

# OPTIMIZATION AND IDENTIFICATION OF A BIAXIAL TENSILE TEST BASED UPON SENSITIVITY TO MATERIAL PARAMETERS

**Morgan Bertin<sup>1</sup>, François Hild<sup>2</sup>, Stéphane Roux<sup>3</sup>, Florent Mathieu<sup>4</sup>, Hugo Leclerc<sup>5</sup>**

LMT (ENS Cachan, CNRS, Université Paris Saclay)  
61, avenue du Président Wilson  
94235 Cachan, France

<sup>1</sup> morgan.bertin@lmt.ens-cachan.fr, <sup>2</sup> hild@lmt.ens-cachan.fr, <sup>3</sup> roux@lmt.ens-cachan.fr,

<sup>4</sup> florent.mathieu@lmt.ens-cachan.fr, <sup>5</sup> hugo.leclerc@lmt.ens-cachan.fr

The objective of the present study is to propose a protocol and a criterion to guide the design of specimen shape but also loading history to lower the sensitivity of the identified parameters to the measurement uncertainties. The identification quality of constitutive parameters is both related to the experimental procedures and the identification methods. In the measurement work, the identification quality is estimated by the covariance matrix of the identified material parameters [1, 2]. It accounts for different aspects of the problem, namely, the geometry of the studied structure, the chosen constitutive law, the set of parameters, the boundary conditions, the measurement uncertainties, the identification method and to a lesser extent the numerical simulation method. It is written relatively to the two identification techniques followed herein (i.e., Finite Element Updating Method (FEMU) and Integrated Digital Image Correlation (I-DIC) [3]). The first one consists of computing the set of constitutive parameters that minimize the displacement error  $\chi_u^2$  [2] such that for each iteration  $i$ , the parameter increment  $\{\delta^u \mathbf{p}\}^{(i)}$  reads

$$\{\delta^u \mathbf{p}\}^{(i)} = \left( ([\mathbf{S}_u]^{(i-1)})^t [\mathbf{M}] [\mathbf{S}_u]^{(i-1)} \right)^{-1} [\mathbf{S}_u]^{(i-1)} [\mathbf{M}] \{\delta \mathbf{u}\}^{(i-1)} \quad (1)$$

where  $[\mathbf{S}_u]$  is the matrix gathering the kinematic sensitivities,  $[\mathbf{M}]$  is the DIC matrix and  $\delta \mathbf{u}$  the displacement uncertainty. The covariance matrix of the identified parameters based on displacement uncertainties becomes [2]

$$[\mathbf{C}_p^u] = 2\gamma_f^2 \left( [\mathbf{S}_u]^t [\mathbf{M}] [\mathbf{S}_u] \right)^{-1} = 2\gamma_f^2 [\mathbf{H}^u]^{-1} \quad (2)$$

where  $[\mathbf{H}^u]$  is the Hessian matrix at convergence of the identification procedure and  $\gamma_f$  the standard deviation of gray levels. It is worth noting that if the solution at convergence is identical for weighted FEMU and I-DIC procedures, the same Hessian is obtained [2]. Thus, the optimization consists of maximizing the Hessian with respect to a chosen parameter. In the present case, it is a geometrical parameter, the fillet radius of the cruciform sample. A so-called triangle loading path is chosen. The procedure has been applied to an experiment with a precipitate hardened stainless steel (17-7 PH grade). The experimental setup with the tested sample is shown in **Figure 1(a)**. Integrated Digital Images Correlation is used to perform the identification of an elastoplastic plastic law with exponential kinematic hardening. The identification is based on the whole experiment including 360 steps (loads and images). The identification process leads to obtain I-DIC and load residuals. The correlation residual is the natural tool to evaluate the registration quality of DIC approaches [2] where

$$\eta^2 = \frac{1}{2\gamma_f^2 |\Omega| \Delta t} \sum_t \int_{\Omega} ((g(\mathbf{x} + \mathbf{u}(\mathbf{x}, t), t) - f(\mathbf{x}))^2 d\mathbf{x} \quad (3)$$

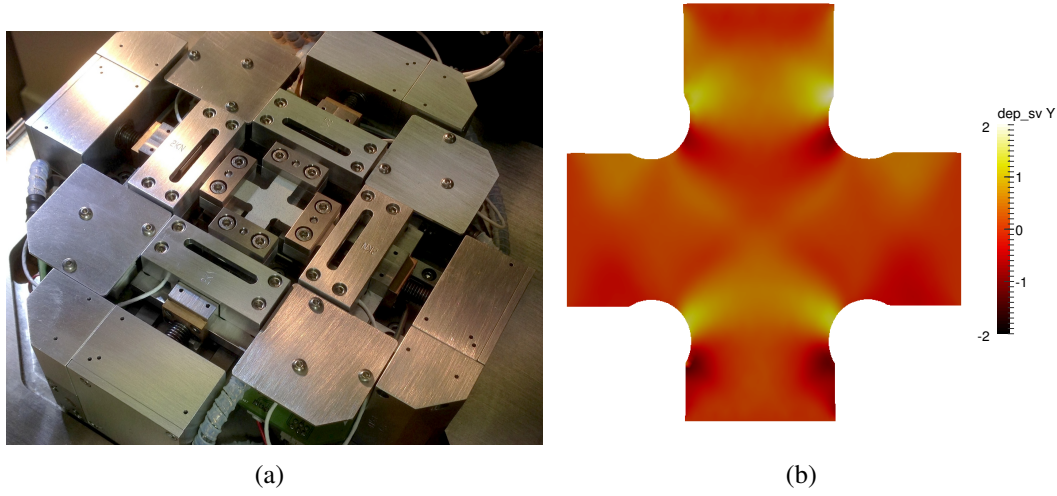


Figure 1: (a) Biaxial testing machine with the chosen sample geometry (b) Displacement difference expressed in pixel between raw DIC and I-DIC.

with  $\gamma_l = 233$  gray levels. An 16-bit camera is used (PCO edge 2.0) with telecentric lens ( $\times 0.5$ ). The correlation residual is equal to  $\eta = 2.36$  pixel for the 360 images whereas the I-DIC residual is greater and equal to  $\eta_{IDIC} = 25.6$ . **Figure 1(b)** shows the displacement difference expressed in pixel (1 pixel =  $13.5 \mu\text{m}$ ) between the result from DIC and I-DIC at convergence for the time step  $t = 240$ . This residual is an indication for assessing of model error. The normalize load residual is the standard deviation of the load residual divided by  $\gamma_l = 1$  Newton, the standard resolution of the load measurement,  $\eta_F = 42.0$ . The global residual is equal to  $\eta_g = 33.8$ .

## References

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