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Robust partial-learning in linear Gaussian systems
Valérien Némesin, and Stéphane Derrode

Abstract—This paper deals with unsupervised and off-line learning of parameters involved in linear Gaussian systems, i.e. the estimation of the transition and the noise covariances matrices of a state-space system from a finite series of observations only. In practice, these systems are the result of a physical problem for which there is a partial knowledge either on the sensors from which the observations are issued or on the state of the studied system. We therefore propose in this work an “Expectation-Maximization” learning type algorithm that takes into account constraints on parameters such as the fact that two identical sensors have the same noise characteristics, and so estimation procedure should exploit this knowledge. The algorithms are designed for the pairwise linear Gaussian system that takes into account supplementary knowledge on the transition and the noise covariances matrices of a state-space system i.e. a PLGS, which is made robust through QR decompositions and the propagation of a square-root of the covariance matrix of a Kalman-like filter. The algorithm is made robust through the physical system to constrain EM, to reduce the dimension of the space of possible configurations of parameters which give a closed-form solution within EM recursions. However, as exposed later, it is still possible to define configurations of parameters which give a closed-form solution covering many practical cases.

Due to well-known identifiability problems in such systems, the restoration of a signal using an EM algorithm can be very erroneous since the estimated set of parameters can be far from the true one. In this work, we propose to exploit possible a priori knowledge on the physical system to constrain EM, to reduce the dimension of search space and so to improve parameter estimation efficiency and robustness. This strategy, known as partial-learning, has only been addressed in Gaussian mixture models [13], [14], in hidden Markov chains [15] and in linear systems [16]. The problem is more complex in PLGS, given the higher dimension of the matrices and the propagation of a square-root of the covariance matrix of a Kalman-like filter. The key point is that it is not possible to force any type of constraint and to get a closed-form solution for parameter re-estimation formulas within EM recursions. However, as exposed later, it is still possible to define configurations of parameters which give a closed-form solution covering many practical cases.

This paper extends the work done by E. Holmes [16] on linear Gaussian systems to PLGS, with two main contributions: (1) the characterization of constraints that still allow strict EM re-estimation formulas and, (2) a robust version based on the computation of triangular square root of covariance matrices instead of the matrices themselves. The remainder of the paper is organized as follows. Firstly, Section II presents constraints that still allow an EM-based partial-learning algorithm for the pairwise linear system. Then, a robust version of this EM algorithm is proposed in Section III, based on a previous work regarding full-learning estimation [11]. Examples of realistic simulations are reported in Section IV to illustrate the interest and the robustness of the partial-learning algorithm for signal restoration. Finally, main conclusions and perspectives are presented in Section V.

II. PARTIAL LEARNING EM—PLGS ALGORITHM

This Section intends to present configurations of parameters for which a constrained estimation of PLGS parameters with strict EM remains possible.

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ROBUST PARTIAL-LEARNING IN LINEAR GAUSSIAN SYSTEMS 1
E-step

\text{filter and smooth}

\text{compute } \tilde{C}

\text{M-step}

\text{estimate } \tilde{Q}

\text{estimate } \tilde{F}

\text{estimated parameters and restored data}

\text{\begin{tabular}{c} \text{observations, } \ i = 0 \\
\text{i = i + 1} \end{tabular}}

\text{\begin{tabular}{c} \text{no} \\
\text{yes} \end{tabular}}

Fig. 1: Flowchart of the EM-PLGS algorithm.

The EM algorithm search to approach the maximum of likelihood function to estimate parameters. It is an iterative algorithm in which each iteration\(^1\) decomposes into two steps [11]:

- the Expectation step (E-step) evaluates the auxiliary likelihood function from probability density functions of smoothing states \(x_{n|N}\)

\[
g_{\tilde{E}}(Q, F) = -\frac{1}{2} [N + 1] \log |Q| - \frac{1}{2} \text{tr} \left[ Q^{-1} [F, -I] \tilde{C} \left[ F^T \right] \right],
\]

(2)

where \(\tilde{C}\) is the sum of correlation matrices of the augmented smoothing states \(\{\tilde{t}_{n|N}, \tilde{e}_{n+1|N}\}\) (see [11], section II) for details).

- the Maximization step (M-step) re-estimates PLGS parameters by maximizing (2).

It is not possible to impose whatever constraint on \(F\) and \(Q\) while keeping strict EM learning principle. So we next present a series of constraints that preserve EM and which only impact its M-step so that the flowchart sketched in Fig. 1 remains valid. The term “constraints” is used to express a priori knowledge on the PGLS model. In order to preserve a strict EM algorithm (exact closed-form re-estimation formulas for PGLS parameters), we impose \(F\) to be processed row-wise (blocks \(F_i\)), while forcing a decorrelation of corresponding sub-noise blocks \(Q_i\). For example, to constrain observations and states separately \(F_{i,x} = [F_{i,x}, F_{i,y}^2] \) and \(F_{i,v} = [F_{i,v,x}, F_{i,v,y}]\), we have to assume \(Q_{i,x} = Q_{i,y} = 0\).

Remark: We must mention that the EM algorithm designed below does not work in case of singular \(Q^{\tilde{E}}\) (e.g. when observations are perfect).

A. Model noise decorrelation

In order to estimate transition matrix \(F\) using row-wise blocks, noise covariance matrix \(Q\) has to be block-diagonal, i.e. noise decomposes into \(n_{1N}\) independent sub-noises with covariance matrices \(\{Q_i\}_{i=1:n_{1N}}\), and \(F\) is estimated accordingly. Hence, matrices \(Q\)

and \(F\) are written

\[
Q = \sum_{i=1}^{n_{1N}} P_i^T Q_i P_i,
\]

(3)

\[
F = \sum_{i=1}^{n_{1N}} P_i^T F_i,
\]

(4)

where \(\{P_i\}_{i=1:n_{1N}}\) are the projectors of diagonal blocks of \(Q\). We recall that projectors are mutually orthogonal (\(\forall i \neq j\), \(P_i P_j = 0\)).

Under these hypotheses, auxiliary likelihood function in (2) decomposes into \(n_{1N}\) independent sub-functions according to

\[
g_{\tilde{E}}(Q, F) = \sum_{i=1}^{n_{1N}} g_{\tilde{E}_i}(Q_i, F_i),
\]

(5)

where

\[
\tilde{C}_i = \begin{bmatrix} I & 0 \\ 0 & P_i \end{bmatrix} \tilde{C}_i \begin{bmatrix} I & 0 \\ 0 & P_i^T \end{bmatrix},
\]

(6)

which can be maximized separately. Let see now how to maximize each auxiliary sub-functions \(g_{\tilde{E}_i}(Q_i, F_i)\) with respect to \(F_i\) and to \(Q_i\) in next subsections.

B. Constraining row-wise blocks of \(F\)

Each matrix \(F_i\) can always be written

\[
F_i = F_{i,v}^0 + F_{i|x}^0,
\]

(7)

where \(F_{i,v}^0\) is a known matrix and \(F_{i|x}^0\) is a matrix to be estimated. The estimation of \(F_{i,v}^0\) is obtained by maximizing the auxiliary sub-function in (5). Maximizing \(g_{\tilde{E}_i}(Q_i, F_i)\) is equivalent to maximize \(g_{\tilde{E}^b_i}(Q_i, F_i^0)\) with

\[
\tilde{C}_i = \begin{bmatrix} I & 0 \\ -F_{i,v}^0 & I \end{bmatrix} \tilde{C}_i \begin{bmatrix} I & 0 \\ -F_{i,v}^0 & I \end{bmatrix}^T.
\]

(8)

Closed-up form solutions were obtained for the four following matrix shapes:

(0) \(F_{i,v}^0 = 0\), nothing has to be done.

(1) \(F_{i,v}^0 = G_i\), where \(G_i\) is a \((n_i, n_i)\)-matrix to be fully estimated. In this case, matrix \(F_{i,v}^0\) which maximizes the auxiliary sub-function \(g_{\tilde{E}^b_i}(Q_i, F_i^0)\) is given by

\[
\tilde{F}_i = \tilde{C}_i^{1,1} \tilde{C}_i^{0,1}^{-1},
\]

(9)

where \(\tilde{C}_i^{0,0}\), \(\tilde{C}_i^{0,1}\) correspond to the matrix views of \(\tilde{C}_i\) with respective sizes \((n_i, n_i)\) and \((n_i, n_{1N})\) according to

\[
\tilde{C}_i = \begin{bmatrix} \tilde{C}_i^{0,0} & \tilde{C}_i^{0,1} \\ \tilde{C}_i^{1,0} & \tilde{C}_i^{1,1} \end{bmatrix}.
\]

(2) \(F_{i,v}^0 = G_i M_i\), where \(G_i\) is a \((n_i, n_i)\)-matrix to be estimated and \(M_i\) is a known and full-rank \((n_i G_i)\) by \(n_{1N}\)-matrix i.e. \(r k(M_i) = n_i G_i\). In this case, matrix \(F_{i,v}^0\) which maximizes the auxiliary sub-function \(g_{\tilde{E}^b_i}(Q_i, F_i^0)\) is given by

\[
\tilde{F}_i = \tilde{C}_i^{1,1} M_i \left[ M_i \tilde{C}_i^{0,1} M_i^T \right]^{-1} M_i.
\]

(10)

Examples:

- Full-learning in PLGS is obtained by setting \(G_i = \begin{bmatrix} F_{i,v,x}^0 & F_{i,v,y}^0 \\ F_{i,v,x}^0 & F_{i,v,y}^0 \end{bmatrix}\), \(M_i = I\) and \(F_{i,v}^0 = 0\), in which case eq. (4) writes \(F = F_i\) and \(\tilde{C}_i = \tilde{C}_i\). So that, eq. (10) is, up to notation, exactly the one given in [10].
ROBUST PARTIAL-LEARNING IN LINEAR GAUSSIAN SYSTEMS

C. Constraining blocks of Q

Then, auxiliary likelihood function can be expressed using \( Q^M \) by

\[
q_{\mathcal{E}}(Q, F) = q_{\mathcal{E}M}(Q^M, F^M),
\]

where \( \{P_i^M\}_{i=1}^{nQ_B} \) are the projectors of diagonal blocks \( Q^M_i \).

Hence, auxiliary likelihood function decomposes into \( nQ_B \) independent sub-functions according to

\[
g_{\mathcal{E}}(Q, \tilde{F}) = \sum_{i=1}^{nQ_B} g_{\mathcal{E}i}(Q^M_i, F^M_i),
\]

where

\[
F^M_i = \begin{bmatrix} 0 & P_i^M \end{bmatrix}, \quad \tilde{C}_i^M = \begin{bmatrix} I & 0 \\ 0 & P_i^M \end{bmatrix} T.
\]

Example: Let assume a system where two identical sensors measure a physical quantity. It is legitimate to want to fix the same measurement noise covariance \( Q^s \) for both sensors. So, the measurement covariance matrix is a type-(C) matrix, with shape

\[
Q^s = Q^s_2 = \begin{bmatrix} Q^s & 0 \\ 0 & Q^s \end{bmatrix},
\]

with

\[
P_1^s = [I, 0], \quad P_2^s = [0, I], \quad M_2^s = M_2^s = I \quad \text{and} \quad R_2 = Q^s.
\]
In this section, we presented an EM algorithm that takes into account constraints on matrices $F$ and $Q$ to allow partial-learning using strict EM principle. Despite constraints may seem restrictive, this algorithm is very general as illustrated in the Section IV. Whatever, this kind of algorithm suffers from numerical instabilities [11]. Next section is devoted to design a robust version of this partial-learning algorithm.

### III. ROBUST ALGORITHM

Following [18], we now propose a robust version of the partial-learning algorithm which makes use of only a triangular square-root $S$ for each positive-definite matrix $P = S^T S$ instead of $P$ itself. Robust algorithm is almost the same as the one presented in [11] except for the M-step which is modified to take into account the constraints for partial-learning. The only difference lies in the way to compute $\hat{Q}_{i}^{1/2}$ from matrices $F$ and $\hat{C}_i^{1/2}$ [11, Section III].

The algorithm decomposes into four steps:\n
1. First, a square root of $\hat{C}_i^{1/2}$ (16) is given by QR decomposition \[ \hat{C}_i^{1/2} = Z^* [C_i^{1/2} \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}] . \] (24)

2. Then each block $Q_i^{MC}$ is estimated using the following algorithm\n
   a) A square root of $\hat{C}_i^{1/2}$ (20) is obtained by QR decomposition \[ \hat{C}_i^{1/2} = Z^* [C_i^{1/2} \begin{bmatrix} I & 0 \\ 0 & P_i^{MC} \end{bmatrix}]^T . \] (25)

   b) $[Q_i^{MC}]^{1/2}$ is computed depending on its block-type.

   - For a type-(O) block, $[Q_i^{MC}]^{1/2} = [Q_i^{O}]^{1/2}$.
   - For a type-(A) block, $[Q_i^{MC}]^{1/2}$ is computed according to QR decomposition \[ [Q_i^{MC}]^{1/2} = \frac{1}{\sqrt{N + 1}} Z^* [C_i^{1/2} \begin{bmatrix} I & 0 \\ 0 & P_i^{MC} \end{bmatrix}]^T . \] (26)

   - For a type-(B) block, $[Q_i^{MC}]^{1/2}$ is computed according to (22).

   - For a type-(C) block, $[Q_i^{MC}]^{1/2}$ is computed according to the following method:
     - A square root of $R_i^{MC}$ (23) is computed according to QR algorithm: $R_i^{MC}^{1/2} = \frac{1}{(W + 1)n} \begin{bmatrix} M_i^{n} \end{bmatrix}^T W_i^{n} \] where \[ W_i^{n} = \sum_{j=1}^{k} \begin{bmatrix} M_i^{n} \end{bmatrix}^{-T} P_i^{n} \begin{bmatrix} M_i^{n} \end{bmatrix} - I \]
   \[ \hat{C}_i^{1/2} = \begin{bmatrix} F_i^{n} \end{bmatrix} - I \end{bmatrix}^T P_i^{n} \begin{bmatrix} M_i^{n} \end{bmatrix}^{-T} . \] (27)

   a) For $k = 0$, $\begin{bmatrix} W_i^{n} \end{bmatrix} = 0$
   b) For $k = 1$ to $n$, $\begin{bmatrix} W_i^{n} \end{bmatrix}^{1/2} = 0$ is computed from QR decomposition \[ \begin{bmatrix} W_i^{k-1} \end{bmatrix}^{1/2} = Z^* \begin{bmatrix} W_i^{k-1} \end{bmatrix}^{1/2} K_i \] where $K_i = \begin{bmatrix} 1 & 0 \\ 0 & M_i \end{bmatrix}$.

3All unitary matrices denoted $Z^*$ that appears in QR decompositions are never used.

### IV. EXPERIMENTS

To illustrate the potential of unsupervised partial-learning in PLGS, we propose two series of experiments which are representative of different learning situations. Please note that initial expectation $t_0$ and covariance matrix $Q_0$ are not estimated.

#### A. Simulated PLGS scalar signal

In this first experiment, all series of PLGS-signals were simulated according to parameters reported in first row of Table I. Values for $t_0$ and $Q_0$ were respectively set to $[0, 0]$ and to $I$. Then, the observations were restored by

- the optimal PLGS (OPLGS),
- an equivalent LGS (ELGS),

We used the method described in [11] in order to select an equivalent filter in which $F^{O,L} = I$ and $F^{O,L,0} = 0$.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\theta_0$</th>
<th>$\theta_LGS$</th>
<th>$\theta_{PLGS}$</th>
<th>$\theta_{PLG}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0.4, 0.3]$</td>
<td>$[1.0, 0.2]$</td>
<td>$[0.2, 0.1]$</td>
<td>$[0.0, 0.1]$</td>
<td>$[0.0, 0.0]$</td>
</tr>
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</tr>
</tbody>
</table>
2. a free PLGS (FPLGS), that is to say full-learning PLGS
3. an equivalent\(^3\) PLGS (EPLGS),
4. and a constrained PLGS (CPLGS).

For the CPLGS, we imposed
• a two-block decomposition of transition matrix \( F \),
• a type-(0) constraint on \( F^{(0)} = [1, 0] \),
• a type-(2) constraint on \( F^{(2)} = \lambda[1, 1] + [1, 0] \) with \( \lambda \) to be estimated,
• a type-(B) constraint on \( Q : Q = \gamma \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \) with \( \gamma \) to be estimated \(^4\).

Initial EM parameters are given in row two, and final mean estimation of 100 experiments in rows three to six for ELGS, FPLGS, EPLGS and CPLGS respectively for a \( N = 1000 \) samples signal. An example of trajectory is reported in Fig. 2a. Mean MSE for increasing signal sizes for the four filters are reported in Fig. 2b.

This experiments allows to draw three main conclusions: (1) As expected, an ELGS is not able to restore correctly PLGS data. Moreover, FPLGS does not give expected results due to the identifiability problem of PLGS. (2) Although we get nice estimation by EPLGS, CPLGS produces better one for short as well as long signals. (3) For signals that exceed 1000 samples, CPLGS has the same performances than the optimal filter!

![Fig. 2: Mean Square Error (MSE) for simulated signals with varying number of samples, for the five filters experimented in Section IV-A. A trajectory is reported in (a).](image)

![Fig. 3: Instantaneous MSE for simulated signals with for \( N = 100 \) samples for the three filters defined in Section IV-B (b). An example of trajectory is reported in (a).](image)

**TABLE II:** Parameters of experiment in Section IV-B. Rows 1 and 2: initial values for EM; Rows 3 to 5: estimated parameters for the three filters described in the text.

<table>
<thead>
<tr>
<th>( \theta^{(0)}_{LGS/PLGS} )</th>
<th>( F )</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \begin{pmatrix} 1 &amp; 0 \ 0 &amp; 0 \end{pmatrix} )</td>
<td>( \begin{pmatrix} 0 &amp; 1 \ 0 &amp; 0 \end{pmatrix} )</td>
<td>( \begin{pmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{pmatrix} )</td>
</tr>
<tr>
<td>( \begin{pmatrix} 0.05 &amp; 0 \ 0.01 &amp; 0 \end{pmatrix} )</td>
<td>( \begin{pmatrix} 0.2 &amp; 0 \ 0 &amp; 0.1 \end{pmatrix} )</td>
<td>( \begin{pmatrix} 0.04 &amp; 0.01 \ 0.07 &amp; 0.15 \end{pmatrix} )</td>
</tr>
<tr>
<td>( \begin{pmatrix} -0.38 &amp; 0.61 \ 0 &amp; 0 \end{pmatrix} )</td>
<td>( \begin{pmatrix} 0.002 &amp; 0 \ 0.001 &amp; 0 \end{pmatrix} )</td>
<td>( \begin{pmatrix} -0.003 &amp; 0 \ 0 &amp; 0.001 \end{pmatrix} )</td>
</tr>
</tbody>
</table>

**B. Sinusoidal signal**

In order to evaluate partial-learning EM-PLGS algorithm, we chose to restore a noisy signal \( y(t) = p(t) + b(t) \), where \( p(t) = \cos(0.2\pi t) \) is a pseudo-sinusoid and \( b(t) \) is a white Gaussian noise with variance 0.1. Noisy signal is sampled between moments \( t = 0s \) and \( t = 90s \) with a sampling period \( T_S = 1s \). Hence, previous model becomes

\[
\begin{aligned}
    y_n &= p(nT_S) + \omega_n^{(y)} \\
    y(nT_S) &= b(nT_S)
\end{aligned}
\]

\(^4\)Constraint on \( Q \) is compatible with constraints on \( F \) since we forced decorrelation between process and measurement noises.
Three filters were used to restore the signal

1. A classical LGS in which there are no correlation between measurement and process noise covariances, and with \( F^{y,x} = \mathbf{I} \) and \( F^{y,y} = \mathbf{0} \).

2. A PLGS in which \( F^{y,x} = \mathbf{I} \) and \( F^{y,y} = \mathbf{0} \).

3. An augmented and constrained LGS (ACLGS). For that, we decomposed \( Q \) and \( F \) into, respectively, two blocks \( Q_1, Q_2 \) and \( F_1, F_2 \) with projectors \( P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \) and \( P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \). Block \( Q_1 \) and \( Q_2 \) are type-(A) or free blocks. We imposed a type-(0) constraint on block \( F_1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \), and a type-(2) constraint on block \( F_2 = \begin{bmatrix} a & b \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \) with \( a \) and \( b \) to be estimated.

Table II reports EM initial parameters in rows 1 and 2 and estimated parameters in rows 3 to 5. An example of generated signal is reported in Fig. 3a, whereas instantaneous mean square restoration errors for \( 1 \) are reported in Fig. 3a, whereas instantaneous mean square restoration errors for \( 1 \) are reported in Fig. 3b. These experiments allow to draw three main conclusions:

1. Classical LGS can not restore noisy sinusoidal data.
2. Although PLGS outperforms restoration by LGS, augmented and constrained LGS gives the best restoration. The difference between PLGS and ACLGS is that PLGS introduces measurement noise in hidden states, which degrades a little bit the restoration of this kind of signals.
3. An augmented LGS seems to be the optimal model for restoring sinusoidal data, as the low process noise covariance matrix and the low MSE suggest.

V. CONCLUSION

The main interest of the Pairwise Linear Gaussian System (PLGS) relies on the possibility for the user to finely tune the amount of interactions between successive observations and states. In turn, this increases the number of parameters to be estimated if no a priori knowledge is available. In practice, due to the underlying physics of the model, some knowledge parameters are often partially known, whether they are fixed or constrained by other parameters. In this context, we have proposed an EM-based approach for unsupervised partial estimation of parameters in a PLGS, by extending the work in [16] regarding classical linear systems. Since the latter is a particular PLGS, our algorithm is directly applicable to robust partial-learning of linear Gaussian system parameters.

We have evaluated our algorithm in different configurations including the difficult case of very short signals (i.e. \( N = 100 \) samples). All experiments confirm the robustness of our approach. As expected, the constrained PLGS-EM algorithm provides a better quality estimation than the full-learning one. It is worth noting that, in some cases, the classical linear system can outperform the PLGS when using an augmented state-space, yet at the larger number of parameters to estimate.

A perspective to this work is to relax strict EM principle, allowing GEM principle instead [3], so that to increase the possibilities to constraint the model with other kind of a priori knowledge. Especially of interest, will be to impose that the model we want to estimate is stationary.

ACKNOWLEDGMENT

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