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# Contrast re-enhancement of Total-Variation regularization jointly with the Douglas-Rachford iterations

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**Abstract**—Restoration of a piece-wise constant signal can be performed using anisotropic Total-Variation (TV) regularization. Anisotropic TV may capture well discontinuities but suffers from a systematic loss of contrast. This contrast can be re-enhanced in a post-processing step known as least-square refitting. We propose here to jointly estimate the refitting during the Douglas-Rachford iterations used to produce the original TV result. Numerical simulations show that our technique is more robust than the naive post-processing one.

## I. INTRODUCTION

We consider the reconstruction of a 2D signal identified as a vector  $u_0 \in \mathbb{R}^N$  from its noisy observation  $f = \Phi u_0 + w \in \mathbb{R}^P$  with  $w \in \mathbb{R}^P$  a zero-mean noise component and  $\Phi \in \mathbb{R}^{P \times N}$  a linear operator accounting for a loss of information (e.g., low-pass filter). Anisotropic TV regularization writes, for  $\lambda > 0$ , as [1]

$$u^{\text{TV}} \in \underset{u \in \mathbb{R}^N}{\operatorname{argmin}} \frac{1}{2} \|\Phi u - f\|^2 + \lambda \|\nabla u\|_1, \quad (1)$$

with  $\nabla u \in \mathbb{R}^{2N}$  being the concatenation of vertical and horizontal components of the discrete gradient vector field of  $u$ , and  $\|\nabla u\|_1 = \sum_i |(\nabla u)_i|$  being a sparsity promoting term. Anisotropic TV is known to recover piece-wise constant signals. However, even though the discontinuities can be correctly recovered in some cases, the amplitudes of  $u^{\text{TV}}$  are known to suffer from a loss of contrast compared to  $u_0$  [2].

## II. LEAST-SQUARE REFITTING PROBLEM

A simple technique to correct this effect, known as least-square refitting, consists in enhancing the amplitudes of  $u^{\text{TV}}$  while leaving unchanged the set of discontinuities, as

$$\tilde{u}^{\text{TV}} \in \underset{u; \operatorname{supp}(\nabla u) \subset \operatorname{supp}(\nabla u^{\text{TV}})}{\operatorname{argmin}} \|\Phi u - f\|^2 \quad (2)$$

where, for  $x \in \mathbb{R}^{2N}$ ,  $\operatorname{supp}(x) = \{i \in [2N]; |x_i| \neq 0\}$  denotes the support of  $x$ . Post-refitting identifies  $\operatorname{supp}(\nabla u^{\text{TV}})$  and solves (2) [3], typically with a conjugate gradient. However,  $u^{\text{TV}}$  is usually obtained thanks to a converging sequence  $u^k$ , and unfortunately,  $\operatorname{supp}(\nabla u^k)$  can be far from  $\operatorname{supp}(\nabla u^{\text{TV}})$  even though  $u^k$  can be made arbitrarily close to  $u^{\text{TV}}$ . Such erroneous support identifications can lead to results that strongly deviates from the solution  $\tilde{u}^{\text{TV}}$ .

## III. JOINT REFITTING WITH DOUGLAS-RACHFORD

To alleviate this difficulty, we build a sequence  $\tilde{u}^k$  jointly with  $u^k$  that converges towards a solution  $\tilde{u}^{\text{TV}}$ . We consider the Douglas-Rachford sequence  $u^k$  applied to the splitting TV reformulation [4] given by

$$u^{\text{TV}} \in \underset{u \in \mathbb{R}^N}{\operatorname{argmin}} \min_{z \in \mathbb{R}^N \times 2} \frac{1}{2} \|\Phi u - f\|^2 + \lambda \|z\|_1 + \iota_{\{z, u; z = \nabla u\}}(z, u)$$

where  $\iota_S$  is the indicator function of a set  $S$ . This leads to the proposed algorithm given, for  $\tau > 0$  and  $\beta \geq 0$ , by Eq. (3) (see right column). The sequence  $u^k$  is exactly the Douglas-Rachford sequence converging towards a solution  $u^{\text{TV}}$  [5]. Regarding  $\tilde{u}^k$ , we prove the following.

**Theorem 1.** *Let  $\alpha > 0$  be the minimum non zero value of  $|(\nabla u)_i|$ ,  $i \in [2N]$ . For  $0 < \beta < \alpha\lambda$ ,  $\tilde{u}^k$  converges towards a solution  $\tilde{u}^{\text{TV}}$ .*

*Sketch of proof:* As  $u^k$  converges towards a solution  $u^{\text{TV}}$ , for  $k$  large enough, we get after few manipulations and triangle inequalities that

$$\begin{cases} \mu^{k+1} = (\operatorname{Id} + \Delta)^{-1}(2u^k - \mu^k - \operatorname{div}(2z^k - \zeta^k))/2 + \mu^k/2, \\ \tilde{\mu}^{k+1} = (\operatorname{Id} + \Delta)^{-1}(2\tilde{u}^k - \tilde{\mu}^k - \operatorname{div}(2\tilde{z}^k - \tilde{\zeta}^k))/2 + \tilde{\mu}^k/2, \\ \zeta^{k+1} = \nabla \mu^{k+1}, & \tilde{\zeta}^{k+1} = \nabla \tilde{\mu}^{k+1}, \\ u^{k+1} = \mu^{k+1} + \tau \Phi^t (\operatorname{Id} + \tau \Phi \Phi^t)^{-1} (f - \Phi \mu^{k+1}), \\ \tilde{u}^{k+1} = \tilde{\mu}^{k+1} + \tau \Phi^t (\operatorname{Id} + \tau \Phi \Phi^t)^{-1} (f - \Phi \tilde{\mu}^{k+1}), \\ z^{k+1} = \Psi_{\zeta^{k+1}}(\zeta^{k+1}, \lambda), & \tilde{z}^{k+1} = \Pi_{\tilde{\zeta}^{k+1}}(\tilde{\zeta}^{k+1}, \lambda) \end{cases} \quad (3)$$

$$\text{where } \Psi_{\zeta}(\zeta, \lambda)_i = \begin{cases} 0 & \text{if } |\zeta_i| \leq \tau\lambda, \\ \zeta_i - \tau\lambda \operatorname{sign} \zeta_i & \text{otherwise} \end{cases}$$

$$\text{and } \Pi_{\tilde{\zeta}}(\tilde{\zeta}, \lambda)_i = \begin{cases} 0 & \text{if } |\zeta_i| \leq \tau\lambda + \beta, \\ \tilde{\zeta}_i & \text{otherwise.} \end{cases}$$



Fig. 1. Damaged image, result of TV, post-refitting and our joint-refitting.

$\{i; |\zeta_i^k| > \tau\lambda + \beta\} = \operatorname{supp}(\nabla u^{\text{TV}})$  (for the given range of  $\beta$ ). As a result, for  $k$  large enough, the sequence (3) can be rewritten by substituting  $\Pi_{\tilde{\zeta}}(\cdot, \lambda)$  by the projector onto  $\{u; \operatorname{supp}(u) \subset \operatorname{supp}(\nabla u^{\text{TV}})\}$  which is exactly the Douglas-Rachford sequence for the refitting problem (2) which is provably converging towards a solution  $\tilde{u}^{\text{TV}}$  [5].  $\square$

## IV. RESULTS AND DISCUSSION

Figure 1 shows results on an 8bits image damaged by a Gaussian blur of 2px and white noise  $\sigma = 20$ . The parameter  $\beta$  is chosen as the smallest positive value up to machine precision. While TV reduces the contrast, refitting recovers the original amplitudes and keep unchanged the discontinuities. Post-refitting offers comparable results to ours except for suspicious oscillations due to wrong support identification.

Being computing during the Douglas-Rachford iterations, our refitting strategy is free of post-processing steps such as support identification. It is moreover easy to implement and can be used likewise for other  $\ell_1$  analysis penalties. Extensions of this approach for isotropic TV or block sparsity regularizations are under investigation.

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