Multiple Redundancy Constants with Trickle
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Abstract—Wireless sensor network protocols very often use the Trickle algorithm to govern information dissemination. For example, the widely used IPv6 Routing Protocol for Low-Power and Lossy Networks (RPL) uses Trickle to emit control packets. We derive an analytical model of Trickle to take into account multiple redundancy constants and the common lack of synchronization among nodes. Moreover, we demonstrate message count unfairness when Trickle uses a unique global redundancy constant because nodes with less neighbors transmit more often. Consequently, we propose a heuristic algorithm that calculates a redundancy constant for each node as a function of its number of neighbors. Our calculated redundancy constants reduce unfairness among nodes by distributing more equally the number of transmitted messages in the network. Our analytical model is validated by emulations of constrained devices running the Contiki Operating System and its IPv6 networking stack. Furthermore, results very well corroborate the heuristic algorithm improvements.

Index Terms—Trickle; protocol fairness; analytical model; probabilistic model; WSN;

I. INTRODUCTION

The Trickle algorithm is a timer based control algorithm relying on recursive doubling time intervals and “polite gossip” policy. It quickly propagates updates in the network but avoids unnecessary transmissions. It was initially proposed for system code versioning [1] in Wireless Sensor Networks (WSN). Due to its wide-spread use, it has been separately standardized in RFC 6206 [2]. Most notably, the Routing Protocol for Low Power and Lossy Networks (RPL) [3] utilizes Trickle for topology maintenance. Also, Multicast Protocol for Low Power and Lossy Networks [4] and other protocols [5], [6], [7] build upon it, leveraging Trickle’s benefits. This makes the understanding of its behavior crucial for performance optimization of control overhead.

In this paper, we model and study the operation of Trickle, which leads to a better understanding of the impact of its redundancy constant. We demonstrate that the usage of a unique redundancy constant for the whole network leads to communication unfairness when the underlying topology density is not homogeneous. The root cause of this unfairness is the increased probability of transmission of nodes with less neighbors in their radio vicinity. This results in uneven transmission load, e.g. message count, across the network. Moreover, in battery powered networks, these nodes with a higher transmission probability will cease functioning sooner because of on board energy depletion as broadcasting is very expensive in WSNs. We model the individual transmission probabilities with individual node redundancy constants across the network. The model with multiple redundancy constants can be numerically resolved for arbitrary topologies but also simplified to closed-form in specific cases, outside of the scope of this paper.

From the model’s results, we propose a simple heuristic algorithm in order to improve Trickle fairness by a local computation of each redundancy constant as a function of the number of neighbors. We demonstrate the resulting improvements in terms of transmission load balance both by leveraging our analytical model results and by emulating constrained-node networks running the full Contiki Operating System network stack.

The main contributions of the paper are the following:
- A new probabilistic model estimating the message count and average transmission probabilities of individual nodes in steady state networks. This model works for arbitrary topologies without any synchronization requirement, and accounts for multiple redundancy constants among nodes,
- A demonstration of transmission load unfairness in networks utilizing a fixed redundancy constant among nodes,
- A new algorithm improving fairness in the network by locally computing the redundancy constants as a function of the number of neighbors in the node’s radio vicinity,
- A validation of the model and an evaluation of the proposed algorithm improvements. The emulation uses highly accurate instruction-level execution of the binary file that contains our code and the Contiki network stack and normally runs on real hardware.

The paper is structured as follows. Section II reviews the Trickle algorithm. In Section III, we discuss the related work in details. Section IV presents our probabilistic model design. We validate the model and discuss unique redundancy constant unfairness in Section V. In Section VI, we present our heuristic algorithm that locally computes each redundancy constant and discuss its achieved improvements. Finally, we conclude and discuss future work in Section VII.

II. THE TRICKLE ALGORITHM

The main idea of the Trickle algorithm is on one hand to exponentially reduce the amount of control traffic in the network, while there are no detected inconsistencies in a given state. On the other hand, once an inconsistency has been detected it quickly propagates the new information state. Naturally, the "consistency notion" is defined by the protocol
or application actually using Trickle. For instance, in the case of IETF RPL routing protocol, consistency is checked by comparing the advertised DODAG state in the network to the local one. In the case of firmware updates a similar verification is usually performed between software versions.

Trickle splits time into intervals of variable length where transmissions may occur following Trickle’s rules. The three parameters to configure Trickle are: i) $I_{\text{min}}$, the minimum interval size; ii) $I_{\text{max}}$, the maximum interval size expressed as the number of times the minimum interval may double; iii) $K$, the redundancy constant.

A node following the Trickle algorithm increments a local counter $c$ for each consistent reception. The node transmits at instant $t$ if:

$$c < K,$$

that is, if the number of consistent receptions is smaller than the redundancy constant. Counter $c$ is reset to zero at the beginning of each interval. Instant $t$ at which Trickle decides if it is going to transmit is selected randomly from the uniform interval $[\frac{1}{2}I, I)$, where $I \in \{I_{\text{min}} \times 2^n \mid n \in \mathbb{N}_0, n \leq I_{\text{max}}\}$. Interval $I$ is doubled upon its expiration by incrementing $n$. When a node detects inconsistency, $n$ becomes 0, which sets interval $I$ to $I_{\text{min}}$. In this paper, though, we model Trickle networks in steady state, such that $I = I_{\text{min}} 2^{I_{\text{max}}}$, and focus on the effect of the redundancy constant.

Fig. 1 illustrates an example Trickle operation in steady state $K = 1$. As soon as $c \geq K$, transmissions are suppressed. Note that the Trickle intervals among nodes are not necessarily synchronized.

III. RELATED WORK

Due to its wide-spread use, the Trickle algorithm has been subject to many studies [8], [9], [10], [11], [12], [13], [14]. Becker et al. [8] develop a model to study the propagation time of new information in a network using Laplace transforms.

Meyfroyt et al. [11] recently published a model generalizing the algorithm by introducing the listen-only parameter $\eta$. In the standardized version of Trickle [2] and the original paper [1] $\eta = \frac{1}{2}$ and is introduced in order to avoid broadcast storms in unsynchronized networks at the beginning of intervals, by forcing nodes to keep listening before attempting transmissions (i.e. listen-only period). The authors demonstrate that using a short listen-only period provides advantage in terms of smaller propagation time, but in the same time increases the number of transmitted messages in the network. They derive the cumulative distribution function of inter-transmission times for large number of nodes in a steady state, unsynchronized, single cell network.

Kermajani et al. [9] approach the problem of estimating the Trickle message count in steady state by deriving the average probability $P$ that a node in the network will transmit in a given interval. Then, the average message count in a given interval is simply $N \times P$, where $N$ denotes the number of nodes in the network. In respect to the model of Meyfroyt et al. [11], the approach implicitly supports multi-cell topologies.

A common point on published Trickle models [8], [11], [9] (and deployments) is that they all consider a unique, fixed redundancy constant among nodes. The unique redundancy constant leads to unfairness as nodes with less neighbors have less incoming packets and thus a higher probability to transmit, and will therefore deplete their available energy source sooner. Note that Trickle messages translate to L2 broadcast frames that are very expensive in WSNs.

Other authors [10], [12], [13], [14] have studied Trickle and its performance in the specific use case of RPL and how it affects the convergence and route optimality of the DODAG building process.

IV. MODELLING THE TRICKLE MESSAGE COUNT

Our model calculates the message count of individual nodes in a steady state Trickle-based network. Similarly to Kermajani et al. [9], we derive the average probability of transmission. However, we use a different decomposition that allows us to extend their approach in order to calculate per node probabilities, rather than the network average. Hence we can give insights on the fairness of the algorithm. To render the model more practical, we do not make explicit assumptions on topology. Rather, we assume that each node $i$ is able to know its number of neighbors $y_i$. Therefore, our model requires for each node its redundancy constant $K_i$, and its neighbors list.

The main idea of our analysis is to express the average probability of transmission of a node as a function of the transmission probabilities of its neighbors. This will yield a system of $N$ equations with $N$ unknowns, where $N$ is the number of nodes in the network, that can be numerically resolved.

Distribution of Transmission Times. In networks with synchronized Trickle intervals, transmission times simply follow a uniform distribution [9]. However, with the lack of synchronization, this does not longer hold. Let $X_1, \ldots, X_{y_i}$ be i.i.d. random variables of uniform distribution modeling the transmission time positions of the $y_i$ nodes into an interval of length $I$. Let $T$ be the selected transmission time of node $i$, $T \sim \frac{1}{2}I$]. Let $Y_T$ denote the number of selected transmission times before $T$. Let $n$ be the positive integer that denotes the position of transmission time of node $i$ in the set of increasingly ordered transmission times. The probability that $n$ is selected by node $i$ and by its neighbors is equal to
Therefore, \( P(B) = \binom{n-1}{y_i} \cdot \frac{y_i}{T} \cdot \left(\frac{T}{T-y_i}\right)^{n-1} \). \( Y_T \) can be shown to follow a binomial distribution with parameters \( y_i \) and \( \frac{T}{T-y_i} \).

### A. Probabilistic Model

The average probability that node \( i \) will send a message in a given interval is denoted \( P_{TX}[i] \). A node will surely transmit in the case where its neighbors \( y_i \) is less than its redundancy constant \( K_i \), because the counter \( c \) can never reach \( K_i \). Otherwise, the node transmits in two cases: i) if it selects any of the first \( K_i \) transmission times, ii) if it selects any of the last \( y_i + 1 - K_i \) transmission times and at most \( K_i - 1 \) of its neighbors have already transmitted. Consequently, \( P_{TX}[i] \), can be written as follows:

\[
P_{TX}[i] = \begin{cases} 1, & y_i < K_i \\ P_F[i] + P_{LO}[i], & y_i \geq K_i. \end{cases}
\]

where:

- \( P_F[i] \) is the probability that node \( i \) selects one of the first \( K_i \) transmission times. We can find this probability simply as: \( P_F[i] = \sum_{n=0}^{K_i-1} P(Y_T = n) \).
- \( P_{LO}[i] \) is the probability that node \( i \) selects any of the last \( y_i + 1 - K_i \) transmission times and at most \( K_i - 1 \) nodes, with a lower transmission time than node \( i \), will transmit before it. We refer to this probability as the last opportunity transmission probability. This probability depends on \( P_{TX} \), where \( j \) is a neighbor of node \( i \).

#### Last Opportunity Transmission Probability

We are considering the case where node \( i \) selected one of the last \( y_i - 1 - K_i \) transmission times. The probability that at most \( K_i - 1 \) nodes, with a lower transmission time than node \( i \) transmit before, depends on transmission time of node \( i \).

We will compute the probability \( P_{LO}[i] \) by conditioning on transmission time of node \( i \). As \( Y_T \in \{ K_i, \ldots, y_i \} \), \( P_{LO}[i] \) can be derived as:

\[
P_{LO}[i] = \sum_{n=K_i}^{y_i} P_{LO}[i \mid Y_T = n] \times P(Y_T = n).
\]

Let \( B \) be the set of \( n \) neighbors of node \( i \), denoted by \( \{a, b, c, d\} \) whose transmission times are lower than the one of node \( i \). Let \( \mathbb{R} \) be the set composed of \( \binom{y_i}{n} \) possible sets of nodes \( B \) that possibly match \( B \).

Therefore, \( P_{LO}[i \mid Y_T = n] \) can be obtained as:

\[
P_{LO}[i \mid Y_T = n] = \frac{1}{\binom{y_i}{n}} \sum_{B \in \mathbb{R}} P_{LO}[i \mid Y_T = n \land B_{\text{set}} = B]
\]

The probability that node \( i \) transmits in this case, \( P_{LO}[i \mid Y_T = n \land B_{\text{set}} = B] \), is the probability that at most \( K_i - 1 \) nodes of \( B \) transmit before:

\[
P_{LO}[i \mid Y_T = n \land B_{\text{set}} = B] = \sum_{j=0}^{K_i-1} \gamma(j, n, B),
\]

where \( \gamma(j, n, B) \) denotes the probability that \( j \) nodes of the set \( B = \{1, \ldots, n\} \) transmit before node \( i \). By definition of \( P_{TX} \), we have:

\[
\gamma(0, n, B) = \prod_{l=1}^{n} (1 - P_{TX}[l]).
\]

More generally:

\[
\gamma(k, n, B) = \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} \cdots \sum_{i_k=1}^{n} [P_{TX}[i_1] \times \cdots \times P_{TX}[i_k] \times \prod_{l=1, l \neq i_1, \ldots, i_k}^{n} (1 - P_{TX}[l])].
\]

By re-organizing the sums, this leads to:

\[
P_{LO}[i] = \sum_{n=K_i}^{y_i} P(Y_T = n) \times \frac{1}{\binom{y_i}{n}} \sum_{B \in \mathbb{R}} \sum_{j=0}^{K_i-1} \gamma(j, n, B).
\]

For example, let us consider the case where node \( i \) has \( y_i = 4 \) neighbors, \( \{a, b, c, d\} \), and \( n = 2 \) of them have selected a transmission time lower than its own. We can have \( \binom{4}{2} \) different sets of neighbors \( B_1 = \{a, b\}, B_2 = \{a, c\}, B_3 = \{a, d\}, B_4 = \{b, c\}, B_5 = \{b, d\}, B_6 = \{c, d\} \), whose nodes have a lower transmission time. For instance, when \( B = B_1 = \{a, b\} \) and \( K_i = 2 \), node \( i \) will transmit either if \( j = 0 \) nodes of \( B \) transmit, or if \( j = 1 = K_i - 1 \) nodes of \( B \) transmit. The probability that \( j \) nodes of the set \( B \) transmit is \( \gamma(j, n, B) \).

We now have \( P_F[i] \) and \( P_{LO}[i] \) expressed according to \( \{P_{TX}[j] \mid j \) is a neighbor of node \( i \} \). In order to find \( P_{TX}[i] \forall i \in \{1, \ldots, N\} \), we need to solve the \( N \) equations with \( N \) unknowns, i.e. the system \( P_{TX}[i] = P_F[i] + P_{LO}[i] \).

The solutions of the system are the average probabilities of transmission for each node in the network and in the same time the average message count per node during one interval. In its general form, the system of equations models arbitrary network topologies and can be resolved numerically. For specific topologies which are outside of the scope of this paper, a closed form solution may be obtained.

Due to its complexity, the general form of the model does not allow a direct practical implementation for constrained devices. However, its numerical resolution gives precise insights on node behavior and where the imbalance in the network occurs. Based on this, in Section VI we propose a practical, heuristic approach that is easily computable locally.

### V. Model Validation and Trickle Unfairness

To validate our model we implemented a tool resolving the model in Python and Sage\(^1\), an open-source computational software program. We emulate the Tmote Sky sensor motes running Contiki Operating System, by using the MSPSim emulator, and the Cooja simulator, in order to obtain real data.
world results of Trickle. We use the Trickle application level library code available in Contiki. The same binary file used for emulations runs on real hardware without any modifications. Note that due to the use of the emulator, some imperfections of results in respect to the real deployments come from the Unit Disk Graph (UDG) radio model in Cooja. We validate the model for 49 node networks: i) using a $7 \times 7$ grid topology, to demonstrate unfairness, ii) using a randomly generated topology, to demonstrate the validity of the model for the general case. Transmission range for the grid topology was $R = \sqrt{2}$, with the average node degree of 6.37. In the case of randomly generated topology, the average node degree was 3.92. We average emulation results over 30 runs, and calculate the model for the same topology based on the list of neighbors of each node. We count the number of transmissions of each node over 10 steady state Trickle intervals of 16 seconds. We calculated 95% confidence intervals but do not present them on the graphs for the sake of clarity, as they are graphically indistinguishable from the plotted averages.

Fig. 2 presents the results for the total message count in the network using a unique, fixed redundancy constant among nodes. The model accurately predicts the number of messages in the network. Numerical values of maximum, minimum probabilities, variance and their comparison with the emulation results are shown in Tables I and II. The imperfections of the model come from the fact that with the lack of synchronization of Trickle intervals among nodes, if we consider a node with $y$ neighbors, $y$ is only the mean number of transmission times that can occur during one interval. Also, the assumption that transmission probabilities of nodes are independent events does not hold. As discussed by Kermajani et al. [9], a transmission performed by a node causes the increment of the counter $c$ and therefore decreases the transmission probability of its neighbors. Nevertheless, emulation results are obtained by running the binary file that normally executes on real hardware. They show that our model provides accurate estimations of the average transmission probabilities of nodes in the network, and consequently of the message count.

**Trickle Unfairness with Fixed Redundancy Constant.** As most real world deployments using Trickle utilize a unique, fixed redundancy constant among nodes, we demonstrate the transmission unfairness that arises due to the heterogeneous network topologies, as nodes do not have the same number of neighbors. Fig. 3 presents the three dimensional graphs on probability of transmission calculated by our model for the grid topology, where the effects are easily noted. We present results for $K \in \{1, 2, 3, 4\}$, as the probabilities quickly approach 1.0 for larger $K$ (average node degree of 6.37). Inside the grid with 8 neighbors ($R = \sqrt{2}$), for $K = 1$, we can see that the nodes have average transmission probabilities of approximately 0.2, while the nodes on the edges with 5 neighbors have around 0.5, and the nodes in the corners with 3 neighbors on the average transmit with the probability of approximately 0.7. Increasing the redundancy constant increases the transmission probabilities in the network. However, as the number of neighbors can be considered fixed, nodes with less neighbors are affected more and their transmission probabilities increase faster in respect to those in the middle of the grid. This can be best seen for the case $K = 4$ in Fig. 3(d), as the nodes in the corners of the grid transmit with probability 1.0, while nodes on the edges have probability 0.85, and nodes inside the grid have probability 0.45.

To validate the estimations of our model, we have confronted the results with emulations. Fig. 4 presents the comparison for $K \in \{2, 3\}$, in the grid and the randomly generated topology.

**VI. LOCAL COMPUTATION OF THE REDUNDANCY CONSTANT TO IMPROVE FAIRNESS**

As discussed, the average transmission probability of a node in the network depends on the number of neighbors and the redundancy constant. The usage of a fixed redundancy constant in the network causes unbalanced transmission load and may cause early depletion of energy sources of nodes with less neighbors. Notice that the number of neighbors is generally available locally due to the common use of either Neighbor
TABLE I: Model and emulation results on $7 \times 7$ grid, $R = \sqrt{2}$, for $K \in \{1, 2, 3, 4, 5, 6\}$

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</tr>
</thead>
<tbody>
<tr>
<td>max probability</td>
<td>0.673</td>
<td>0.606</td>
<td>0.887</td>
<td>0.896</td>
<td>0.980</td>
<td>0.983</td>
<td>0.999</td>
<td>1.0</td>
<td>0.999</td>
<td>1.0</td>
<td>0.999</td>
<td>1.0</td>
</tr>
<tr>
<td>min probability</td>
<td>0.070</td>
<td>0.05</td>
<td>0.084</td>
<td>0.05</td>
<td>0.116</td>
<td>0.153</td>
<td>0.173</td>
<td>0.22</td>
<td>0.295</td>
<td>0.38</td>
<td>0.501</td>
<td>0.497</td>
</tr>
<tr>
<td>variance</td>
<td>0.03217</td>
<td>0.02466</td>
<td>0.06402</td>
<td>0.05030</td>
<td>0.08261</td>
<td>0.05736</td>
<td>0.08553</td>
<td>0.06077</td>
<td>0.06401</td>
<td>0.05158</td>
<td>0.03268</td>
<td>0.03359</td>
</tr>
</tbody>
</table>

TABLE II: Model and emulation results on 49 node random topology and 3.92 average node degree, for $K \in \{1, 2, 3, 4, 5, 6\}$

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</tr>
</thead>
<tbody>
<tr>
<td>max probability</td>
<td>0.833</td>
<td>0.773</td>
<td>0.999</td>
<td>0.986</td>
<td>0.999</td>
<td>1.0</td>
<td>0.999</td>
<td>1.0</td>
<td>0.999</td>
<td>1.0</td>
<td>0.999</td>
<td>1.0</td>
</tr>
<tr>
<td>min probability</td>
<td>0.070</td>
<td>0.056</td>
<td>0.033</td>
<td>0.083</td>
<td>0.095</td>
<td>0.146</td>
<td>0.233</td>
<td>0.28</td>
<td>0.427</td>
<td>0.396</td>
<td>0.663</td>
<td>0.023</td>
</tr>
<tr>
<td>variance</td>
<td>0.04783</td>
<td>0.03346</td>
<td>0.08453</td>
<td>0.07687</td>
<td>0.08518</td>
<td>0.07773</td>
<td>0.05276</td>
<td>0.04963</td>
<td>0.0237</td>
<td>0.02392</td>
<td>0.00772</td>
<td>0.00750</td>
</tr>
</tbody>
</table>

Fig. 4: Transmission probability of nodes in the grid and random topology.

We show the effect estimated by the model of the locally computed redundancy constant for the most interesting combinations of parameters in Fig. 5. We also confront the estimations with emulation results in Table III.

We can see that the use of the locally computed redundancy constant greatly reduced the effects observed in Fig. 3. Depending on the parameters passed to the procedure, we note that the effect can be either reversed such that the nodes in the corners transmit with smaller probability than the nodes inside the grid, or reduced which is the case for nodes on the edges of the grid. Clearly, the absolute value of the ideally balanced transmission probability in the network depends on the requirements of the application actually using Trickle.

Parameter Selection. In original Trickle, the redundancy constant $K$ is a parameter that effectively depends on the application requirements. With our algorithm, we extend this concept in order to catch the topology characteristics and to provide a better transmission load distribution among nodes. However, both redundancy step and redundancy offset effectively depend on the application using Trickle and are

Algorithm 1 Local calculation of the redundancy constant $K_i$

1: procedure CALCULATE $K_i$(num_neighbors, step, offset)
2: if num_neighbors $\leq$ offset then return 1
3: else return $\lceil\frac{\text{num_neighbors} - \text{offset}}{\text{step}}\rceil$
Fig. 5: Transmission probability of nodes in the grid estimated by the model and compared to emulation results for locally computed redundancy constant.

TABLE III: Model and emulation results for the locally computed redundancy constant on the grid. To obtain $K \in \{1, 2\}$, we used $offset = 2, step = 3$, and for $K \in \{2, 3\}$, $offset = 0, step = 3$.

<table>
<thead>
<tr>
<th>Results / Redundancy Constant</th>
<th>Model: $K \in {1, 2}$</th>
<th>Emulation: $K \in {1, 2}$</th>
<th>Model: $K \in {1, 2, 3}$</th>
<th>Emulation: $K \in {1, 2, 3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Message Count</td>
<td>15.734</td>
<td>15.326</td>
<td>21.587</td>
<td>21.66</td>
</tr>
<tr>
<td>Max Probability</td>
<td>0.479</td>
<td>0.493</td>
<td>0.520</td>
<td>0.586</td>
</tr>
<tr>
<td>Min Probability</td>
<td>0.011</td>
<td>0.015</td>
<td>0.239</td>
<td>0.213</td>
</tr>
<tr>
<td>Variance</td>
<td>0.01188</td>
<td>0.00947</td>
<td>0.00511</td>
<td>0.00800</td>
</tr>
</tbody>
</table>

We used offset = 2, step = 3 in parallel, specifies that a transmission should be suppressed when at least half of the neighbors in the network achieved better transmission load distribution. However, deriving an optimal value of the redundancy constant that will perfectly balance the transmission probabilities of nodes in heterogeneous topologies remains an open problem that we plan to study as future work.

REFERENCES


