Contrast re-enhancement of Total-Variation regularization jointly with the Douglas-Rachford iterations
Charles-Alban Deledalle, Nicolas Papadakis, Joseph Salmon

To cite this version:
Charles-Alban Deledalle, Nicolas Papadakis, Joseph Salmon. Contrast re-enhancement of Total-Variation regularization jointly with the Douglas-Rachford iterations. Signal Processing with Adaptive Sparse Structured Representations, Jul 2015, Cambridge, United Kingdom. hal-01187061

HAL Id: hal-01187061
https://hal.archives-ouvertes.fr/hal-01187061
Submitted on 26 Aug 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Contrast re-enhancement of Total-Variation regularization jointly with the Douglas-Rachford iterations

Charles-Alban Deledalle
CNRS – Université Bordeaux
Institut de Mathématiques de Bordeaux, Talence, France
Email: firstname.lastname@math.u-bordeaux.fr

Joseph Salmon
CNRS LTCI – Télécom ParisTech
Institut Mines-Télécom, Paris, France
Email: joseph.salmon@telecom-paristech.fr

Abstract—Restoration of a piece-wise constant signal can be performed using anisotropic Total-Variation (TV) regularization. Anisotropic TV may capture well discontinuities but suffers from a systematic loss of contrast. This contrast can be re-enhanced in a post-processing step known as least-square refitting. We propose here to jointly estimate the refitting during the Douglas-Rachford iterations used to produce the original TV result. Numerical simulations show that our technique is more robust than the naive post-processing one.

I. INTRODUCTION

We consider the reconstruction of a 2D signal identified as a vector $u_0 \in \mathbb{R}^N$ from its noisy observation $f = \Phi u_0 + w \in \mathbb{R}^p$ with $w \in \mathbb{R}^p$ a zero-mean noise component and $\Phi \in \mathbb{R}^{p \times N}$ a linear operator accounting for a loss of information (e.g., low-pass filter). Anisotropic TV regularization writes, for $\lambda > 0$, as [1]

$$\min_{u \in [0]} \frac{1}{2}\|\Phi u - f\|^2 + \lambda \|\nabla u\|_1,$$

(1)

with $\nabla u \in \mathbb{R}^{2N}$ being the concatenation of vertical and horizontal components of the discrete gradient vector field of $u$, and $\|\nabla u\|_1 = \sum_i |\nabla u_i|$ being a sparsity promoting term. Anisotropic TV is known to recover piece-wise constant signals. However, even though the discontinuities can be correctly recovered in some cases, the amplitudes of $u^{TV}$ are known to suffer from a loss of contrast compared to $u_0$ [2].

II. LEAST-SQUARE REFITTING PROBLEM

A simple technique to correct this effect, known as least-square refitting, consists in enhancing the amplitudes of $u^{TV}$ while leaving unchanged the set of discontinuities, as

$$\hat{u}^{TV} \in \arg\min_{u \in \mathbb{R}^N} |\Phi u - f|^2$$

(2)

where, for $x \in \mathbb{R}^N$, $\supp(x) = \{i \in [2N] ; |x_i| \neq 0\}$ denotes the support of $x$. Post-refitting identifies $\supp(\nabla u^{TV})$ and solves (2) [3], typically with a conjugate gradient. However, $u^{TV}$ is usually obtained thanks to a converging sequence $u^k$, and unfortunately, $\supp(\nabla u^k)$ can be far from $\supp(\nabla u^{TV})$ even though $u^k$ can be made arbitrarily close to $u^{TV}$. Such erroneous support identification can lead to results that strongly deviates from the solution $\hat{u}^{TV}$.

III. JOINT REFITTING WITH DOUGLAS-RACHFORD

To alleviate this difficulty, we build a sequence $\tilde{u}^k$ jointly with $u^k$ that converges towards a solution $\hat{u}^{TV}$. We consider the Douglas-Rachford sequence $u^k$ applied to the splitting TV regularization [4] given by

$$u^k \in \arg\min_{u \in [0]} \frac{1}{2}\|\Phi u - f\|^2 + \lambda \|\nabla u\|_1$$

where $\mathbb{S}_1$ is the indicator function of a set $S$. This leads to the proposed algorithm given, for $\tau > 0$ and $\beta > 0$, by Eq. (3) (see right column). The sequence $u^k$ is exactly the Douglas-Rachford sequence converging towards a solution $u^{TV}$ [5]. Regarding $\tilde{u}^k$, we prove the following.

**Theorem 1.** Let $\alpha > 0$ be the minimum non zero value of $|\nabla u_i|$, $i \in [2N]$. For $0 < \beta < \alpha\lambda$, $u^k$ converges towards a solution $\hat{u}^{TV}$.

**Sketch of proof:** As $u^k$ converges towards a solution $u^{TV}$, for $k$ large enough, we get after few manipulations and triangle inequalities that

$$\begin{align*}
\mu^{k+1} &= (Id + \Delta)^{-1}(2\mu^k - \mu^k - \mu^k - \mu^k) / 2 + \mu^k / 2, \\
\tilde{u}^{k+1} &= (Id + \Delta)^{-1}(2\tilde{u}^k - \tilde{u}^k - \tilde{u}^k - \tilde{u}^k) / 2 + \mu^k / 2, \\
\zeta^{k+1} &= \nabla u^{k+1}, \\
\tilde{\zeta}^{k+1} &= \nabla u^{k+1}, \\
\tilde{u}^{k+1} &= \mu^{k+1} + \tau \Phi^T (Id + \tau \Phi^T)^{-1} (f - \Phi u^{k+1}), \\
\tilde{u}^{k+1} &= \mu^{k+1} + \tau \Phi^T (Id + \tau \Phi^T)^{-1} (f - \Phi u^{k+1}), \\
\tilde{\zeta}^{k+1} &= \Psi (\zeta^{k+1}, \lambda), \\
\tilde{\zeta}^{k+1} &= \Psi (\zeta^{k+1}, \lambda), \\
\mu^{k+1} &= \Pi_{\zeta^{k+1}} (\zeta^{k+1}, \lambda), \\
\mu^{k+1} &= \Pi_{\zeta^{k+1}} (\zeta^{k+1}, \lambda),
\end{align*}$$

(3)

**IV. RESULTS AND DISCUSSION**

Figure 1 shows results on an 8bits image damaged by a Gaussian blur of 2px and white noise $\sigma = 20$. The parameter $\beta$ is chosen as the smallest positive value up to machine precision. While TV reduces the contrast, refitting recovers the original amplitudes and keep unchanged the discontinuities. Post-refitting offers comparable results to ours except for suspicious oscillations due to wrong support identification.

Being computing during the Douglas-Rachford iterations, our refitting strategy is free of post-processing steps such as support identification. It is moreover easy to implement and can be used likewise for other $\ell_1$ analysis penalties. Extensions of this approach for isotropic TV or block sparsity regularizations are under investigation.

**REFERENCES**


