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# Minimal Synchrony for Asynchronous Byzantine Consensus

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## Abstract

Solving the consensus problem requires in one way or another that the underlying system satisfies some synchrony assumption. Considering an asynchronous message-passing system of  $n$  processes where (a) up to  $t < n/3$  may commit Byzantine failures, and (b) each pair of processes is connected by two unidirectional channels (with possibly different timing properties), this paper investigates the synchrony assumption required to solve consensus, and presents a signature-free consensus algorithm whose synchrony requirement is the existence of a process that is an *eventual  $\langle t + 1 \rangle$ bisource*. Such a process  $p$  is a correct process that eventually has (a) timely input channels from  $t$  correct processes and (b) timely output channels to  $t$  correct processes (these input and output channels can connect  $p$  to different subsets of processes). As this synchrony condition was shown to be necessary and sufficient in the stronger asynchronous system model (a) enriched with message authentication, and (b) where the channels are bidirectional and have the same timing properties in both directions, it follows that it is also necessary and sufficient in the weaker system model considered in the paper. In addition to the fact that it closes a long-lasting problem related to Byzantine agreement, a noteworthy feature of the proposed algorithm lies in its design simplicity, which is a first-class property.

**Keywords:** Adopt-commit, Asynchronous message-passing, Byzantine process, Consensus, Distributed algorithm, Eventual timely channel, Feasibility condition, Lower bound, Optimal resilience, Reliable broadcast, Signature-free algorithm, Synchrony assumption.

# 1 Introduction

**Byzantine consensus** A process has a *Byzantine* behavior when it behaves arbitrarily [26]. This bad functioning can be intentional (malicious behavior, e.g., due to intrusion) or simply the result of a transient fault that altered the local state of a process, thereby modifying its execution in an unpredictable way.

We are interested here in the *consensus* problem in message-passing distributed systems prone to Byzantine process failures whatever their origin. Consensus is an agreement problem in which each process first proposes a value and then decides on a value [26]. In a Byzantine failure context, the consensus problem is defined by the following properties: every non-faulty process decides (termination), no two non-faulty processes decide differently (agreement), and the decided value is not arbitrary, i.e., it is related in one way or another to values proposed by non-faulty processes (validity).

**Context of the paper** A synchronous distributed system is characterized by the fact that both processes and communication channels are synchronous (or timely) [3, 21, 28]. This means that there are known bounds on process speed and message transfer delays. Let  $t$  denote the maximum number of processes that can be faulty in a system made up of  $n$  processes. In a synchronous system, consensus can be solved (a) for any value of  $t$  (i.e.,  $t < n$ ) in the crash failure model, (b) for  $t < n/2$  in the general omission failure model, and (c) for  $t < n/3$  in the Byzantine failure model [20, 26]. Moreover, these bounds are tight.

Differently, when all channels are asynchronous (i.e., when there is no bound on message transfer delays), it is impossible to solve consensus even if we consider the weakest failure model (namely, the process crash failure model) and assume that at most one process may be faulty (i.e.,  $t = 1$ ) [15]. It trivially follows that Byzantine consensus is impossible to solve in a failure-prone asynchronous distributed system.

As Byzantine consensus can be solved in a synchronous system and cannot in an asynchronous system, a natural question that comes to mind is the following “*When considering the synchrony-to-asynchrony axis, which is the weakest synchrony assumption that allows Byzantine consensus to be solved in a message-passing system?*” This long-lasting question is the issue addressed in this paper. To that end, the paper considers a synchrony assumption capturing both the structure and the number of eventually synchronous channels among correct processes.

**Related work** Several approaches to solve Byzantine consensus have been proposed. We consider here only deterministic approaches<sup>1</sup>. One consists in enriching the asynchronous system (hence the system is no longer fully asynchronous) with a failure detector, namely, a device that provides processes with (possibly unreliable) hints on failures [10]. Basically, in one way or another, a failure detector encapsulates synchrony assumptions. Failure detectors suited to Byzantine behavior have been proposed and used to solve Byzantine consensus (e.g., [13, 16, 19]).

Another approach proposed to solve Byzantine consensus consists in directly assuming that some channels satisfy a synchrony property (“directly” means that the synchrony property is not hidden inside a higher level abstraction such as a failure detector). This approach, which relies on the notion of an  $\diamond\langle x+1 \rangle$ bisource (read “ $\diamond$ ” as “eventual”), was introduced in [1]. Intuitively, this notion states that there is a correct process that has  $x$  input channels from correct processes and  $x$  output channels to correct processes that are eventually timely [12, 14] (the “+1” comes from the fact that it is assumed that each process has a “virtual” input/output channel from itself to itself, which is always timely).

Considering asynchronous systems with Byzantine processes without message authentication, it is shown in [1] that Byzantine consensus can be solved if the system has an  $\diamond\langle n-t \rangle$ bisource (all other channels being possibly fully asynchronous). Moreover, the process that is the  $\diamond\langle n-t \rangle$ bisource can never be explicitly known by the whole set of processes. Considering systems with message authentication, a Byzantine consensus algorithm is presented in [25] that requires an  $\diamond\langle t+1 \rangle$ bisource only. As for Byzantine consensus in synchronous systems, all these algorithms assume  $t < n/3$ . Finally, it has been shown in [4] that the “ $\diamond\langle t+1 \rangle$ bisource” synchrony assumption is a necessary and sufficient condition to solve Byzantine consensus in asynchronous bi-directional message-passing systems, enriched with message authentication.

**Content of the paper** This paper presents a signature-free Byzantine consensus algorithm for asynchronous message-passing systems, which requires only the two assumptions:  $t < n/3$  and the existence of an  $\diamond\langle t+1 \rangle$ bisource. As these assumptions are necessary and sufficient to solve Byzantine consensus in the asynchronous model enriched with message authentication [4], it follows that (a) the existence of an  $\diamond\langle t+1 \rangle$ bisource

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<sup>1</sup>Enriching the system with random numbers allows for the design of randomized Byzantine consensus algorithms. These algorithms are characterized by a probabilistic termination property (e.g., [5, 9, 22, 27]).

1)bisource is necessary and sufficient to solve Byzantine consensus in an asynchronous signature-free system, and (b) the proposed algorithm is optimal with respect to underlying synchrony assumptions.

The proposed algorithm, which is round-based, assumes that at most  $m \leq \lfloor \frac{n-(t+1)}{t} \rfloor$  different values can be proposed by the correct processes (see also the paragraph “variant” in the conclusion). To attain its goal, it relies on a modular construction involving two communication abstractions and two distributed objects. More precisely, we have the following.

- The communication abstractions are the one-to-all *reliable broadcast* (RB) abstraction introduced in [7], and a very simple new communication abstraction that we call *cooperative broadcast* (CB). As suggested by its name, it is an all-to-all broadcast abstraction. This abstraction, which uses RB as an underlying subroutine, is particularly simple. It actually captures important cooperation properties, which make easier the design of upper layer distributed agreement algorithms.
- The two distributed objects are the following ones (the implementation of each of them use the underlying CB broadcast abstraction).
  - The first object is a message-passing version of the *adopt-commit* (AC) object (introduced in [17]) appropriately modified to cope with up to  $t < n/3$  Byzantine processes. Each round of the consensus algorithm uses a specific AC object. The aim of these objects is to prevent the consensus safety property from being violated.
  - The second object is a round-based object called *eventual agreement* (EA) object. Its aim is to ensure the consensus termination property. Hence, its implementation relies on the  $\diamond\langle t + 1 \rangle$ bisource assumption.

It is important to emphasize that, when designing the algorithm presented in the paper, modularity and simplicity were considered as first class design criteria. The algorithm presented is only the last step of a long quest: “Simplicity does not precede complexity, but follows it” (Alan Perlis, First Turing Award).

**Road map** The paper is made up of 7 sections. Section 2 presents the basic underlying asynchronous Byzantine computation model, the RB broadcast abstraction and the new CB broadcast abstraction. Section 3 presents an AC object suited to message-passing systems prone to Byzantine failures. Then, Section 4 presents the  $\diamond\langle t + 1 \rangle$ bisource behavioral assumption. Section 5 presents the round-based eventual agreement object. Section 6 pieces together the previous abstractions to obtain the synchrony-optimal Byzantine consensus algorithm. Finally, Section 7 concludes the paper. Due to page limitation, the missing proofs can be found in [6].

## 2 Basic Model, Reliable Broadcast, and Cooperative Broadcast

### 2.1 Processes, communication network, and failure model

**Asynchronous processes** The system is made up of a finite set  $\Pi$  of  $n > 1$  sequential processes, namely  $\Pi = \{p_1, \dots, p_n\}$ . As local processing times are negligible with respect to message transfer delays, they are considered as being equal to zero. Both notations  $i \in Y$  and  $p_i \in Y$  are used to say that  $p_i$  belongs to the set  $Y$ .

**Communication network** The processes communicate by exchanging messages through an asynchronous reliable point-to-point network. “Asynchronous” means that there is no bound on message transfer delays. “Reliable” means that the network does not lose, duplicate, modify, or create messages. “Point-to-point” means that any pair of processes is connected by two uni-directional channels (one in each direction). Hence, when a process receives a message, it can identify its sender. Moreover, as there is no message loss, all message transfer delays are finite.

A process  $p_i$  sends a message to a process  $p_j$  by invoking the primitive “send TAG( $m$ ) to  $p_j$ ”, where TAG is the type of the message and  $m$  its content. To simplify the presentation, it is assumed that a process can send messages to itself. A process  $p_i$  receives a message by executing the primitive “receive()”. Then say that the message is *received* by  $p_i$ .

**Failure model** Up to  $t$  processes can exhibit a *Byzantine* behavior. A Byzantine process is a process that behaves arbitrarily: it can crash, fail to send or receive messages, send arbitrary messages, start in an arbitrary state, perform arbitrary state transitions, etc. Moreover, Byzantine processes can collude to “pollute” the

computation (e.g., by sending messages with the same content, while they should send messages with distinct content if they were non-faulty).

A process that exhibits a Byzantine behavior is called *faulty*. Otherwise, it is *correct* or *non-faulty*. Given an execution,  $\mathcal{C}$  denotes the set of processes that are correct in this execution.

Let us notice that, as each pair of processes is connected by a channel, no Byzantine process can impersonate another process. Moreover, it is assumed that the Byzantine processes do not control the network (they can neither corrupt the messages sent by non-faulty processes, nor modify the message reception schedule).

**Discarding messages from Byzantine processes** If, according to its algorithm, a process  $p_j$  is assumed to send a single message  $\text{TAG}()$  to a process  $p_i$ , then  $p_i$  processes only the first message  $\text{TAG}(v)$  it receives from  $p_j$ . This means that, if  $p_j$  is Byzantine and sends several messages  $\text{TAG}(v)$ ,  $\text{TAG}(v')$  where  $v' \neq v$ , etc., all of them except the first one are discarded.

**Unreliable (best effort) broadcast** This simple broadcast is defined by a pair of operations denoted  $\text{broadcast}()$  and  $\text{receive}()$ , where  $\text{broadcast TAG}(m)$  is used as a shortcut for

**for each**  $j \in \{1, \dots, n\}$  **send**  $\text{TAG}(m)$  **to**  $p_j$  **end for**.

This means that a message *broadcast* by a correct process is *received* at least by all the correct processes. Differently, while it is assumed to send the same message to all the processes, a faulty process can actually send different messages to distinct processes and no message to others.

**Notation** The notation  $\mathcal{BZ\_AS}_{n,t}[\emptyset]$  is used to denote the previous basic Byzantine asynchronous message-passing computation model.

## 2.2 Reliable broadcast abstraction

This broadcast abstraction (in short, RB-broadcast) was proposed by G. Bracha [7]. It is a one-shot one-to-all communication abstraction, which provides processes with two operations denoted  $\text{RB\_broadcast}()$  and  $\text{RB\_deliver}()$ . When  $p_i$  invokes the operation  $\text{RB\_broadcast}()$  (resp.,  $\text{RB\_deliver}()$ ), we say that it “RB-broadcasts” a message (resp., “RB-delivers” a message). An RB-broadcast instance where process  $p_x$  is the sender is defined by the following properties.

- **RB-Validity.** If a non-faulty process RB-delivers a message  $m$  (from  $p_x$ ), then, if  $p_x$  is correct, it RB-broadcast  $m$ .
- **RB-Unicity.** A correct process RB-delivers at most one message from  $p_x$ .
- **RB-Termination-1.** If  $p_x$  is non-faulty and RB-broadcasts a message  $m$ , all the non-faulty processes eventually RB-deliver  $m$  from  $p_x$ .
- **RB-Termination-2.** If a non-faulty process RB-delivers a message  $m$  from  $p_x$  (possibly faulty) then all the non-faulty processes eventually RB-deliver the same message  $m$  from  $p_x$ .

The RB-Validity property relates the output to the input, while RB-Unicity states that there is no message duplication. The termination properties state the cases where processes have to RB-deliver messages. The second of them is what makes the broadcast reliable. It is shown in [8] that  $t < n/3$  is an upper bound on  $t$  when one has to implement such an abstraction. An algorithm implementing RB-broadcast is described in [6, 7]).

**Notation** The basic computing model strengthened with the additional constraint  $t < n/3$  is denoted  $\mathcal{BZ\_AS}_{n,t}[t < n/3]$ . RB-broadcast can consequently be implemented in this model.

## 2.3 Cooperative broadcast abstraction

**Definition** This new communication abstraction (in short CB-broadcast) is a one-shot all-to-all broadcast defined by an operation, denoted  $\text{CB\_broadcast}()$ , plus a read-only set at every process  $p_i$ , denoted  $\text{cb\_valid}_i$ . “All-to-all” means that it is assumed that all correct processes invoke  $\text{CB\_broadcast}()$ . When a process  $p_i$  invokes  $\text{CB\_broadcast}(v)$ , we say that “it cb-broadcasts  $v$ ”.

An invocation of  $\text{CB\_broadcast}()$  by a process  $p_i$  has an input parameter, namely the value that  $p_i$  wants to broadcast, and returns a value, which is a value CB-broadcast by a correct process. The CB-broadcast abstraction is formally defined by the following properties.

- **CB-Operation Termination.** The invocation of the operation  $\text{CB\_broadcast}()$  by a correct process terminates.
- **CB-Operation Validity.** If the invocation of  $\text{CB\_broadcast}()$  returns  $v$  to a correct process  $p_i$ ,  $v \in \text{cb\_valid}_i$ .

- **CB-Set Termination.** The set  $cb\_valid_i$  of a correct process  $p_i$  is eventually non-empty.
- **CB-Set Validity.** The set  $cb\_valid_i$  of any correct process  $p_i$  contains only values cb-broadcast by correct processes.
- **CB-Set Agreement.** The set  $cb\_valid_i$  and  $cb\_valid_j$  of any two correct processes  $p_i$  and  $p_j$  are eventually equal.

**Feasibility condition in the presence of up to  $t$  Byzantine processes** Let  $m$  be the number of different values that can be cb-broadcast by correct processes. It follows from the previous specification that, even when the (at most)  $t$  Byzantine processes propose a same value  $w$ , not proposed by a correct process,  $w$  can neither be returned, nor belong to the set  $cb\_valid_i$  of a correct process  $p_i$ . This can be ensured if and only if there is a value cb-broadcast by at least  $(t + 1)$  correct processes. This feasibility condition is captured by the predicate  $n - t > mt$ . (A proof of this feasibility condition can be found in [18]).

```

operation CB_broadcast( $v_i$ ) is
(1)  RB_broadcast CB_VAL( $v_i$ );
(2)  wait ( $cb\_valid_i \neq \emptyset$ );
(3)  return (any value in  $cb\_valid_i$ ).

when CB_VAL( $v$ ) is RB_delivered from  $p_j$  do
(4)  if ( $v$  RB_delivered from  $(t + 1)$  diff. processes)
      then add  $v$  to  $cb\_valid_i$  end if.

```

Figure 1: An algorithm implementing  $m$ -valued CB-broadcast in  $\mathcal{BZ\_AS}_{n,t}[t < n/3]$

Hence, we assume in the following that at most  $m \leq \lfloor \frac{n-(t+1)}{t} \rfloor$  different values can be cb-broadcast by the set of correct processes, and the corresponding abstraction is called  $m$ -valued CB-broadcast.

```

operation AC_propose( $v_i$ ) is
(1)   $est_i \leftarrow$  CB_broadcast AC_PROP( $v_i$ );
(2)  RB_broadcast AC_EST( $est_i$ );
(3)  wait (AC_EST( $est$ ) messages have been RB-delivered from  $(n - t)$ 
         different processes, and their  $est$  values belong to  $cb\_valid_i$ );
(4)   $MFA_i \leftarrow$  most frequent value in the previous  $(n - t)$  AC_EST() messages;
(5)  if (each of the previous  $(n - t)$  AC_EST() messages carries  $MFA_i$ )
(6)    then return ((commit,  $MFA_i$ ))
(7)    else return ((adopt,  $MFA_i$ ))
(8)  end if.

```

Figure 2: An algorithm implementing an  $m$ -valued adopt-commit object in  $\mathcal{BZ\_AS}_{n,t}[t < n/3]$

**An algorithm implementing CB-broadcast** A simple algorithm implementing CB-broadcast is described in Figure 1. When  $p_i$  invokes  $CB\_broadcast(v_i)$ , it first invokes the underlying  $RB\_broadcast\ CB\_VAL(v_i)$  for all correct processes to be eventually aware of  $v_i$  (line 1). Then, it waits until its set  $cb\_valid_i$  becomes non-empty (line 2). When this occurs,  $p_i$  takes any value from  $cb\_valid_i$  and returns it (line 3). Finally,  $p_i$  adds to  $cb\_valid_i$  all the values it RB-delivers from  $(t + 1)$  different processes (i.e.,  $v$  was RB-broadcast by at least one correct process). It is important to notice that, after the predicate  $cb\_valid_i \neq \emptyset$  became satisfied, new values can still be added to  $cb\_valid_i$ .

**Theorem 1.** *The algorithm described in Figure 1 implements the  $m$ -valued CB-broadcast abstraction in  $\mathcal{BZ\_AS}_{n,t}[t < n/3]$ .*

**Proof** Proof of the CB-Termination properties.

It follows from the feasibility condition, that there is a value  $v$  that is proposed by at least  $(t + 1)$  correct processes. Hence, these processes RB-broadcast  $CB\_VAL(v)$ . It then follows from line 4 and the RB-termination property that  $v$  will be added to the set  $cb\_valid_i$  of each correct process  $p_i$ . Hence, the CB-Set Termination property is satisfied, and no correct process can be blocked forever at line 2, from which follows the CB-Operation Termination property.

Proof of the CB-Validity properties.

To prove the CB-Set Validity property, let us consider a value  $v$  cb-broadcast only by Byzantine processes.

It follows that a correct process  $p_i$  can RB-deliver  $v$  from at most  $t$  different processes. Hence,  $p_i$  cannot add  $v$  to  $cb\_valid_i$  at line 4, which proves the property. The CB-Operation Validity property is then a trivial consequence of the CB-Set Validity property.

Proof of the CB-Set Agreement property.

Let us consider a value  $v \in cb\_valid_i$ . This means that  $p_i$  RB-delivered the message  $CB\_VAL(v)$  from  $(t + 1)$  different processes (line 4). It then follows from the RB-termination property of RB-broadcast that each correct process  $p_j$  RB-delivers these  $(t + 1)$  messages  $CB\_VAL(v)$ . Consequently, any correct process  $p_j$  adds  $v$  to its local set  $cb\_valid_j$ , which concludes the proof.  $\square_{Theorem 1}$

### 3 Adopt-Commit in the Presence of Byzantine Processes

This object was introduced in [17] in the context of read/write communication. Here we slightly modify its definition to cope with Byzantine processes (which by definition can decide anything).

**Definition** An adopt-commit (AC) is a one-shot object which encapsulates the safety part of agreement problems. It provides processes with a single operation denoted  $AC\_propose()$ . This operation takes a value as input parameter (we say that the invoking process proposes this value), and returns a pair  $\langle d, v \rangle$  (we say that the invoking process decides  $\langle d, v \rangle$ ), where  $d$  is a control tag and  $v$  a value. An AC object is defined by the following properties.

- AC-Termination. An invocation of  $AC\_propose()$  by a correct process terminates.
- AC-Validity. This property is made up of two parts.
  - AC-Output domain. If a correct process decides  $\langle d, v \rangle$ ,  $d \in \{\text{commit}, \text{adopt}\}$ , and  $v$  is a value that was proposed by a correct process.
  - AC-Obligation. If all the correct processes propose the same value  $v$ , only  $\langle \text{commit}, v \rangle$ , can be decided.
- AC-Quasi-agreement. If a correct process decides  $\langle \text{commit}, v \rangle$ , no other correct process can decide  $\langle -, v' \rangle$  where  $v' \neq v$ .

Implementations of an AC object in the presence of process crash failures can be found in [17, 23, 29, 30]. The implementations of [17, 30] are for asynchronous systems where any number of processes may crash and communication is by atomic read/write registers. The implementations of [23, 29] are for asynchronous message-passing systems where a minority of processes may crash.

It follows from the AC-Output domain property, that a value proposed only by Byzantine processes cannot be decided by a correct process. This means that an AC object has the same feasibility condition as CB-broadcast (let us also notice that this is independent from the fact that an AC object can be built on top of CB-broadcast). Hence, we assume that at most  $m \leq \lfloor \frac{n-(t+1)}{t} \rfloor$  values can be proposed by the correct processes, and the corresponding object is called an  $m$ -valued adopt-commit object.

**Implementation of an  $m$ -valued adopt-commit object** A distributed algorithm implementing an AC object in the presence of up to  $t < n/3$  Byzantine processes is described in Figure 2, for a correct process  $p_i$ . This algorithm is based on an underlying CB-broadcast, which means that each process has a read-only local set  $cb\_valid_i$  (initially empty).

When a process  $p_i$  invokes  $AC\_propose(v_i)$ , it first issues the operation  $CB\_broadcast AC\_PROP(v_i)$  from which it obtains a value that it saves in  $est_i$  (line 1). It then RB-broadcasts the message  $AC\_EST(est_i)$  (line 2), and waits until (a) it has RB-delivered messages  $AC\_EST()$  from  $(n - t)$  different processes, and (b) the values carried by these messages belong to the set  $cb\_valid_i$  supplied by CB-broadcast (line 3). Let us remember that, after the predicate  $cb\_valid_i \neq \emptyset$  became satisfied, new values can still be added to  $cb\_valid_i$ .

When this predicate becomes satisfied,  $p_i$  computes the most frequent value  $MFA_i$  carried by the previous  $(n - t)$   $AC\_EST()$  messages (line 4). If there are several “most frequent” values,  $p_i$  takes any of them. Finally, if all the messages who made satisfied the predicate of line 3 carried the same value  $MFA_i$ ,  $p_i$  returns the pair  $\langle \text{commit}, MFA_i \rangle$  (line 6); otherwise it returns the pair  $\langle \text{adopt}, MFA_i \rangle$  (line 7).

**Theorem 2.** Assuming that each correct process invokes the operation  $AC\_propose()$ , the algorithm of Figure 2 implements an  $m$ -valued adopt-commit object in  $BZ\_AS_{n,t}[t < n/3]$ .

**Proof** Proof of the AC-termination property.

Due to the CB-Operation termination property, no correct process blocks forever at line 1. So, we have only to show that no correct process can block forever at line 3. It follows from CB-Set Termination and CB-Set Validity that the sets  $cb\_valid_i$  of the correct processes are eventually not empty and contain only values proposed by correct processes. As (i) the value RB-broadcast by each correct process at line 2 is a value of its set  $cb\_valid_i$ , (ii) there are at least  $(n - t)$  correct processes, and (iii) the sets  $cb\_valid_i$  of the correct processes are eventually equal (CB-Set Agreement property), it follows that the predicate of line 3 is eventually satisfied at each correct process, which concludes the proof of AC-termination property. Proof of the AC-Output domain property.

Let us first observe that a correct process can decide only the pair  $\langle \text{commit}, v \rangle$  or the pair  $\langle \text{adopt}, v \rangle$  (lines 6-7). Hence, we have only to show that  $v$  is a value proposed by a correct process. A value  $v$  decided by a correct process  $p_i$  was RB-delivered in a message  $\text{AC\_EST}(v)$ . It follows from the predicate of line 3 that  $v \in cb\_valid_i$ . Finally, it follows from the CB-Set Validity property that  $v$  is a value proposed by a correct process. Proof of the AC-Obligation property.

If all correct processes propose the same value  $v$ , it follows from the CB-Set (Termination, Validity, and Agreement) properties that the set  $cb\_valid_i$  of each correct process  $p_i$  is eventually equal to  $\{v\}$ . Hence, each correct process RB-broadcasts the message  $\text{AC\_EST}(v)$  at line 2. It then follows from the predicate of line 3 that no value different from  $v$  can be decided. Proof of the AC-Quasi-agreement property.

Let  $p_i$  and  $p_j$  be two correct processes such that  $p_i$  decides the pair  $\langle \text{commit}, v \rangle$  while  $p_j$  decides  $\langle -, v' \rangle$ . As  $p_i$  decides  $\langle \text{commit}, v \rangle$ , it follows from line 3 that it RB-delivered the message  $\text{AC\_EST}(v)$  from  $(n - t)$  different processes. As, due to the RB-Unicity and RB-Termination-2 properties, no two correct processes RB-deliver different values from the same process, it follows that, among the  $(n - t)$  messages  $\text{AC\_EST}()$  RB-delivered by  $p_j$ , at most  $t$  of them may carry a value different from  $v$ , i.e., at least  $n - 2t \geq t + 1$  carry the value  $v$ . It follows that  $v$  is the most frequent value RB-delivered by  $p_j$ , and consequently  $v' = v$ .  $\square_{\text{Theorem 2}}$

## 4 The $\diamond\langle t + 1 \rangle$ Bisource Assumption

**Eventually timely channel** Let us consider the channel connecting a process  $p_i$  to a process  $p_j$ . This channel is *eventually timely* if there is a finite time  $\tau$  and a bound  $\delta$ , such that any message sent by  $p_i$  to  $p_j$  at time  $\tau'$  is received by  $p_j$  by time  $\max(\tau, \tau') + \delta$ . Let us observe that neither  $\tau$ , nor  $\delta$ , is known by the processes.

As already indicated, there is an input/output channel from each process to itself.

**$\diamond\langle k \rangle$ sink,  $\diamond\langle k \rangle$ source, and  $\diamond\langle k \rangle$ bisource** A correct process  $p_i$  is an  $\diamond\langle k \rangle$ sink if it has eventually timely input channels from  $k$  correct processes (including itself). This set of processes is denoted  $X_i^-$ . Similarly, a correct process is an  $\diamond\langle k \rangle$ source if it has  $k$  eventually timely output channels to correct processes (including itself). This set of processes is denoted  $X_i^+$ .

An  $\diamond\langle k \rangle$ bisource is a correct process  $p_i$  that is both  $\diamond\langle k \rangle$ sink and  $\diamond\langle k \rangle$ source. Let us remark that the timely input channels and the timely output channels do not necessarily connect  $p_i$  to the same subset of processes.

**Notation for system models** The system model  $\mathcal{BZ}\text{-}\mathcal{AS}_{n,t}[t < n/3]$  enriched with an  $\diamond\langle t + 1 \rangle$ bisource is denoted  $\mathcal{BZ}\text{-}\mathcal{AS}_{n,t}[t < n/3, \diamond\langle t + 1 \rangle\text{bisource}]$ .

**Discussion** The previous notions were introduced in [1, 14]. Our definition of an  $\diamond\langle t + 1 \rangle$ bisource is slightly different from the original definition introduced in [1]. The difference is that it considers only eventually timely channels connecting correct processes, while [1] considers eventually timely channels connecting a correct process to correct or faulty processes. Hence, an  $\diamond\langle t + 1 \rangle$ bisource is an  $\diamond\langle 2t + 1 \rangle$ bisource in the parlance of [1]. We consider only eventually timely channels connecting pair of correct processes for the following reason: an eventually timely channel connecting a correct process and a Byzantine process can always appear to the correct process as being an asynchronous channel.

## 5 Eventual Agreement Object

### 5.1 Motivation and definition

This object, which is round-based, will be used to ensure the termination of the consensus algorithm, namely, its aim is to allow the correct processes to eventually converge on a single value. To this end, it provides the processes with a single operation denoted  $\text{EA\_propose}(r, v)$  where  $r$  is a round number and  $v$  is the value proposed at this round by the invoking process. Each invocation of  $\text{EA\_propose}()$  by a correct process returns



a value. It is assumed that each correct process invokes this operation once per round, and its successive invocations are done according to consecutive round numbers. When a process invokes  $\text{EA\_propose}(r, v)$ , we say that it “ea-proposes  $v$  at round  $r$ ”.

**Definition** An eventual agreement (EA) object is defined by the following properties.

- **EA-Termination.** For any  $r$ , if all correct processes invoke  $\text{EA\_propose}(r, -)$ , each of these invocations terminates.
- **EA-Validity.** For any  $r$ , if all correct processes invoke the operation  $\text{EA\_propose}(r, v)$  no correct process returns a value different from  $v$ .
- **EA-Eventual agreement.** If the correct processes execute an infinite number of rounds, there is an infinite number of rounds  $r$  at which all the correct processes return the same value  $v$ , where  $v$  is such that a correct process invoked the operation  $\text{EA\_propose}(r, v)$ .

It is important to notice that the EA-Validity property is particularly weak. More precisely, if, during a round  $r$ , two correct processes invoke  $\text{EA\_propose}(r, v1)$  and  $\text{EA\_propose}(r, v2)$ , with  $v1 \neq v2$ , the invocation of  $\text{EA\_propose}(r, -)$  by any correct process is allowed to return an arbitrary value (i.e., even a value proposed neither by a correct nor by a Byzantine process).

As the implementation that follows uses at every round an instance of CB-broadcast, we assume that at most  $m \leq \lfloor \frac{n-(t+1)}{t} \rfloor$  different values are ea-proposed by correct processes.

## 5.2 Implementation of an $m$ -valued eventual agreement object

**Definitions** The algorithm presented below uses the following sets and functions.

- There are  $\alpha = \binom{n}{n-t}$  possible combinations of  $(n-t)$  processes among the  $n$  processes  $p_1, \dots, p_n$ . Let us call them  $F_1 \dots F_\alpha$ .
- Given any round number  $r \geq 1$ :
  - $\text{coord}(r)$  denotes the function  $((r-1) \bmod n) + 1$ .  
Given a round  $r$ ,  $\text{coord}(r)$  defines its coordinator process. As we can see, if there is an infinite number of rounds, each process is infinitely often round coordinator.
  - $F(r)$  denotes the function  $F_{\text{index}(r)}$ , where  $\text{index}(r) = ((\lfloor \frac{r}{n} \rfloor - 1) \bmod \alpha) + 1$ .  
Hence, each set  $F(r)$  returns a set made up of  $(n-t)$  processes. During each round, its coordinator strives to decide a value. To this end, it requires the help of the processes in  $F(r)$  to broadcast the value it champions.  $F_1$  is used by the coordinators of the rounds 1 to  $n$ ;  $F_2$  is used by the coordinators of the rounds  $(n+1)$  to  $2n$ ; ...,  $F_\alpha$  is used by the coordinators of the rounds  $((\alpha-1)n+1)$  to  $\alpha n$ ;  $F_1$  is used by the coordinators of the rounds  $((\alpha n+1)$  to  $(\alpha+1)n$ ; etc.

Considering an infinite sequence of rounds, it is important to notice that there is an infinite number of rounds  $r$  and  $r'$  such that  $(\text{coord}(r) = \text{coord}(r')) \wedge F(r) = F(r')$  and an infinite number of rounds  $r$  and  $r'$  such that  $(\text{coord}(r) = \text{coord}(r')) \wedge (F(r) \neq F(r'))$ .

**Local variables** Each process  $p_i$  manages the following local variables.

- $\text{timer}_i[1..]$  is an array of timers, such that  $\text{timer}_i[r]$  is the timer used by  $p_i$  for round  $r$ .
- $\text{CB}[1..]$  is an array of CB-broadcast instances shared by all processes.  $\text{CB}[r]$  is the instance associated with round  $r$ . Hence,  $\text{CB}[r].\text{cb\_valid}_i$  is the set of values supplied to  $p_i$  by  $\text{CB}[r]$ .

To distinguish messages which have the same tag but are sent at different rounds, a message  $\text{XXX}()$  associated with round  $r$  is denoted  $\text{XXX}[r]()$ .

**Algorithm: first part of  $\text{EA\_propose}()$**  (Lines 1-5) The algorithm executed by a correct process  $p_i$  is described in Figure 3. Let us remind that, it is assumed that each correct process invokes  $\text{EA\_propose}()$  at every round.

When a correct process  $p_i$  invokes  $\text{EA\_propose}(r_i, \text{val}_i)$  ( $r_i$  is a round number and  $\text{val}_i$  the value it ea-proposes at this round), it first invokes  $\text{CB}[r_i].\text{CB\_broadcast EA\_PROP1}(\text{val}_i)$ , and saves the value returned in  $\text{aux}_i$  (line 1). Then,  $p_i$  broadcasts the message  $\text{EA\_PROP2}[r_i](\text{aux}_i)$  (line 1) and waits until (a) it has received messages  $\text{EA\_PROP2}[r_i]()$  from  $(n-t)$  different processes, and (b) the values carried by these messages belong to the set denoted  $\text{CB}[r_i].\text{cb\_valid}_i$ , which is locally supplied by the CB-broadcast instance  $\text{CB}[r_i]$  (line 3). If

```

operation EA_propose( $r_i, val_i$ ) is
(1)  $aux_i \leftarrow CB[r_i].CB\_broadcast\ EA\_PROP1(val_i)$ ;
(2) broadcast EA_PROP2[ $r_i$ ]( $aux_i$ );
(3) wait (EA_PROP2[ $r_i$ ]() messages have been received from  $(n - t)$ 
    different processes, and their  $aux$  values belong to  $CB[r_i].cb\_valid_i$ );
(4) if (the  $(n - t)$  previous messages carry the same value  $v$ ) then return( $v$ ) end if;
(5) set  $timer_i[r_i]$  to  $r_i$ ;
(6) wait (EA_RELAY[ $r_i$ ]( $aux$ ) messages received from  $(n - t)$  different processes);
(7) if (EA_RELAY[ $r_i$ ]( $v$ ) where  $v \neq \perp$  received from a process in  $F(r_i)$ )
(8)   then return( $v$ )
(9)   else return( $val_i$ )
(10) end if.

when EA_PROP2[ $r$ ]() is received from a process in  $F(r)$  do
(11) if ( $(i = coord(r) \wedge (EA\_COORD[r]() \text{ not already broadcast}))$ )
(12)   then let  $w$  be the value carried by the message EA_PROP2[ $r$ ]();
(13)   broadcast EA_COORD[ $r$ ]( $w$ )
(14) end if.

when EA_COORD[ $r$ ]( $v$ ) is received from  $p_{coord(r)}$  or ( $timer_i[r]$  expires) do
(15) if (EA_RELAY[ $r$ ]() not already broadcast)
(16)   disable  $timer_i[r]$ ;
(17)   if ( $timer_i[r]$  expired) then  $v\_coord_i \leftarrow \perp$  else  $v\_coord_i \leftarrow v$  end if;
(18)   broadcast EA_RELAY[ $r$ ]( $v\_coord_i$ )
(19) end if.

```

Figure 3: An algorithm implementing an  $m$ -valued EA object in  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \diamond\langle t + 1 \rangle\text{bisource}]$

all these messages carry the same value  $v$ ,  $p_i$  returns  $v$  as result of its invocation  $EA\_propose(r_i, val_i)$  (line 4)<sup>2</sup>. Otherwise,  $p_i$  sets the timer associated with the round  $r_i$  to the value  $r_i$  (line 5)<sup>3</sup>.

**Algorithm: message processing and role of the round coordinator** (Lines 11-19) Each round  $r$  uses a round coordinator, defined by  $coord(r)$ . As we have also seen, the set of  $(n - t)$  processes denoted  $F(r)$  is associated with round  $r$ .

When  $p_i$  is the coordinator of round  $r$  and receives for the first time a message  $EA\_PROP2[r]()$  from a process in the set  $F(r)$ , it champions the value  $w$  carried by this message to become the value returned by the invocations of  $EA\_propose(r, -)$ . To that end, it simply broadcasts the message  $EA\_COORD[r](w)$  (lines 11-14).

When a process  $p_i$  receives a message  $EA\_COORD[r](v)$  from the coordinator of round  $r$ , if not yet done, it broadcasts the message  $EA\_RELAY[r](v)$  to inform the other processes that it has received the value  $v$  championed by the coordinator of round  $r$ . If the local timer associated with this round ( $timer_i[r]$ ) has already expired,  $p_i$  broadcasts the message  $EA\_RELAY[r](\perp)$ , to inform the other processes that it suspects the coordinator of round  $r$  not to be an  $\diamond\langle t + 1 \rangle\text{bisource}$  (this suspicion can be due to the asynchrony of the channel connecting  $p_{coord(r)}$  to  $p_i$ , or the fact that –while  $p_{coord(r)}$  is an  $\diamond\langle t + 1 \rangle\text{bisource}$ – the link from  $p_{coord(r)}$  to  $p_i$  is not yet synchronous, or the fact that  $p_{coord(r)}$  has a Byzantine behavior). In all cases, as  $timer_i[r]$  will no longer be useful,  $p_i$  disables it. This behavior of  $p_i$  is captured by the lines 15-19.

**Algorithm: second part of  $EA\_propose()$**  (Lines 6-10) After it has set  $timer_i[r_i]$  (line 5),  $p_i$  waits until it has received a message  $EA\_RELAY[r_i]()$  from  $(n - t)$  different processes (line 6). When this occurs, the invocation of the operation  $EA\_propose(r_i, val_i)$  by  $p_i$  returns a value. This value is  $v \neq \perp$  if  $p_i$  received a message  $EA\_RELAY[r_i](v)$  from a process in the set  $F(r_i)$  (lines 7-8). Otherwise, no process of  $F(r_i)$  witnesses the value championed by the coordinator of round  $r$ . In this case,  $p_i$  returns the value  $val_i$ , i.e., the value it ea-proposed (line 9).

<sup>2</sup>Let us remark that lines 1-3 of Figure 3 and lines 1-3 of Figure 2 differ only in the fact that an RB-broadcast is used at line 2 for the AC object, and a simple broadcast is used at line 2 for the EA object. These lines have not been encapsulated to define a higher level object because the messages  $EA\_PROP2[r_i]()$  are explicitly used in lines 11-14 of Figure 3, while their counterparts in an AC object –messages  $AC\_EST$ – are not used by the upper layer.

<sup>3</sup>The important point here is that the value of the timer increases; as  $r_i$  increases at every round, it is used as a timeout value. More generally, it is possible to assign to  $timer_i[r_i]$  the value returned by an increasing function  $f_i(r_i)$ , which can be specific to each process  $p_i$ .

### 5.3 Proof

Let us remember that, by assumption, all correct processes invoke  $\text{EA\_propose}(r, -)$ , where  $r = 1$ . Moreover, they ea-propose at most  $m$  different values.

**Lemma 1.** *Whatever the round  $r$ , if all correct processes invoke  $\text{EA\_propose}(r, v)$  no correct process returns a value different from  $v$ . (Proof in [6].)*

**Lemma 2.** *Let  $r \geq 1$ . If all correct processes invoke the operation  $\text{EA\_propose}(r, -)$ , then each of these invocation terminates. (Proof in [6].)*

**Lemma 3.** *If the correct processes execute an infinite number of rounds, there is an infinite number of rounds  $r$  at which all the correct processes return the same value  $v$ , where  $v$  is such that a correct process invoked  $\text{EA\_propose}(r, v)$ .*

**Proof** Let us define the following rounds:

- Let  $r_1$  be the first round that is strictly greater than  $2\delta$ .
- Let  $p_\ell$  be an  $\diamond\langle t + 1 \rangle$ bisource. There exists a round  $r_2$  such that in every subsequent round:
  - Each message sent by any  $p_x \in X_\ell^-$  to  $p_\ell$  is received within an interval of at most  $\delta$  time units.
  - Each message sent by  $p_\ell$  to any  $p_y \in X_\ell^+$  is received within an interval of at most  $\delta$  time units.
- Let  $r > \max(r_1, r_2)$  be any round coordinated by  $p_\ell$  such that  $X_\ell^+ \subseteq F(r)$  and  $F(r) \subseteq \mathcal{C}$ . Let us notice that, due to the definition of  $F(r)$ , an infinity of such rounds  $r$  exists.

Claim C. For every process  $p_i \in X_\ell^+$ , we have  $v\_coord_i \neq \perp$  in round  $r$  (line 18).

Proof of claim C. Let  $p_i$  be any process in  $X_\ell^+$ . Let  $\tau$  be the time at which  $p_i$  sets the timer at line 5 of round  $r$ . At this moment, since  $p_i$  finished executing line 4, there are at least  $(n - t)$  processes from which  $p_i$  received an  $\text{EA\_PROP2}[r]()$  message. Since  $|X_\ell^-| \geq t + 1$ , it follows that among these  $(n - t)$  processes, there is at least one, say  $p_k$ , that belongs to  $X_\ell^-$ . Observe that  $p_k$  necessarily broadcast the message  $\text{EA\_PROP2}[r]()$  before  $\tau$ . Since  $r > r_2$ , this message is received by  $p_\ell$  before time  $\tau + \delta$ .

Therefore, if  $p_\ell$  did not broadcast a message  $\text{EA\_COORD}[r]()$  before receiving  $\text{EA\_PROP2}[r]()$  from  $p_k$ , as  $p_k \in X_\ell^- \subseteq F(r)$ , the condition of line 11 and the when statement preceding it are both satisfied, and  $p_\ell$  broadcasts  $\text{EA\_COORD}[r]()$  at line 13. Consequently, in all cases,  $p_\ell$  broadcasts a message  $\text{EA\_COORD}[r]()$  by time  $\tau + \delta$ . Finally, since  $p_i \in X_\ell^+$  and  $r > r_2$ , this message is received by  $p_i$  before time  $\tau + 2\delta$ .

Let us recall that, as  $r > r_1$ , it holds that  $r > 2\delta$ , and consequently, since  $p_i$  set  $\text{timer}_i[r]$  to  $r$  (line 8) at time  $\tau$ , the timeout occurs after time  $\tau + 2\delta$ . Therefore,  $p_i$  receives the message  $\text{EA\_COORD}[r]()$  from  $p_\ell$  before the timeout. Consequently, when evaluated by  $p_i$ , the predicate of line 17 is necessarily false, and  $v\_coord_i \neq \perp$ . This proves the claim.

We show in the following that all correct processes return the same value in round  $r$ . Let us first observe that every correct process broadcasts an  $\text{EA\_PROP2}[r]()$  message that carries a value which was necessarily ea-proposed by a correct process. Therefore, since  $p_\ell$  is correct (and is the coordinator of  $r$ ), the message  $\text{EA\_COORD}[r]()$  it broadcasts in round  $r$  contains a value, say  $w$ , that was sent to it by a correct process. Therefore, since (due to the definition of  $r$ ), the processes of  $F(r)$  are correct, the  $\text{EA\_RELAY}[r]()$  messages broadcast by them carry either  $w$  or  $\perp$ . Consequently, every correct process  $p_i$  can either returns  $w$  or  $val_i$  after executing the lines 7-10. To finish the proof, it remains to show that no correct process  $p_i$  returns  $val_i$  (if  $val_i \neq w$ ).

Let us observe that each correct process waits at line 6 until it receives  $(n - t)$   $\text{EA\_RELAY}[r]()$  messages. Since  $|X_\ell^+| > t$ , it follows that at least one of these messages was broadcast by a process in  $X_\ell^+$ . Due to Claim C, this message cannot carry  $\perp$ . It then follows from the predicate of line 7 that any correct process executes line 8 and returns  $w$ , which proves the lemma.  $\square$ Lemma 3

**Theorem 3.** *The algorithm of Figure 3 implements an  $m$ -valued eventual agreement object in  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \diamond\langle t + 1 \rangle$ bisource]. (The proof follows from Lemma 1, Lemma 2, and Lemma 3.)*

```

operation CONS_propose( $v_i$ ) is
(1)   $est_i \leftarrow CB[0].CB\_broadcast\ VALID(v_i);$            % safety: validity %
(2)  repeat forever
(3)     $r_i \leftarrow r_i + 1;$ 
(4)     $v \leftarrow EA\_OBJECT.EA\_propose(r_i, est_i);$            % liveness %
(5)    if ( $v \in CB[0].cb\_valid_i$ ) then  $est_i \leftarrow v$  end if; % safety: validity %
(6)     $\langle tag, est_i \rangle \leftarrow AC\_OBJECT[r_i].AC\_propose(est_i);$  % safety: agreement %
(7)    if ( $tag = commit$ ) then RB.broadcast DECIDE( $est_i$ ) end if
(8)  end repeat.

when DECIDE( $v$ ) is RB-delivered do
(9)  if (DECIDE( $v$ ) RB-delivered from  $(t + 1)$  diff. processes) then return( $v$ ) end if.

```

Figure 4: An algorithm for  $m$ -valued Byzantine consensus in  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \diamond\langle t + 1 \rangle\text{bisource}]$

## 5.4 Looking for efficiency: Parameterized eventual agreement

**Time complexity of the EA algorithm** The aim of the previous algorithm was to attain a round  $r$  during which all correct processes return the same value (ea-proposed by one of them). Hence its time complexity can be measured by the value of this round number. As the underlying synchrony assumption is *eventual*, we only know that this number  $r$  is finite.

Hence, to eliminate the noise created by the “eventual” attribute, and consequently be able to compute a time complexity of the algorithm, let us replace the  $\diamond\langle t + 1 \rangle$ bisource synchrony assumption by the  $\langle t + 1 \rangle$ bisource assumption, i.e., we consider that there is a  $\langle t + 1 \rangle$ bisource from the very beginning. The corresponding system model is denoted  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \langle t + 1 \rangle\text{bisource}]$ .

The uncertainty created by the “eventual” attribute is consequently eliminated, and the only uncertainty is the identity of the bisource and its associated input and output timely channels. As there are  $n$  processes and  $\alpha = \binom{n}{n-t}$  combinations for the sets  $F(r)$ , it follows that the algorithm, which works in  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \diamond\langle t + 1 \rangle\text{bisource}]$ , terminates in at most  $\alpha n$  rounds when the system behaves as  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \langle t + 1 \rangle\text{bisource}]$ .

**Improving the time complexity** One way to improve the time complexity of the algorithm (as measured previously) is to consider a “tuning” parameter  $k$ ,  $0 \leq k \leq t$ , and use it in both the synchrony assumption and the size of the sets  $F(r)$ , as follows.

- The assumption  $\langle t + 1 \rangle$ bisource is replaced by the stronger assumption  $\langle t + 1 + k \rangle$ bisource.
- Instead of  $(n - t)$ , the size of the sets  $F(r)$  is now  $n - t + k$ .

An algorithm, parameterized with  $k$ , extending the basic algorithm of Figure 3 and based on the previous definition is described in [6]. Designed for the system model  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \diamond\langle t + 1 + k \rangle\text{bisource}]$ , this algorithm has a time complexity of  $\beta n$  where  $\beta = \binom{n}{n-t+k}$  when executed in  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \langle t + 1 + k \rangle\text{bisource}]$ . As simple instances of this parameterized algorithm, let us consider two particular values of  $k$ . For  $k = 0$ , we obtain the basic algorithm. For  $k = t$ , the time complexity is  $n$ , which is the best that can be obtained with a round coordinator-based algorithm (up to  $n$  rounds can be needed to benefit from the  $\langle t + 1 + k \rangle$ bisource).

## 6 Byzantine Consensus Algorithm

**$m$ -Valued Byzantine consensus** In the  $m$ -valued Byzantine consensus, the correct processes propose values from a set of at most  $m$  values. The corresponding object is a one-shot object, that provides the processes with a single operation denoted  $\text{CONS\_propose}(v)$ , where  $v$  is the value proposed by the invoking process. This operation returns a value to the invoking process. If  $p_i$  obtains the value  $v$ , we say that it “decides”  $v$ . The consensus object is defined by the following properties.

- **CONS-Termination.** The invocation of  $\text{CONS\_propose}()$  by a correct process terminates.
- **CONS-Validity.** If a correct process decides  $v$ , a correct process invoked  $\text{CONS\_propose}(v)$ .
- **CONS-Agreement.** No two correct processes decide different values.

**An algorithm solving  $m$ -valued Byzantine consensus** Assuming  $m \leq \lfloor \frac{n-(t+1)}{t} \rfloor$ , the algorithm described in Figure 4 implements an  $m$ -valued consensus object in  $\mathcal{BZ\_AS}_{n,t}[t < n/3, (t+1)\text{bisphere}]$ . This algorithm, which –thanks to the previous abstractions– is simple, uses the following underlying objects.

- Each process  $p_i$  manages a round number  $r_i$  (initialized to 0), and a current estimate denoted  $est_i$ .
- $EA\_OBJECT$  is a shared  $m$ -valued EA object. Its aim is to allow processes to eventually converge to the same estimate value. Hence, the associated line 4 is related to CONS-Termination.
- $AC\_OBJECT[1..]$  is an unbounded array of  $m$ -valued adopt-commit objects, shared by all processes.  $AC\_OBJECT[r]$  is the adopt-commit object used at round  $r$ . The aim of these objects (line 6) is to allow correct processes to decide a value proposed by one of them, and prevent them from deciding different values, i.e., to guarantee consensus safety.
- $CB[0]$  is a CB-broadcast instance, used at the very beginning to obtain a value proposed by a correct process and allow a process  $p_i$  to use the associated set  $CB[0].cb\_valid_i$  to check the validity of the values returned by the  $EA\_OBJECT$  object (i.e., check if this value is from a correct process).

When a correct process  $p_i$  invokes  $CONS\_propose(v_i)$ , it first invokes  $CB[0].CB\_broadcastVALID(v_i)$  to obtain a value that was proposed by a correct process (line 1)<sup>4</sup>. As already indicated, this invocation also ensures that the sets  $CB[0].cb\_valid_i$  of correct processes are eventually equal and contain values proposed only by correct processes.

Then process  $p_i$  enters an infinite loop (lines 2-8). After it has entered its current round (line 3), process  $p_i$  proposes its current estimate of the decision value  $est_i$  to the EA object, namely, it invokes  $EA\_OBJECT.EA\_propose(r_i, est_i)$  (line 4). If the value returned by this invocation is a value that it knows as proposed by a correct process, it adopts it as new estimate, otherwise it keeps its previous estimate (line 5).

Process  $p_i$  proposes then the current value of  $est_i$  to the adopt-commit object associated with the current round, from which it obtains a pair  $\langle tag, est_i \rangle$  (line 6). If the value of the tag is `commit` (line 7),  $p_i$  RB-broadcasts the message  $DECIDE(est_i)$  to inform the other processes that the value of  $est_i$  can be decided. Then, whatever the value of the tag,  $p_i$  proceeds to the next round with its (possibly new) estimate value  $est_i$ .

Finally, as soon as a process, that not yet decided, has RB-delivered the same message  $DECIDE(v)$  from  $(t + 1)$  different processes, it decides  $v$  and stops (line 9). Let us notice that at least one of these messages is from a correct process.

**Theorem 4.** *The algorithm of Figure 4 solves the  $m$ -valued Byzantine consensus problem in the system model  $\mathcal{BZ\_AS}_{n,t}[t < n/3, \diamond(t + 1)\text{bisphere}]$ .*

**Proof** We say that a process  $p_i$  starts round  $r$  when it assigns value  $r$  to its local variable  $r_i$  (line 3).

Proof of the CONS-Termination property.

If a process decides at line 9, it previously RB-delivered the message  $DECIDE(v)$  from  $(t + 1)$  different processes. Due to the RB-termination property of the corresponding  $(t + 1)$  RB-broadcasts, each correct process RB-delivers this message from the same set of  $(t + 1)$  processes, and consequently decides. So, let us assume by contradiction that no correct process decides at line 9.

Let us first observe that, due to the CB-Operation Termination property that no correct process  $p_i$  blocks forever at line 1. Moreover, it follows from the CB-Operation Validity property that the set  $CB[0].cb\_valid_i$  is not empty when this invocation terminates.

As no correct process decides, and all correct processes invoke  $EA\_propose(1, -)$ , it follows from the EA-Termination and AC-Termination properties that they all terminate the first round, and consequently start the second. Moreover, if the estimate  $est_i$  of a correct process  $p_i$  is updated at line 5, its new value is a value proposed by a correct process. It follows that the correct processes start the second round with estimate values  $est_i$  containing values proposed by correct processes. As no correct process decides, the same reasoning applies to all rounds  $r > 1$ .

Let us observe that the local variables  $CB[0].cb\_valid_i$  of the correct processes eventually converge to the same content (CB-Set Agreement and Termination of  $CB[0]$ ). Hence, there is a round  $r_0$  such that, for every correct process  $p_i$ , the set  $CB[0].cb\_valid_i$  is never updated after it starts  $r_0$ .

It then follows from the EA-Eventual Agreement property of  $EA\_OBJECT$ , that there is a round  $r > r_0$  during which all correct processes obtain the same value  $v$  at line 4, where  $v$  is a value proposed by a correct process. Hence, since  $r > r_0$ , they all succeed the test of line 5 and adopt  $v$  as their new estimate  $est_i$ .

<sup>4</sup>Even if, up to now, a process behaved “correctly”, it may crash in the future and become then faulty. Hence, no process can a priori consider the value it proposes as a value proposed by a correct process.

Therefore, all correct processes invoke  $AC\_OBJECT[r].AC\_propose(v)$  at line 6. Due to the AC-Obligation property of  $AC\_OBJECT[r]$ , all correct processes obtain  $\langle commit, v \rangle$  at line 6. Consequently, they all RB-broadcast the same message  $\langle commit, v \rangle$  at line 7. An  $n - t \geq t + 1$ , the decision predicate of line 9 becomes eventually true at every correct process, which contradicts the initial assumption.

Proof of the CONS-Validity property.

Let us consider the first round. Let  $p_i$  be a correct process. It follows from the CB-Operation Validity property of  $CB[0]$  that  $est_i$  is a value proposed by a correct process. Moreover, it follows from the CB-Set Validity property, that  $CB[0].cb\_valid_i$  contains only values proposed by correct processes. It follows from these observations that, be or not  $est_i$  modified at line 5, when  $p_i$  invokes  $AC\_OBJECT[1].AC\_propose(est_i)$  at line 6,  $est_i$  contains a value proposed by a correct process. It then follows from the AC-Validity property of  $AC\_OBJECT[1]$  that the value assigned to  $est_i$  at line 6 is a value proposed by a correct process. The same reasoning applies iteratively to all rounds, from which it follows that a value that is RB-broadcast by a correct process at line 7 is a value proposed by a correct process.

If a correct process  $p_i$  decides a value  $v$  at line 9, it follows from the decision predicate used at this line that  $v$  was RB-broadcast at line 7 by at least one correct process  $p_j$ . The previous paragraph has shown that such a value  $v$  was proposed by a correct process.

Proof of the CONS-Agreement property.

Let us first observe that, if a correct process decides at line 9, it decides a value RB-broadcast by a correct process at line 7. Hence, the proof consists in showing that no two correct processes RB-broadcast different values at line 7.

Let  $r$  be the first round at which a correct process  $p_i$  RB-broadcast a message  $DECIDE()$  at line 7. Let  $v$  the value carried by this message. It follows that, at line 6,  $p_i$  obtained the pair  $\langle commit, v \rangle$  from the object  $AC\_OBJECT[r]$ . Let us consider another correct process  $p_j$ . There are two cases.

- $p_j$  RB-broadcast  $DECIDE(w)$  at line 9 of round  $r$ . This means that it obtained  $\langle commit, w \rangle$  from  $AC\_OBJECT[r]$ . It then follows from the AC-agreement property of  $AC\_OBJECT[r]$  that  $v = w$ . Moreover,  $p_j$  proceeds to the next round with  $est_j = v$ .
- $p_j$  did not RB-broadcast the message  $DECIDE(w)$  at line 9 of round  $r$ . It then follows from the AC-agreement property of  $AC\_OBJECT[r]$  that  $p_j$  obtained the pair  $\langle adopt, v \rangle$ . Hence, at line 6,  $p_j$  assigned the value  $v$  to  $est_j$ .

It follows that the estimate values of all the correct processes that progress to the next round are equal to  $v$ . Let  $p_x$  be any correct process executing round  $(r + 1)$ . It follows from the EA-Validity property of  $EA\_OBJECT$ , that the invocation by  $p_x$  of  $EA\_OBJECT.EA\_propose(r + 1, est_x)$  returns  $v$ , and from the AC-Obligation property of  $AC\_OBJECT[r + 1]$  that this object returns  $\langle -, v \rangle$  to  $p_x$ . This means that the estimates of all the correct processes remain forever equal to  $v$ . Hence, no value different from  $v$  can be RB-broadcast at line 7 by a correct process during a round  $r' \geq r$ .  $\square_{Theorem 4}$

## 7 Conclusion

**A variant** To ensure that a value decided by a correct process is always a value that was proposed by a correct process, the paper considered  $m$ -valued consensus, i.e., at most  $m \leq \lfloor \frac{n-(t+1)}{t} \rfloor$  different values can be proposed by the correct processes (i.e., there is a value that is proposed by at least  $(t + 1)$  correct processes). To ensure that no value proposed only by Byzantine processes is ever decided, some Byzantine consensus algorithms (e.g., [11, 24]) do not have such an “ $m$ -valued” requirement. They instead allow the correct processes to decide a default value  $\perp$  when they do not propose the same value. The algorithms proposed in the paper can be modified to satisfy this different validity requirement.

**The aim and the content of the paper** This paper presented a consensus algorithm for asynchronous Byzantine message-passing systems, that is optimal with respect to the underlying synchrony assumption. This assumption is the existence of a process that is an *eventual  $\langle t + 1 \rangle$  bisource*. Such a process  $p$  is a non-faulty process that eventually has (a) timely input channels from  $t$  correct processes and (b) timely output channels to  $t$  correct processes. Moreover these input and output channels can connect  $p$  to different subsets of processes.

In addition to a reliable broadcast abstraction, the design of the algorithm, which is very modular, is based on simple abstractions: a new broadcast abstraction called *cooperative broadcast*, adopt-commit objects that cope with Byzantine processes (as far as we know, the paper presented the first implementation of such objects

in the presence of Byzantine processes), and a new round-based object called *eventual agreement*, whose definition involves a pretty weak validity property.

This paper answered a long-lasting problem, namely, solving Byzantine consensus with the weakest underlying synchrony assumptions. Finally, as claimed in the introduction, and in addition to its optimality with respect to synchrony requirements, a very important first class property of the proposed algorithm lies in its *design simplicity*. “Simplicity  $\Rightarrow$  easy” is rarely true for non-trivial problems [2].

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