Although the supra-grid scale methods can now account for the frequency dependence of the energy absorption, they do not account for the radiation of energy by the WECs.

4.2 Sub-grid models

A sub-grid scale method of representing a WEC array is implemented in the TOMAWAC model and includes both frequency dependent energy absorption and the radiation of energy by WECs [39]. This is done by treating each WEC (located at a computation grid point) as a source and sink of wave energy. The energy absorption and radiation, which can be dependent on the incident wave, is therefore incorporated at each WEC location into the wave action conservation equation. This technique is similar to how the existing wave processes such as wind generation and wave dissipation are treated in spectral wave models.

As the development of WECs matures, and the possibility of deploying devices in large arrays becomes closer to a reality, it is important to develop numerical tools that can be used to investigate both the annual power production of a WEC array and the potential impacts it may have downstream on the wave climate. It is advantageous to use a spectral wave model for this task because it is possible to cover a relatively large domain (tens of kilometres square) with a large WEC array (tens of devices) in a relatively short computational time. The parameterisations of WECs in spectral wave models which have been developed can represent the energy absorbed and radiated by individual WECs, and are also capable of representing nonlinear processes [40]. Of course, phase-averaging models cannot resolve phase-dependent processes, so near-field effects around each individual WEC are not explicitly modelled in a spectral wave model. It is therefore important to carry out comparison studies between phase-resolving numerical models, experimental results, and spectral wave models in order to ensure the best possible representation of a WEC in a spectral wave model.

5. CFD MODELS

The term Computational Fluid Dynamics (CFD) model is commonly used for codes that seek to resolve the Navier-Stokes equations. The Navier-Stokes equations are derived from mass and momentum conservation, and often regarded as the most fundamental set of fluid flow equations. Both viscous effects and turbulence are accounted for.

From a practical point of view, Navier-Stokes solvers may be classified into two distinct categories: (i) Direct Numerical Simulation (DNS), resolving turbulence at the smallest relevant length scale, and (ii) CFD codes, where turbulence is not directly resolved but dealt with in a parametric representation. For the modelling of WEC arrays only the latter is relevant, DNS being prohibitively expensive in terms of computational demand. CFD models are based on Finite Element or Finite Volume implementations, and often referred to as such.

In contrast to the potential flow models discussed above, CFD models include viscous effects and two-phase flow (air entrainment in breaking waves) making them the ideal tool for the simulation of extreme wave loading. Furthermore, the CFD approach retains the full nonlinearity of the underlying hydrodynamics. An additional benefit of CFD codes (particularly when compared to potential flow models) is that marine currents are easily described. In the near-shore environment, currents may add significantly to the overall loading and also affect the device dynamics.

Unfortunately, CFD models are often prone to internal dissipation, particularly when resolving gravity water waves (free surface flow). Maguire examined the free-surface modelling capability of a number of commercially available codes [41]. The overall conclusion from this extensive study is that none of the tested tools may readily be used to model gravity water waves; however, more recently, a number of tools under development appear to be more reliable in terms of their free surface prediction [42, 43].

To overcome this difficulty of internal dissipation, a decomposition of variables can be used. This consists in splitting all unknowns of the problem (pressure, fluid velocity and free-surface elevation) into the sum of an incident term and a diffracted term. The incident terms are described explicitly using a linear or nonlinear potential flow model. Thus only the part of the grid in the vicinity of the structure needs to be refined. The method is called SWENSE (Spectral Wave Explicit Navier Stokes Equations). It has been already successfully applied and validated in 3D cases [44-46]

The disadvantage common to all CFD codes is their computational demand. To minimise computational demand, many codes offer non-uniform (at times also adaptive) meshes, where the grid in the area of interest (the free surface and the vicinity of the device) is defined with a finer spatial resolution. Particularly in deep water this may offer significant benefits.

The most extensive WEC CFD study to date has been reported by Westphalen *et al.* [47]. Recently, an array of 2 heaving WECs was considered by Agamloh *et al.* [48] using CFD. A number of very recently funded research projects (UK EPSRC funded SUPERGEN Marine Challenge - Accelerating the Deployment of Marine Energy) propose the use of CFD for the modelling of small arrays (5-10 devices). Guidelines for the use of CFD codes in the modelling of WEC arrays are expected to emerge within the next 2-3 years.

6. COMPARATIVE ANALYSIS

The fundamental modelling characteristics, computational processing characteristics and usability characteristics are used for the comparative analysis of the numerical techniques used to model WEC arrays. The results of the analysis are summarised in Table 1, located at the end of the paper. These characteristics are then used to consider the suitability of each numerical technique for four different modelling tasks; localised effects, dynamic control, annual energy production (AEP) - separated into small and large arrays, and distal environmental impacts. Of course, other modelling tasks exist; however, these are considered to be representative of a range of tasks and demonstrate the comparative performances of the different numerical modelling techniques. Table 1 also includes an estimation of suitability for each numerical modelling technique for these four modelling tasks.

The designations of the modelling technique suitability are discussed below; however, prior to this, it is important to recognise that the characteristics of the different modelling techniques make them suitable for modelling particular WECs. For example, a linear BEM model may be suitable for a large WEC in deep water that sheds minimal vortices, but is less suitable for a small WEC that sheds significant amount of vortices, whose motions are significantly nonlinear and with a complex control strategy. This aspect will not be investigated further; however, it is clearly an additional consideration in determining the most suitable modelling technique for a particular WEC array.

Returning to the modelling of WEC arrays, the suitability for modelling localised effects is considered first. This refers to the extent that near-field effects, such as evanescent waves and vortex shedding, from one WEC may influence another WEC nearby. In close proximity, phase correlations between two WECs are high and so phase-averaged models, i.e. those based on spectral wave models, are not suitable. In addition, the semi-analytical techniques based on further simplifying approximations to linear wave theory (the point absorber and plane wave methods) are not suitable because these focus on modelling the far-field. On the other hand, provided that the modelling and computational effort can be justified CFD is highly suitable for modelling localised effects because the model may include both evanescent waves and vortex shedding implicitly. However, in many cases it is possible that potential flow models (excluding some semi-analytical techniques) would be adequate, with significantly less computation effort. Finally, whilst Boussinesq/Mild-slope models resolve phase, they are unlikely to accurately model the near-field and so are poorly suitable.

The suitability for modelling dynamic control, whereby the motion of each WEC is controlled to maximise power capture, is now considered. Again, the spectral wave models are not suitable because they are based on phase-averaging, together with the Linear BEM and semi-analytical techniques, because these are based on frequency-domain representations, whilst dynamic control requires a solution in the time-domain. The Boussinesq/Mild-slope models are poorly suitable because whilst they provide a solution in the time-domain, it is not clear how to change the absorption layers dynamically to correctly model the control. CFD is poorly suitable as well because of the CPU time. The remaining methods (Time-domain formulation, nonlinear BEM) are suitable for modelling dynamic control, with the most suitable approach depending on the particular WEC array being modelled.

Modelling the annual energy production (AEP) requires power capture to be calculated for a large number of irregular sea-states. This means that CFD and Nonlinear BEM are poorly suited because of their high computational requirements. In addition, Linear BEM models and the Time-domain formulations derived from them, rapidly become unsuitable as the number of WECs increase due to the quadratic relationship between the computation effort and the number of WECs. In addition, the supra-grid spectral wave model is not suitable for modelling the AEP because WEC array interactions are subsumed within the explicit definition of the absorption layer; the model itself cannot calculate array power performance directly. The Boussinesq/Mild-slope models and sub-grid spectral wave models are all suitable for the calculation of the AEP. The most suitable method will depend on conditions (e.g. water depth, marine currents, bathymetry, etc) and also the accuracy with which the WEC and its interactions with the sea (e.g. WEC radiation, diffraction, nonlinear dynamics, etc.) can be modelled.

Finally, suitability for determining the distal environmental impact is considered. Unfortunately, none of the potential flow models are suitable because of the assumption of constant water depth, which makes them unsuitable for propagating the waves to the shoreline, where the environmental impact is typically most significant. Furthermore, the large propagation distances mean that CFD models are poorly suited due to their high computational requirements. The remaining modelling techniques, Boussinesq/Mild-slope Models and Spectral Wave Models, are all suitable for determining environmental impact and have been used extensively for this task in applications other than WECs. In addition to the model differences discussed above, the larger cell size in Spectral Wave Models means that these are most suited for modelling more remote impacts, whilst Boussinesq/Mild-slope Models are most suitable for situations where reflections and resonances may be significant.

7. DISCUSSION

This review paper is a snap-shot of the currently available numerical modelling techniques for WEC arrays. Whilst it is not expected that the results of this comparative analysis will change in the short-term, there will be long-term changes. It is clear from the descriptions of the different numerical modelling techniques that in many cases, potential remains for improvement, by either increasing their accuracy and/or reducing their computational requirements. Improvements in readily available computing power are also likely to change what can be done practically.

Finally, it is clear from the comparative analysis described above that there is no single best numerical modelling techniques for WEC arrays. The most appropriate numerical modelling technique being that which best matches the required characteristics of the particular modelling task. Unfortunately, it is not always clear which modelling technique this may be, as each model has different strengths and weaknesses, which rarely match the characteristics of the modelling task exactly. However, it is expected that identification of the most appropriate numerical modelling technique for a particular task will become clearer with experience and by experimental validation. Although, the lack of suitable validation data for these numerical modelling techniques is a significant issue that needs to be addressed urgently.

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TABLE 1: COMPARATIVE ANALYSIS OF NUMERICAL MODELLING TECHIQUES FOR WEC ARRAYS

	Potential flow models						**Spectral wave models**		
	Linear BEM	**Semi-analytical techniques**	**Time-domain formulation**	**Nonlinear BEM**	**Boussinesq**	**Mild-slope**	**Supra-grid**	**Sub-grid**	**CFD**
Fundamental									
Definition of hydrodynamics	Implicit body surfaces Explicit coefficients				Explicit absorption layers		Explicit absorption layer	Explicit source strength	Implicit fluid flow
Nonlinear wave dynamics	Not capable			Implicitly capable	Implicitly capable	Not capable	Implicitly capable for phase-averaged dynamics		Implicitly capable
Nonlinear dynamics	Not capable		Implicit solver		Explicit absorption layers		Explicit absorption layer	Explicit source strength	Implicit solver
Vortex shedding	Explicit inclusion by linearisation		Explicit inclusion		Explicit inclusion		Explicit inclusion		Implicit inclusion
WEC radiation	Implicitly capable				Explicitly capable		Not capable	Explicitly capable	Implicitly capable
Diffraction	Implicitly capable				Explicitly capable		Approximated by phase-decoupled refraction-diffraction		Implicitly capable
Variable bathymetry and marine currents	Not capable				Implicitly capable[†]	Implicitly capable[‡]	Implicitly capable		Implicitly capable
Computational									
Primary dependent	Number of panels	Complexity of function	Number of panels and complexity of equations	Number of panels	Number of cells		Number of cells		Number of cells
Secondary dependent	Number of frequencies and directions		Number of time-steps		Number of time-steps		Number of frequencies and directions		Number of time-steps
Determinate of array "size"	Quadratic increase with number of WECs				Linear increase with spatial area		Linear increase with spatial area		Linear inc. with spatial volume
Solver	Simple and stable		Simple and poss. unstable	Complex and stable	Simple and poss. unstable	Simple and stable	Simple and stable		Complex and poss. unstable
Usability									
Required skill	Low	High	Medium	High	Medium	Low	Low	Medium	High
Software availability in 2012	Commercial code available	Research code only	Commercial code available	Research code only	Commercial code available, WEC model required		Open-source code available, WEC model required		Commercial and open-source code available
Suitability (** - highly suitable, *** - moderately suitable, ** - poorly suitable, * - not suitable)**									
Localised effects	***	* to ***	***	***	**	**	*	*	****
Dynamic control	*	*	****	****	*	*	*	*	**
AEP (small WEC array)	***	***	**	**	***	***	**	***	**
AEP (large WEC array)	**	***	**	**	***	***	**	***	**
Environmental impact	*	*	*	*	***	***	****	****	**

[†] Limited to shallow water
[‡] Limited to mild-slopes