L2 -induced gain for discrete-time switched Lur’e systems via a suitable Lyapunov function
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Abstract: This paper deals with discrete-time switched Lur’e systems including an additional saturated input and a bounded energy disturbance. Two control design issues are investigated related to the $L_2$-induced gain and the rejection of disturbance. An common optimization problem under LMI conditions and differing from the cost function to optimize, is provided to offer a solution to the both problems. The main approach is to upper bound a criterion with a suitable Lur’e type Lyapunov function. Numerical examples are given to underline the efficiency of our approach and to allow a discussion with the literature.

Keywords: Switched systems; Lur’e type systems; $L_2$-induced gain; Lur’e type Lyapunov functions.

1. INTRODUCTION

Since its introduction in the literature by Lur’e (Lur’e and Postnikov, 1944; Kalman, 1963), the Lur’e problem, that is the stability analysis of the interconnection between a linear system and a nonlinearity verifying a cone bounded sector condition has attracted considerable attention. This should be probably explained by the large class of practical systems, which can be modeled by such a nonlinear system. One of the widely encountered nonlinearity in practice is the saturation, which generates a specific field of research (Khalil, 2002; Hu and Lin, 2001; Kapila and Grigoriadis, 2002; Tarbouriech et al., 2011). Taking into account the presence of the saturation implies the issues of local stability associated with estimating the basin of attraction of the origin. Recently a new Lur’e type Lyapunov function has been provided. It is particularly suitable for studying the global or local stability or control synthesis of discrete-time Lur’e systems and it exhibits possibly non-convex and disconnected level sets (Gonzaga et al., 2012b). The property of disconnection of the level sets generates several interesting issues for discrete-time Lur’e systems obtained by the discretization of continuous-time ones. One can cite among them the questions treated in (Louis et al., 2013, 2015b,a).

A switched version of the discrete-time Lur’e systems has been proposed in (Jungers et al., 2011). It consists of a set of finite number of Lur’e systems and a switching law that indicates at each time which Lur’e system is active. This type of systems is of interest because the considered non-linearity depends on the active mode and is characterized by a switched cone bounded sector condition. The Lur’e problem provided in (Gonzaga et al., 2012b, 2011b) has been extended to the case of switched systems: when the switching law is a perturbation (Gonzaga et al., 2012a, 2011a), or when it is a control input (Jungers et al., 2013). See (Gonzaga et al., 2013) in (Daafouz et al., 2013) for an overview about this topic.

To go beyond the stability issues, the performance aspect of nonlinear systems has been considered, with $L_2$-induced gain and disturbance rejection (Hindi and Boyd, 1998), or quadratic performance index explicitly depending on the saturation (Schnitter, 2000). This last type of quadratic performance index including a saturated control input has been used to formalize the issue of global stabilization of a linear system with a saturated input (Goebel, 2005). Iterative procedure using sum-of-square problems to obtain the optimal control for input-saturated system with saturation dependent cost function could be also provided (Baldi et al., 2015).

In this paper, a switched discrete-time Lur’e system with additional saturated input and $L_2$-bounded disturbance is considered. The contribution is twofold. More precisely, two switched control design problems are investigated aiming for the first one at obtaining the largest set of initial conditions ensuring the stability and for the second one at minimizing the $L_2$-induced gain. The main originality is here to upper bound a quadratic cost function depending explicitly on the nonlinearity and the saturation by using a suitable Lur’e type Lyapunov function in order to improve the results with respect to the literature.
The paper is organized as follows. Section 2 defines the studied system and settles the two problems which are coped with. Furthermore some technical lemmas are presented in order to build the main result given in Section 3. Section 4 is devoted to the numerical illustration of the contribution and allowing a large discussion with respect to the aforementioned literature, before concluding remarks in Section 5.

**Notation.** For a matrix $A \in \mathbb{R}^{n \times n}$, $A'$ denotes its transpose. If $A = A' \in \mathbb{R}^{n \times n}$, then $A < 0$ ($A \leq 0$) means that $A$ is negative- (semi-)definite. The components of any vector $x \in \mathbb{R}^n$ are denoted by $x_{(i)}$, $\forall i = 1, \ldots, n$. Inequalities between vectors are component-wise: $x \leq 0$ means that $x_{(i)} \leq 0$ and $x < y$ means that $x_{(i)} < y_{(i)}$. $I_n$ (resp. $0_n$) denotes the $n \times n$ identity (resp. null) matrix. The symbol $\ast$ stands for symmetric blocks in matrices. The Hermitian operator is denoted $\text{He}(\cdot)$.

**2. PROBLEM STATEMENT AND PRELIMINARIES**

Consider the discrete-time switched Lur’e system, with $N$ nonlinear modes, including a saturated input and a disturbance:

$$x_{k+1} = A_{\sigma_k} x_k + G_{\sigma_k} \varphi_{\sigma_k}(y_k) + B_{\sigma_k} \text{sat}(u_k) + E_{\sigma_k} w_k,$$

$$y_k = C_{\sigma_k} x_k,$$

$$z_k = C'_{\sigma_k} x_k + L_{\sigma_k} \varphi_{\sigma_k}(y_k) + D_{\sigma_k} \text{sat}(u_k) + E'_{\sigma_k} w_k,$$

$$u_k = K_{\sigma_k} x_k + \Gamma_{\sigma_k} \varphi_{\sigma_k}(y_k),$$

where $x_k \in \mathbb{R}^n$ is the state, $y_k \in \mathbb{R}^p$ is the output, $z_k \in \mathbb{R}^p$ is the performance output, $w_k \in \mathbb{R}^q$ is the disturbance of bounded energy and $u_k \in \mathbb{R}^m$ is the control input. The switching law $\sigma(\cdot)$ is defined to take arbitrarily its value in a finite set, $\sigma : \mathbb{N} \rightarrow \{1, \ldots, N\}$.

The nonlinearities $\varphi_{\sigma_k}(\cdot) : \mathbb{R}^p \rightarrow \mathbb{R}^p$ are decentralized and satisfy their own cone bounded sector condition defined by $N$ diagonal positive definite matrices $\Omega_k \in \mathbb{R}^{p \times p}$ by

$$\varphi_{\sigma_k}(y_k) S_{\sigma_k}(\varphi_{\sigma_k}(y_k) - \Omega_k y_k) \leq 0, \quad \forall k \in \mathbb{N},$$

where $S_{\sigma_k}$ and $\Omega_{\sigma_k} \in \mathbb{R}^{p \times p}$ are diagonal positive definite matrices (see Khalil, 2002).

The saturation operator is defined by

$$\text{sat}(u)_{(i)} = \text{sign}(u_{(i)}) \min(\rho_{(i)}, |u_{(i)}|), \quad \forall i \in \{1, \ldots, m\},$$

with $\rho > 0$ is the symmetric saturation level.

The performance index related to the system (1)-(4) is defined to be quadratic. This paper will investigate more particularly the $L_2$-induced gain associated with the criterion:

$$J = \sum_{k \in \mathbb{N}} (z_k' z_k - \gamma w'_k w_k).$$

Thanks to the definition of the performance output (3), the criterion $J$ given by (7) is an explicit function of the nonlinearity and of the saturation.

The disturbance $\{w_k\}_{k \in \mathbb{N}}$ is assumed to have a bounded energy less than or equal to $\delta^{-1} \in \mathbb{R}^+$, that is it belongs to the set

$$W_\delta = \left\{ w : \mathbb{N} \rightarrow \mathbb{R}^p ; \sum_{k \in \mathbb{N}} w'_k w_k \leq \delta^{-1} \right\}.$$
In addition, we have
\[ y_{k+1} = C_y^y M_i \eta_k, \]
\[ z_k = Z_i \eta_k. \]

The issues treated in this paper are the following Problem 1 and Problem 2.

**Problem 1. (Disturbance).** Consider the closed-loop system (1)–(4) and a fixed scalar \( \delta > 0 \). The purpose is to design a control law (4) in order to ensure that the trajectories starting from \( x_0 \) in a bounded set, as large as possible, are bounded for any disturbance belonging to \( \mathcal{W}_b \).

**Problem 2. (Disturbance rejection).** Consider the closed-loop system (1)–(4) and a fixed scalar \( \delta > 0 \). The purpose is to design a control law (4) in order to ensure that the trajectories starting from \( x_0 \) are bounded for any disturbance belonging to \( \mathcal{W}_b \) and, in addition, to design the lowest estimation of the \( \mathcal{L}_2 \)-induced gain from the disturbance \( w_k \) to the performance output \( z_k \).

Before presenting the main result in Section 3, technical lemmas are given.

**Lemma 1.** Consider the switching matrices \( K_i, J_{i,j} \in \mathbb{R}^m \times n, \Gamma_i, J_{i,j} \in \mathbb{R}^m \times p \) and the following notations \( \tilde{K}_i = [K_i^T \Gamma_i^T]^T \) and \( \tilde{J}_i = [J_{i,j}^T \Gamma_j]^T, \) with \( i \in \mathcal{I}_N. \) If \( \tilde{x}_k = (x_k^T \varphi_k^T)^T \) is an element of \( L_Y(\delta^{-1}) \subset \mathcal{S}(\tilde{K}_i) \) then \( \tilde{x}_k \) defined by (4), the nonlinearity \( \Psi(u_k) \) satisfies the following inequality
\[ 2\Psi(u_k)^T \tilde{T}_i^{-1} (\Psi(u_k) - J_{i,j}x_k - J_{i,j}\varphi_i) \leq 0, \]
for any diagonal positive definite switching matrix \( T_i \in \mathbb{R}^m \times m. \)

**Proof 1.** See (Gonzaga et al., 2012a).

In order to clarify the rest of the paper and more precisely Lemma 2, the following notations are defined, \( \Lambda = \bigcup_{j 
exists i} \Lambda_i \bigcup_{j \in j} \Lambda_j \), \( Y_i = (S_i - \Lambda_j)\Omega_i C_j^j \),
\[ M_{i,j}^a = \begin{bmatrix} P_j & 0 & 0 & 0 & 0 & 0 & 0 \\ -\gamma I_{p_j} & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{i,j}^{cl} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & T_{i,j}^p C_{i,j}^{cl} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \]
\[ M_{i,j}^b = \begin{bmatrix} I_n & 0_{p_x \times n} & 0_{p_y \times n} & 0_{p_z \times n} & 0_{p_y \times n} & 0_{p_z \times n} & 0_{p_y \times n} \\ -I_{n} & 0_{p_y \times n} & 0_{p_z \times n} & 0_{p_y \times n} & 0_{p_z \times n} & 0_{p_y \times n} & 0_{p_z \times n} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \]
\[ M_{i,j}^c = \begin{bmatrix} 0_{n \times p_y} & 0_{p_y \times n} \end{bmatrix}, \]
\[ M_{i,j}^d = \begin{bmatrix} 0_{n \times p_y} & 0_{p_y \times n} \end{bmatrix}. \]

\[ M_{i,j}^e = \begin{bmatrix} 0_{n \times p_y} & 0_{p_y \times n} \end{bmatrix}. \]

**Lemma 2.** For a given scalar \( \gamma > 0 \), \( i,j \in \mathcal{I}_N \), consider the existence of matrices \( K_i \) and \( J_{i,j} \in \mathbb{R}^m \times n, \Gamma_i \) and

\[ J_{2,i} \in \mathbb{R}^{m \times p_y}, \mathcal{F}_j^U \text{ and } X^U \in \mathbb{R}^{n \times n}, \text{ diagonal positive definite matrices } \mathcal{F}_j^\Theta \text{ and } X^\Theta \in \mathbb{R}^{p_y \times p_y}, \text{ symmetric positive definite matrices } \mathcal{F}_j \in \mathbb{R}^{m \times m}, \text{ positive diagonal matrices } \Delta_i \in \mathbb{R}^{p_y \times p_y}, S_i \in \mathbb{R}^{p_y \times p_y}, T_i \in \mathbb{R}^{m \times m} \] and a scalar \( \gamma \) such that

\[ \begin{bmatrix} \mathcal{M}_{i,j}^a - \mathbf{He}(D) \times \times 0 \\ \mathcal{M}_{i,j}^b - X_{i,j}^U + \mathcal{F}_j^U \times \times 0 \\ \mathcal{M}_{i,j}^c - \mathcal{F}_j^\Theta \times \times 0 \\ \mathcal{M}_{i,j}^d - \mathcal{F}_j^\Theta \times \times 0 \end{bmatrix} < 0 \]

with \( D = \text{diag}(X^U, 0_q, X^\Theta) \) and \( q = n + p_x + p_y + m + r \), implies that

\[ V_j(x_{k+1}) - V_i(x_k) + \frac{z_{i,k}z_k}{\gamma} - w_k w_k \]

\[ -2 \Psi'(u_k)(T_j)^{-1} (\Psi(u_k) - J_{i,j}x_k - J_{i,j}\varphi_i) - 2\varphi_i S_i (\varphi_i - \Omega_i y_k) - 2\varphi_i W_j (\varphi_j - \Omega_j y_k + 1) < 0. \]
where \( \Pi_j = (W_j + \Delta_j) \Omega_j C_j^\rho \).

By multiplying this last inequality on the left by \( \eta_k \) and on the right by its transpose, the Inequality (29) is ensured.

**Lemma 3.** Consider for \( i \in \mathcal{N}_i \), that there exist symmetric positive definite matrices \( P_i \in \mathbb{R}^{n \times n} \), matrices \( K_i, J_{1,i} \in \mathbb{R}^{m \times n} \) and \( \Gamma_i, J_{2,i} \in \mathbb{R}^{m \times p} \), a positive scalar \( \alpha \) and a fixed positive scalar \( \delta \) such that for all \( i \in \mathcal{N}_i \) and all \( l \in \{1, \ldots, m\} \),

\[
\begin{bmatrix}
-P_i & * & * & * & * & * & * \\
\Pi_j A_i & -2S_i & * & * & * & * & * \\
[\Pi_j A_i] & [\Pi_j G_i] & [-\Pi_j B_i \Pi_j E_i^\gamma - 2W] \\
\end{bmatrix} + \frac{\gamma}{\alpha} \sum_i \sum_j \gamma_j < 0,
\]

(35)

with \( \Pi_j = (\Delta_i - S_j^\rho) \Omega_i C_j^\rho \). Then

\[
L_V(\mu^{-1}) \subset S(\hat{K}_i(\cdot) - \hat{J}_i(\cdot), \rho),
\]

with \( \mu^{-1} = \delta^{-1} + \alpha^{-1} \), which is defined in Lemma 1.

**Proof 3.** The Schur complement, applied on LMI (36), leads to the following inequality

\[
\begin{bmatrix}
P_i & * & * & * \\
\Pi_j & \gamma_i & * & * \\
\end{bmatrix} > 0
\]

(36)

In the sequel, by multiplying the previous Inequality on the right by \( \hat{x}_k = (x_k^T, \varphi_i^T C_j^\rho x_k) \) and on the left by its transpose, we get the following inequality

\[
x_k^T P_k x_k + 2\varphi_i^T \Delta_i \Omega_i C_j^\rho x_k + 2\varphi_i^T S_i^\rho (\varphi_i - \Omega_i C_j^\rho x_k)
\]

\[
> \frac{1 + \alpha}{1 + \delta} \gamma_i \left\| (\hat{K}_i - \hat{J}_i) \hat{x}_k \right\|^2,
\]

(38)

where \( \hat{K}_i = [K_i', \Gamma_i'] \) and \( \hat{J}_i = [J_{1,i}', J_{2,i}'] \). Thus

\[
V_j(x_{k+1}) - V_j(x_k) > \frac{1}{\rho_i(\delta)} \left\| (\hat{K}_i - \hat{J}_i) \hat{x}_k \right\|^2.
\]

(39)

Then \( \forall x_k \in L_V(\mu^{-1}), V_j(x_k) < \mu^{-1} \), which implies that \( x_k \in S(\hat{K}_i - \hat{J}_i, \rho) \). Moreover the inclusion (37) is obtained.

### 3. MAIN RESULTS

We gather in the following theorem the solutions of the two problems 1 and 2.

**Theorem 1.** For a given scalar \( \delta > 0 \), \( i, j \in \mathcal{N}_i \), by considering matrices \( K_i, J_{1,i} \in \mathbb{R}^{m \times n}, \Gamma_i, J_{2,i} \in \mathbb{R}^{m \times p}, \mathcal{F}_j^U \) and \( \mathcal{X}^U \in \mathbb{R}^{m \times n} \), diagonal positive definite matrices \( \mathcal{F}_j^\rho \) and \( \mathcal{X}_j^\rho \), symmetric positive definite matrices \( P_i \in \mathbb{R}^{n \times n} \), symmetric positive definite matrices \( S_i \in \mathbb{R}^{m \times m} \), positive scalars \( \gamma \) and \( \alpha \), the convex optimization problem

\[
\min_{L_k, \Gamma_i, J_{1,i}, J_{2,i}, \mathcal{F}_j^U, \mathcal{X}^U} \xi_1 \gamma + \xi_2 \alpha,
\]

subject to LMIs (28) and (36) leads to a switching control law, solution of Problem 2, related to the maximization of the disturbance rejection with the weighting factors \( \xi_1 = 1 \) and \( \xi_2 = 0 \). The solution of Problem 1 with \( \xi_1 = 0 \) and \( \xi_2 = 1 \).

**Proof 4.** The LMIs (28) being satisfied, Lemma 2 allows to write Inequality (29). Since LMIs (36) are verified, Lemma 3 implies Inequalities (25). After simplification, using the global cone bounded sector condition (5) and the local bounded sector condition (25), we have

\[
V_j(x_{k+1}) - V_j(x_k) > \frac{x_k^T z_k}{\gamma} - w_k^T w_k < 0.
\]

(40)

By recurrence,

\[
V_{\sigma_{k+1}}(x_{k+1}) - V_{\sigma_0}(x_0) + \sum_{k=0}^{k_0} \frac{x_k^T z_k}{\gamma} - w_k^T w_k < 0.
\]

(41)

On assuming that \( x_0 \in L_V(\alpha^{-1}) \) and with assumption of the Inequality (8) the last inequality leads to

\[
V_{\sigma_{k+1}}(x_{k+1}) \leq V_{\sigma_0}(x_0) + \sum_{k=0}^{k_0} w_k^T w_k \leq \frac{1}{\alpha} + \frac{1}{\delta},
\]

(42)

Thus the limit \( \lim_{k_0 \to +\infty} V_{\sigma_{k+1}}(x_{k+1}) \) exists and is positive and bounded by \( \mu^{-1} \). Then the system (1)–(4) is stable. Moreover \( \forall k \in \mathbb{N} w_k = 0 \), then Inequality (42) leads to prove the local asymptotic stability of the system (1)–(4).

In a second time, on assuming that \( x_0 = 0 \) (which implies that \( V_{\sigma_0}(x_0) = 0 \)) and with assumption of the Inequality (8) the last inequality leads to

\[
V_{\sigma_{k+1}}(x_{k+1}) \leq \sum_{k=0}^{k_0} w_k^T w_k \leq \frac{1}{\delta}.
\]

(43)

Thus the limit \( \lim_{k_0 \to +\infty} V_{\sigma_{k+1}}(x_{k+1}) \) exists and is positive and bounded by \( \frac{1}{\gamma} \). Then, on summing Inequality (42), we prove that

\[
\frac{1}{\gamma} \sum_{k=0}^{N} z_k^T z_k < \sum_{k=0}^{N} w_k^T w_k.
\]

(44)

In other words, the \( L_2 \)-induced gain of the closed-loop system (1)–(4) is bounded by \( \sqrt{\gamma} \). The optimization problem
provides the minimal $\gamma$ which gives us an estimate of the $\mathcal{L}_2$-induced gains.

4. ILLUSTRATION

Let us consider the following numerical example to illustrate the solution to Problem 1 and 2.

**Example 1:**

\[
A_1 = \begin{bmatrix} 0.4 & 0.4 \\ 0.2 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.5 & -0.3 \\ 0.5 & -1.4 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},
\]

\[
B_2 = \begin{bmatrix} 1.2 \\ 1.3 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 1 \\ 1.2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 1.2 \\ 1.3 \end{bmatrix}, \quad E^x_1 = \begin{bmatrix} 0.4 \\ 0.48 \end{bmatrix},
\]

\[
E^x_2 = \begin{bmatrix} 0.4 \\ 0.44 \end{bmatrix}, \quad C^u_1 = [1 -0.9], \quad C^u_2 = [-0.5 0.9],
\]

\[
C^i_1 = [2 1], \quad C^i_2 = [1 1], \quad \rho = 1.5, \quad L^i_1 = 1,
\]

\[
L^i_2 = 2, \quad D^i_1 = 1, \quad D^i_2 = 1, \quad E^i_1 = 0.3, \quad E^i_2 = 0.22,
\]

\[
\varphi_1(y_k) = \frac{\Omega_1 y_k}{2} (1 + \sin(10y^2_k)), \quad \Omega_1 = 0.8,
\]

\[
\varphi_2(y_k) = \frac{\Omega_2 y_k}{2} (1 + \sin(3y_k)), \quad \Omega_2 = 0.9.
\]

The disturbance considered is defined by

\[
w_k = \begin{cases} 
0.6 \sin(17k), & 0 \leq k \leq 10 \\
0, & k > 10.
\end{cases}
\]

(47)

The energy of this disturbance is equal to 1.7616. Then Equation (8) implies that $\delta = 0.5$. In this case, the method investigated in (Jungers et al., 2011) is not able to give an estimate of the $\mathcal{L}_2$-induced gain. We can note that the disturbance is rejected. In order to compare, with the previous work of (Jungers et al., 2011), Fig. 1 presents the evolution of the $\mathcal{L}_2$-induced gain obtained by the both method for different values of $\delta$. As we can see, on the example, the method presented in this paper improves the estimate of the $\mathcal{L}_2$-gain around 55% for $\delta \in [2; 100]$. For $\delta < 2$, the advantage is more important. We can also notice that for $\delta < 0.6$, the quadratic method is not feasible, when the new method gives result for $\delta > 0.3$. This can be graphically pointed out by the fact that the both curves admit a vertical asymptote respectively around $\delta = 0.3$ and $\delta = 0.6$.

However, the method introduced on this paper does not always give a better estimate of the $\mathcal{L}_2$-induced gain. Even if the advanced Lyapunov function (9) is an extension of the quadratic one, the tools and manipulations which are used to obtain the LMI conditions to solve the problem are different. More precisely, the change of variables, the Schur complement and the Finsler's lemma are not handled in the same way due to the presence of a sum of two terms in the Lur’e type Lyapunov function. Hence, the different LMI's are associated with sufficient conditions that are different and not comparable. As a consequence, the method provided here is not always more efficient than the quadratic version. In order to emphasize this remark, we consider the following example, based on a single modification of Example 1.

**Example 2:** same as Example 1 but with

\[
E^x_1 = \begin{bmatrix} 0.4 \\ 0.08 \end{bmatrix},
\]

\[
E^x_2 = \begin{bmatrix} 0.4 \\ 0.44 \end{bmatrix}, \quad C^u_1 = [1 -0.9], \quad C^u_2 = [-0.5 0.9],
\]

\[
C^i_1 = [2 1], \quad C^i_2 = [1 1], \quad \rho = 1.5, \quad L^i_1 = 1,
\]

\[
L^i_2 = 2, \quad D^i_1 = 1, \quad D^i_2 = 1, \quad E^i_1 = 0.3, \quad E^i_2 = 0.22,
\]

\[
\varphi_1(y_k) = \frac{\Omega_1 y_k}{2} (1 + \sin(10y^2_k)), \quad \Omega_1 = 0.8,
\]

\[
\varphi_2(y_k) = \frac{\Omega_2 y_k}{2} (1 + \sin(3y_k)), \quad \Omega_2 = 0.9.
\]

The disturbance considered is defined by

\[
w_k = \begin{cases} 
0.6 \sin(17k), & 0 \leq k \leq 10 \\
0, & k > 10.
\end{cases}
\]

(47)

In Fig. 2, we can see that for some $\delta$, the quadratic method leads to a better result. The different sufficient conditions are not straightforwardly comparable.

The second illustration concerns the solution of Problem 1 in Fig. 3. We can see that the level set $L_V(\delta^{-1})$ is the intersection of the two modal level sets and is disconnected. Here the trajectory starting from $x_0 = 0$ with a bounded energy disturbance remains in the main part of the level set $L_V(\delta^{-1})$ containing the origin. Nevertheless the trajectory may jump in the other parts (which do not contain the origin), even if it is not the case here in Fig. 3 by considering Example 2.

5. CONCLUSION

A suitable Lur’e type Lyapunov function has been used to consider the performance aspect of switched discrete-time Lur’e system affected by a saturated input control and a bounded energy disturbance. With several techniques among the Finsler’s lemma, LMI constraints are obtained for an optimization problem leading to find state and nonlinearity feedbacks law minimizing the $\mathcal{L}_2$-induced gain or maximizing the set of initial condition ensuring the
stability under a bounded energy disturbance. Numerical illustrations are also given to underline the applicability of our results. Finally a discussion and a comparison with the results of the literature are performed.

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