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# Optimization Of Future Drinking Water Pipe Renewal Under Uncertainty

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**ABSTRACT:** Developed countries have opted to transport drinking water to households via long networks of pipes, which are expensive to install and maintain. Their management is therefore an important issue for water utilities. Water asset management is a complex multicriteria problem since managers have lots of different objectives. This article focuses on “long term” strategic methods. Particularly it is centered on a new way to estimate the “number of future pipe breaks in the long term at the scale of the water utility territory”. This paper first estimate prospective pipe age distribution at a given network section location over time. Then equations of deterioration process of pipe sections are build-up. Finally both models are mixed. Our case study is eauservice Lausanne, the third water utility of Switzerland. The proposed approach is different from existing “long-term” models because it is based on actual historical survival function.

## 1. INTRODUCTION

Developed countries have opted to transport drinking water to households via long networks of pipes, which are expensive to install and maintain. The total length of water pipes installed in France is approximately 900,000 km, compared with around 1,600,000 km in the USA. Underground pipe networks represent more than 80 % of the total asset value for water distribution systems, and their management is therefore an important issue for water utilities (Folkman 2012). One objective of a water distribution system is to supply each consumer with sufficient good-quality water. Nevertheless other objectives relating to performance and risk should also be

met, and the overall costs (direct and environmental) should be acceptable. Thus, in practice, water asset management is a complex multicriteria problem since managers must prevent or minimize performance losses: service outages, disruption of surface traffic, flooding...

There is a high variability of pipe service lifetimes. While some pipes installed 130 years ago are still in full working order, some younger pipes have a high failure rate and need to be replaced soon. Pipe condition is the cumulative effect of many factors (physical, environmental, operational) acting on the pipe (Liu et al. 2012). Furthermore some younger (than 60 years) pipes may have been renewed not because they were in bad condition but in order to follow other

operational constraints like pavement rehabilitation (Al-Barqawi and Zayed 2006). The broad range of explanatory factors for renewal calls for the use of a powerful prediction model of pipe lifetimes.

To optimize resources and performance, it is important to try to renew the pipes at the best possible time. (Large et al. 2014b) demonstrate that the most relevant is to start from a "long term" vision (more than 50 years, possible median of pipe service life) in order to predict the pipe linear that must be renewed each year to meet strategic objectives in terms of performance. Then deduce the annual need for "medium term" investment ( $\approx 10$  years, across the "master plan"). Finally, water utility should then apply "short term" methods ( $< 3$  years) in order to obtain for the coming year a list of pipes prioritized by level of renewal need or renewal opportunity, ensuring that these works actually achieve the target performance.

In this context, this article will present the key part of a new "long term" approach to get a global view of pipes renewal need, in order to improve their management during their whole life cycle. This approach is tested using real data of Lausanne city, the third biggest water utility of Switzerland. Our long term model is an aggregation of selected short term criteria (level of degradation, level of coordination with pavement rehabilitation, degree of risk aversion) which allow us to make different simulations and scenarii. In this model, we statistically analyze the past chronicle of pipe decommissioning ages (with the Kaplan-Meier method). After we create future scenarii and deduce probabilistic performance indicators such as future renewal rates, future failure rates (thanks to Markov Chain model) and financial indicators such as future investment needs (maintenance OPEX, renewal CAPEX). Our "long term" method allows water utilities to make consistent their "short term" decision-rules and their "long-term" objectives.

This article will only focus on the tricky part of our model: the calculation of the performance indicator "number of future pipe breaks in the

long term at the scale of the water utility territory". It is an indicator of robustness and sustainability of the network.

Deterioration models allow the estimation of future probable failure rates. (Rajani and Kleiner 2001), (Kleiner and Rajani 2001), (Marlow et al. 2009), (Ugarelli and Bruaset 2010) and (Large et al. 2014a) made good reviews of existent deterioration models. A distinction can be made between deterministic models and probabilistic models. For example (McMullen, 1982) model, is the simplest deterministic model, the future number of failure is a linear equation depending only on pipe age. The LEYP (Linear Extension of the Yule Process) model, developed by (Le Gat 2013), is one of the most probabilistic comprehensive models because it can take into account all the variables (material, corrosive soil, pressure, etc.) that have an impact on breaks.

All these models include the pipe age as an explanatory variable. However when a water utility stakeholder wants to estimate the "number of future pipe breaks in the long term at the scale of its territory", there is a problem while using these models. Indeed some pipes will be renewed therefore the age of pipe at the same location will change a lot in the future. Thus we need a powerful model to predict the distribution of the future age at each location of the water utility territory. We introduce in this article the concept of "pipe section location". It is a location which has the same length of the pipe section observed in 2012, but the physical pipe can be renewed over time.

The innovative idea in this paper consists in considering the pipe age distribution at a given network section location over time. The "number of times" that pipes at a given location will be replaced and "when" are studied here like random variables.

## 2. METHODOLOGY

### 2.1. Build-up

The building up of the "number of future pipe breaks in the long term at the scale of the water utility territory" is done through three stages. First

stage we estimate the future probability distribution of ages at section pipe location scale over time. Second stage we deduce equations of deterioration process of physical section of pipes depending on their ages and other explanatory variables. Third stage we mix probable ages of “pipe section locations” with physical equations of deterioration process.

## 2.2. Developments

### 2.2.1. Probable future age of “pipe section locations” over time

The estimation of future probability distribution of ages of “pipe section location” over time is done through five steps.

#### **Step 1: Past survival function estimation**

During the first step we calculate the past survival function of the water utility. This empirical past survival function  $\hat{S}(t)$  is constructed with actual historical data of the water utility over an observation window. In our case study the observation window of decommissioned pipe sections starts in January 2001 and ends in december 2012.  $\hat{S}(t)$  estimates  $S(t)$ .  $S(t)$  is the probability that a pipe has not be decommissioned before age  $t$  [equ. (1)].  $\hat{S}(t)$  is constructed with left truncated and right censored data. We used the (Kaplan and Meier 1958) method adapted by (Claude and Lyon 1997) [equ. (2)] in order to correct this bias.

$$S(t) = \mathbb{P}(T > t) \quad (1)$$

with :

- $\mathbb{P}$  = Probability
- $T$  = random variable, decommissioning age
- $S(t)$  = Survival function
- $t$  = age of pipe section

$$\hat{S}_K(t) = \prod_{g \leq t} \left( 1 - \frac{\text{Card}\{u, b_u = g \ \& \ c_u = 0\}}{\text{Card}\{u, a_u \leq g \leq b_u\}} \right) \quad (2)$$

with :

- $u$  = pipe section number
- $\hat{S}(t)$  = empirical survival function
- $K$  = Kaplan-Meier method

- $a$  = age of the pipe at the beginning of the observation window (2001)
- $b$  = decommissioning age (for pipe out of service) or age in 2012 (for pipe still in service in 2012), classified in ascending hierarchical order, with  $b_{min} = 0$  and  $b_{max} = 134$  years in our case study.
- $c$  = censorship which takes 0 if the pipe has been decommissioned during the observation window, or 1 if the pipe is still in service at the end of the observation window.
- $g$  = age
- $\text{Card}$  = cardinality of the set between brace

#### **Step 2: Future survival function estimation**

In the second step we have to create a prospective future scenario about pipe age evolution.

Here we decided to use the “same as in the past” scenario. It works on the hypothesis that in the past, decommissioning ages were well chosen (good past survival curve) and the same distribution (survival curve) is applicable for future decisions.

Among classical parametric distribution, we choose to use a (Weibull 1951) survival function,  $S_w(t)$  [equ. (3)] based on the past survival function.

$$\hat{S}_W(t) = e^{-\left(\frac{t}{\alpha}\right)^\gamma} \quad (3)$$

with :

- $W$  = Weibull equation
- $\alpha$  and  $\gamma$  = parameters estimated by the (Nelder and Mead 1965) optimization method which optimises the objective function of the least squares method (Legendre 1805) based on the outputs of the (Kaplan and Meier 1958) method.

#### **Step 3: Transformation of prospective survival function in Markov transition matrix**

In order to model the entire life-cycle of pipe section, in step 3, we build the Markov transition matrix  $Q$  (Markov 1971), with the prospective survival function. The probability that a “pipe section location” has to go from age  $t$  to age  $t + 1$  (in years) is quantified. We assumed that the

**maximum age** of a pipe section is 150 years. To this Q matrix is associated the state diagram of Figure 1 and equ. (4):

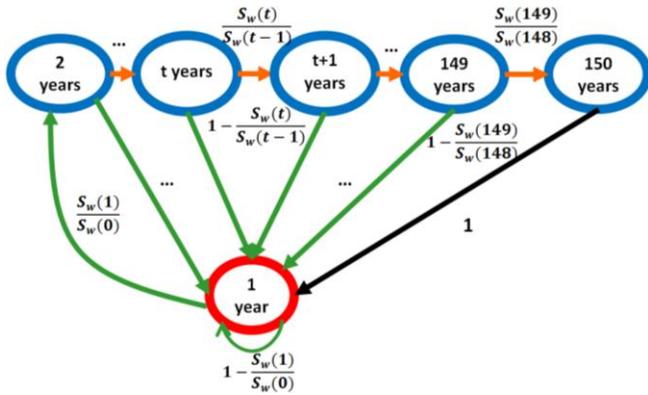


Figure 1: Markov state diagram

$$Q = \begin{cases} i < 150, & \begin{cases} j = 1 & Q_{i,j} = 1 - \frac{S_w(i)}{S_w(i-1)} \\ j = i + 1 & Q_{i,j} = \frac{S_w(i)}{S_w(i-1)} \\ j \neq 1, \neq i + 1 & Q_{i,j} = 0 \end{cases} \\ i = 150, & \begin{cases} j = 1 & Q_{i,j} = 1 \\ j \neq 1 & Q_{i,j} = 0 \end{cases} \end{cases} \quad (4)$$

With

- i = index of a line
- j = index of a column

The Q matrix looks like equ. (5).

$$Q = \begin{matrix} & \begin{matrix} 1 & 2 & \dots & t+1 & \dots & 149 & 150 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \dots \\ t \\ \dots \\ 149 \\ 150 \end{matrix} & \begin{bmatrix} 1 - \frac{S_w(1)}{S_w(0)} & \frac{S_w(1)}{S_w(0)} & \dots & 0 & \dots & 0 & 0 \\ 1 - \frac{S_w(2)}{S_w(1)} & 0 & \dots & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & 0 & \dots & \dots & \dots \\ 1 - \frac{S_w(t)}{S_w(t-1)} & 0 & 0 & \frac{S_w(t)}{S_w(t-1)} & 0 & 0 & 0 \\ \dots & \dots & \dots & 0 & \dots & \dots & \dots \\ 1 - \frac{S_w(149)}{S_w(148)} & 0 & \dots & 0 & \dots & 0 & \frac{S_w(149)}{S_w(148)} \\ 1 & 0 & \dots & 0 & \dots & 0 & 0 \end{bmatrix} \end{matrix} \quad (5)$$

**Step 4: Transformation of ages of “segment of pipe locations” in 2012 in Markov initial vectors**

To complete this phase successfully, in step 4 we created for each “pipe section location” the vector  $X_u(2012)$  (with 150 columns) which takes 0

everywhere except at the age  $b_u$  of the physical pipe section in service in 2012 in its “pipe section location” [equ. (6)]. By convention all the vectors in this article are row vectors.

$$X_u(2012) = \begin{bmatrix} 1 & 2 & \dots & b_u & \dots & 149 & 150 \\ 0 & 0 & \dots & 1 & \dots & 0 & 0 \end{bmatrix} \quad (6)$$

**Step 5: Markov Chain: probabilities of future ages of each “pipe section location”**

Finally in step 5 in order to obtain the probabilities of future ages of “pipe section location” we use a Markov Chain (Markov 1971) at each prospective calendar year (2012 + k) [equ. (7)].

$$X_u(2012 + k) = X_u(2012) \times Q^k \quad (7)$$

2.2.2. Equation of deterioration process of pipe sections depending on their ages

Physical pipe sections have an inherent vulnerability. Due to chemical attacks and physical mechanisms during its service life deterioration process occurs. The path followed in order to model this process may be decomposed into the following two steps.

**Step 1: With actual historical data, construction of prospective equation of deterioration process**

First, we decided to use a probabilistic model: the NHPP (Non Homogeneous Poisson Process) (Røstum 2000) to model the deterioration process of physical section of pipe [equ. (8)]. This model is inside especially “Aware-P” and “Cases” software. This model is based on the counting process of breaks (see Figure 2).

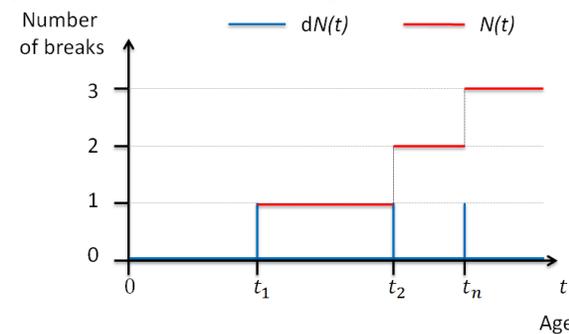


Figure 2: Counting process of breaks  $N(t)$  and dirac function  $dN(t)$

$$\begin{aligned} \frac{E[dN(t)]}{dt} &= \lambda_{u,\theta}(t) \\ &= \delta t_u^{(\delta-1)} e^{\beta_0} e^{\sum_{r=1}^R \beta_r \times Z_{r,u}} \end{aligned} \quad (8)$$

With:

- $t$  = age of pipe section
- $N(t)$  = counting process of breaks (as specified in Figure 2)
- $dN(t)$  = Dirac function takes 1 if a break is observed, 0 otherwise (cf. Figure 2)
- $E$  = expectation of the random variable
- $dt$  = infinitesimal age interval
- $Z$  = explanatory variable (covariate) (other than age)
- $R$  = cardinality of the set “explanatory variables other than age”
- $\delta, \beta_0, \dots, \beta_R$  = parameters estimated with the NHPP (Non Homogeneous Poisson Process) model.
- $\theta = (\delta, \beta_0, \dots, \beta_R)$  = vector of parameters
- $\lambda_{u,\theta}(t)$  = number of break per time unit (break intensity) at age  $t$  for pipe section  $u$

### Step 2: Estimation of the number of breaks at each age for each pipe

Then, this equation enables us to calculate the number of failures at any age  $t$  for all physical pipe sections. “Casses” software developed at IRSTEA was used (Renaud et al. 2012). We store the results of  $\lambda_{u,\theta}(t)$  in  $V_u$  vector [equ. (9)].

$$V_u = [\lambda_{u,\theta}(t)]_{1 \leq t \leq 150} \quad (9)$$

#### 2.2.3. Mixing probable age and equation of deterioration process

Finally in order to obtain the “number of future pipe breaks in the long term at the scale of the water utility territory” we have to mix the two previous models.

Fist we estimate the number of breaks at the scale of segment pipe location for prospective years [equ. (10)].

$$\Psi_u(2012 + k) = X_u(2012 + k) \times V_u' \quad (10)$$

with

- $V_u'$  = transpose of the vector  $V_u$

- $\Psi_u(2012 + k)$  = number of breaks at segment pipe location  $u$  in year (2012+k) per time unit

Then we estimate the number of future breaks at the scale of the water utility territory for prospective years [equ. (11)].

$$\Omega(2012 + k) = \sum_{u=1}^{\sigma} \Psi_u(2012 + k) \quad (11)$$

with

- $\sigma$  : cardinality of the set of “pipe section locations”
- $\Omega(2012 + k)$  = number of breaks at the scale of the water utility territory in (2012+k)

## 3. APPLICATION

### 3.1. Case selection

The results shown below relate to eauservice Lausanne, which had around 900 km of pipes in service at the end of 2012 (see Figure 3). Data relating to around 150 km of out-of-service pipes have been correctly computationally archived since 2001 (see Figure 4). The observation period for decommissioned pipes was 12 years (2001 - 2012). The decommissioning years are right-censored after 2012 and left-truncated before 2001.

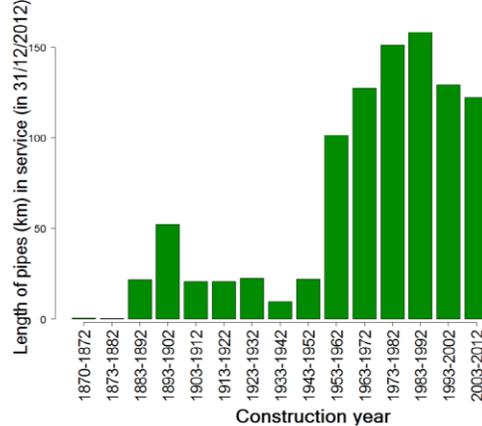


Figure 3: Length in service at the end of 2012 in Lausanne per decade of installation year.

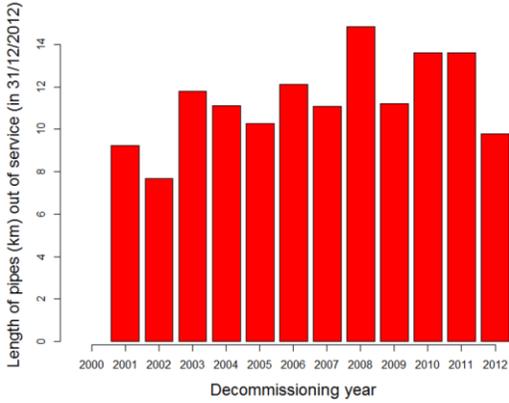


Figure 4: Length out of service available from archives at the end of 2012.

We partitioned the set of pipe sections into four strata according to their materials (see Figure 5): 1) Ductile cast Iron 2) Grey cast Iron 2) Steel and 4) Other materials.

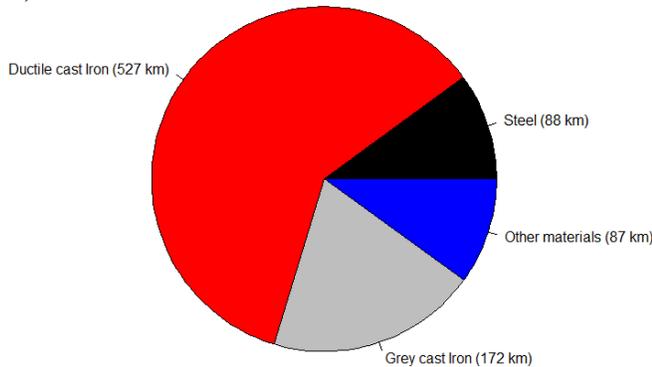


Figure 5: Distribution of pipe materials in eauservice Lausanne

### 3.2. Demonstration and visualization

#### 3.2.1. Probable age of section of pipes over time

First we estimated the past survival functions for each stratum and we adjusted on it Weibull survival functions [see Table 1] (see Table 1 and Figure 6).

Table 1: Parameter values of Weibull survival function depending on pipe materials

Parameters	$\alpha$	$\gamma$
Ductile cast iron	70.7	3.1
Grey cast iron	54.6	4.1
Steel	79.8	2.9
Other materials	86.3	2.4

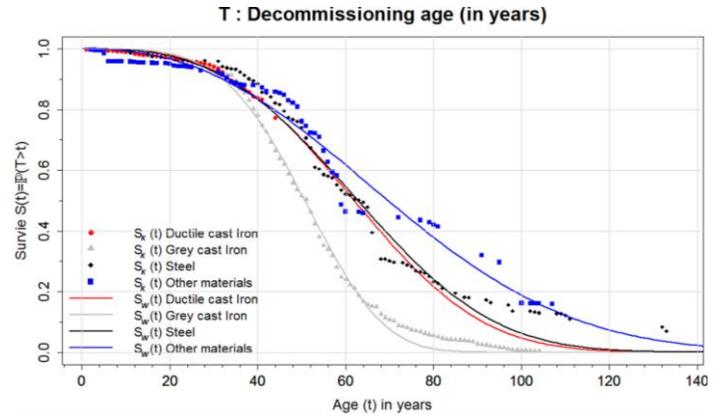


Figure 6: Past (K) and prospective (W) survival functions, Lausanne stratified by pipe materials

The Weibull survival functions are our future survival functions in our prospective scenario “same as in the past”. For example the grey cast iron pipes are renewed younger than other materials (see Figure 6).

#### 3.2.2. Equation of deterioration process of section of pipes depending on their ages

Then we statistically determined four equations of deterioration process using the NHPP model in “Casses” software [see Table 3]. The inputs of the NHPP model are the characteristics of section of pipes (diameter, material, length, etc.), the environment of section of pipe (pressure, soil, roads, etc.) and actual historical pipe breaks (number, shape, etc.). In our models three covariates maximum were statistically significant (cf. Table 2).

Table 2: Covariates and units in our NHPP models

Covariates	Age (t)	Length ( $Z_1$ )	Diameter ( $Z_2$ )
Units	Century	m	mm

Table 3: Parameter values of NHPP models depending on pipe materials

Parameters	$\delta$	$\beta_0$	$\beta_1$	$\beta_2$
Ductile cast iron	3.8	1.1	0.6	-0.005
Grey cast iron	1	-0.06	0.5	-0.007
Steel	1.4	-0.8	0.6	-0.004
Other materials	1.9	-1.2	0.3	0

The age is a parameter not significant with cast iron pipes and diameter is not significant for other materials.

### 3.2.3. Mix of probable ages and equations of deterioration process

Finally using the method described in the previous part we calculated the total number of breaks between 2013 and 2085 at the scale of eauservice Lausanne (see Figure 7).

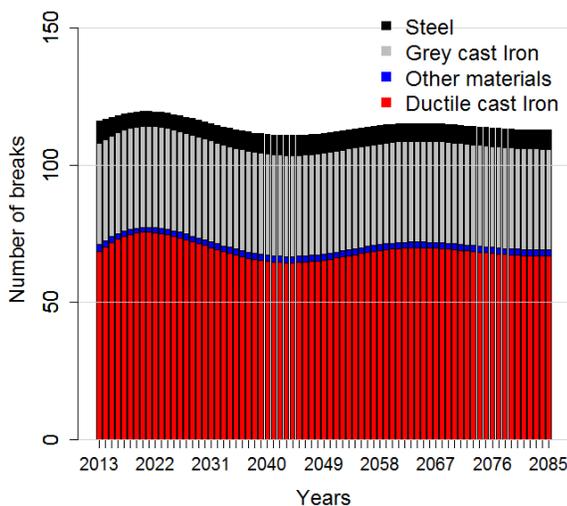


Figure 7: Number breaks in eauservice Lausanne from 2013 to 2085, stratified by materials

### 3.3. Discussion

#### 3.3.1. Probable age of section of pipes over time

One advantage of our model is that it is based on actual historical data in phase 1 (survival functions) and phase 2 (deterioration equations).

In phase 1-step 2 we tested other classical survival functions (normal, lognormal, Gumble, Frechet, etc.), but the Weibull function fits the best with our actual data. However we will use in our future research the “Kernel density estimation” in order to fit closer than the Weibull function.

In phase 1-step 3 (Markov Matrix) we choose 150 years as a maximum age for a pipe segment. Indeed it is an age which seems realistic with Lausanne practices and actual data. However if for another water utility this age is too small or too high it is possible to change this hypothesis

(for example 170 years). You just need to have 170 lines and/or columns for vectors and matrix.

A limitation (see phase 1-step3) of our model is that it is based on the hypothesis that a physical section of pipe (with a material, a diameter, etc.) will be replaced by another pipe section with exactly the same material and the same diameter etc. However it is well known that some materials like grey cast iron will not be used in the future. Very recently we succeed in taking into account this switch. Indeed you have to increase the Markov matrix and then switching pipes characteristics become possible.

#### 3.3.2. Equation of deterioration process of section of pipes depending on their ages

In phase 2 we used the NHPP model which is one of the most comprehensive probabilistic deterioration models. However this model has two limitations. 1) Past failures cannot become an explanatory variable of future failures. However literature review revealed that the more a pipe has broken, the more it is likely to break in the future. 2) The second part of the NHPP model is a Cox model which means that covariates  $Z$  are supposed to have a constant impact during pipe service life. However in practice it can be false. Then in our future research we will try better models, for example the LEYP model.

In Lausanne here, in order to have a short model, we tested only three variables (length, diameter and age) in order to explain the deterioration process. Yet we have for Lausanne also the water pressure, the type of soil (corrosive or not), the road traffic, etc. Therefore next time we will also test these variables in order to see if they are significant in our case study to explain deterioration process.

#### 3.3.3. Mix of probable ages and equations of deterioration process

The past average of the annual renewal rate between 2004 and 2012 in eauservice Lausanne is around 1.4%. It is quite a high figure compare to the French average in 2012 which is 0.6%.

In our scenario “same as in the past” we predicted that the “number of future pipe breaks in

the long term at the scale of eauservice Lausanne territory” is around 110 breaks per year. There is no tendency to increase. Moreover in the past in Lausanne the number of break was around 110.

#### 4. CONCLUSIONS

In order to create a useful long term model, we have to estimate key strategic performance indicators depending on prospective scenario. With prospective survival functions, in (Large et al. 2014) we explained how it is possible to calculate future annual length, renewal rate, average age and annual investment cost at network scale. Here in this article we focused on the tricky part, the estimation of the “future annual number of breaks at the scale of the water utility territory”. It is important to point out that this method can also be used on smaller or bigger networks without issue.

Our next step is to calculate some risk indicators (service outages, disruption of surface traffic, flooding). Risk indicator can be constructed as followed: probability of future break "times" intensity of the break "times" at least one consumer related characteristic (vulnerable element): their quantity, their sensitivity or their value. Then it would be possible to calculate indicators relating to expected maintenance costs.

Another way in which this particular method could be developed would be to move away from the "same as in the past" scenario and focus instead on optimum scenarios. On this basis, it would be possible to estimate how different strategies (for example that of taking greater advantage of roadworks to access pipes) affect "long-term" performance.

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