The VRP with time windows, synchronization and precedence constraints: Application in home health care sector

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Abstract : *This article addresses a new routing problem encountered in the field of Home Health Care. Researches in this emerging area share the goal of establishing fine coordination and optimized planning of human and material resources to provide support and quality monitoring while controlling costs. The problem introduced in this study is a particular vehicle routing problem with time windows and side timing constraints. Two mixed integer programming models are proposed and numerical results are shown on a new benchmark derived from the literature.*

KEYWORDS: *Vehicle Routing, Mixed Integer Linear Programming, Home Health Care, Synchronization Constraints, Precedence Constraints.*

1 INTRODUCTION

Due to high complexity and size, the problem of home health care covers a wide variety of decision-making challenges in the field of Operational Research and Decision Support. Many improvements are possible in terms of the goal efficiency (trends in quality of care or ensuring the mission at least cost). Research on this emerging problem has the intent to establish a fine coordination of human and material resources to provide an optimized planning that maximize the quality of home health care while controlling costs.

Informally, we consider a fleet of vehicles available to serve a set of geographically dispersed customers. The aim of solving such problems is to determine for each vehicle, the customers that it must visit, with respect to the availability of clients and/or vehicles, and so that the activity is planned in the most effective way. Therefore, it corresponds to a vehicle routing problem with time windows where the customers are the patients, vehicles in this case will be associated to caregivers providing specific cares. This type of service allows keeping people who are not entirely dependent at home, or to facilitate the return to normal life for people who have suffered serious illnesses (like cancer) by permitting them to leave traditional health institutions.

Often, the Home Health Care service must be performed at specific times and require the intervention of several caregivers, sometimes linked by precedence constraints. In our study, they are mostly timing constraints that impose the coordination of several caregivers. Generally, around 10 % of the customers that use the Home Health Care Structure ask of this kind of services. This problem is modeled as a variant of the VRPTW "*Vehicle Routing Problem With Time Windows*" where some patients require more than one visit simultaneously or in a given priority order.

2 LITERATURE REVIEW

The vehicle routing problem (VRP) is one among the most studied combinatorial optimization problems in Operational Research. In the field of Home Care, several variants are studied and used to model applications such as the transportation of elderly or disabled persons, delivery of drugs or medical devices to patients' homes. Many researches on problems related to the VRP in home care context have demonstrated their importance. We classify these problems into two broad categories:

Planning And Scheduling Problems: this family addresses the problem of assigning tasks to personal care and planning the schedule of visits (often daily). We can identify the works from (Bertels and al., 2006) on the allocation of nurses, (Eveborn and al., 2006), (Bredström and al., 2007, 2008), (Braysay and al., 2009) and (Rasmussen and al., 2010) on the daily planning of care services where each personal care is characterized by a set of skills (linguistic, medical certificate, sex ...) and specific work hours (part-time, full-time). All personal cares begin and end their days in a database (depot) where the visit reports are delivered. Each visit has a time window and a duration time and requires special skills. Generally, a patient is a person who requires special assistance such as cleaning, washing, cooking or medical attention. The patients may need also special requirements such as two assistants at a time (due to heavy loads during the bath for example) or in a given order (when a customer needs to undergo treatment before or after meals).

Pick-up And Delivery Problems: this family addresses the issues that consider a service of picking up some products or patients and then delivering them at another place. Many researches exists in this subject, we can cite for example the work of (Liu and al., 2013) or (Ceselli and al., 2013) which were primarily interested in such problems with the aim of providing medicines or medical devices to patients' homes and the collection of biological samples, drugs or medical devices not used by patients.

Other authors are also interested in combining these two cases such as (Rousseau and al., 2003) on "*Dial A Ride*" DAR problems where some patients require specific services before being transported. (Kergosien and al., 2013) have also introduced a two levels problem applied to the complex hospital located at Tours (in France). The first level concerns the routing problem for a fleet of vehicles serving several hospital units that delivers medicines, clean linen, meals, various supplies, patient files and picks up waste and dirty linen. The second level concerns the problem of routing employees between buildings within a large hospital unit.

3 PROBLEM DESCRIPTION AND MATHEMATICAL MODEL FOR THE VRPTW-PS

This work studies a version of vehicle routing problems with time window and specific synchronization constraints applied in Home Health Care Systems. More particularly, it focuses on cases for which some customers may require a service accomplished by more than one caregiver who have not the same skills. We call this variant of VRP as "*Vehicle Routing Problem With Time Windows-Precedence and Synchronization constraints*" VRPTW-PS.

Formally, the studied problem can be defined on an undirected graph $G = (V, E)$ with a node-set $V = N \cup D$ where $D = \{d, f\}$ are special nodes called initial and final depots, while $N = \{1, .., n\}$ corresponds to customers, and E is an edge-set. Each edge $e = [i, j]$ is associated with a travel cost C_e and a traversal duration T_e. Each customer is characterized by a time duration D_{is} and must be visited within an available time window $[a_i, b_i]$, where a_i and b_i are respectively the earliest start time of service and the latest start time of service at the customer i. The initial depot d offers a set of possible services $S = \{1, ..., s\}$ to customers who can demand a subset of services provided by a set $S_i = \{s \in S : m_{is} = 1\}$ where m_{is} equals to 1 if customer i demands service $s \in S$, 0 otherwise. The duration D_{is} of each service $s \in S_i$ required by customer i is assumed to be known in advance. A limited number of vehicles are available at the initial depot. Let K be this set of vehicles and each $k \in K$ provides a specific service s. In the benchmark, we assumed that o_{ks} equals to 1 if vehicle k offers the service s, 0 otherwise. Each vehicle k is qualified to perform a unique service. This means that $\forall k \in K,\ \sum_{s\in S} o_{ks} = 1$. $[\alpha_k, \beta_k]$ is the available time for vehicle k. It means that the vehicle k can only leave the initial depot d at α_k and must return to the final depot f before β_k. Each vehicle k is associated to a real number $Pref_{ik}$ related to non preference of customer i to the vehicle (caregiver) k which is endowed to perform the requested service.

The problem consists in building a set of routes starting at the initial depot d and ending at the final depot f, such that each vehicle route does not exceed the maximal imposed duration, and each customer receives the required services either simultaneously or in the imposed order as stated in the data. We define gap_{isr} as the maximum time between the starting times of services s and r required by customer i when service s is requested before service r. Each kind of service must be accomplished by the corresponding qualified vehicle. This study considers the minimization of the total traveling time and the sum of non preferences. For simplicity, we denote $k \in K_s$ if $o_{ks} = 1$ and $s \in S_i$ if $m_{is} = 1$. (Labadie and al., 2014) have studied a relatively similar problem, however, they only consider the case of simultaneous synchronization. Furthermore, only the travel costs are minimized in their objective function. This works extends their model by considering two types of synchronization and a more complex optimization criteria.

The problem under consideration is NP-Hard because it includes the VRPTW who is known to be NP-Hard. The problem studied here can be modeled as a mixed integer linear program (MILP) using the following decisions variables: binary variable x_{ijk} equal to 1 if and only if the vehicle $k \in K$ goes from i to j, 0 otherwise. Real variables: $tdeb_{ik}$ that indicate the starting time of service at customer i if this latter is visited by vehicle k.

The problem can be modeled by the following MILP:

$$\min \sum_{i\in V\setminus\{f\}} \sum_{j\in V\setminus\{d\}} \sum_{k\in K} C_{ij}.x_{ijk} + \sum_{i\in N} \sum_{j\in V\setminus\{d\}} \sum_{k\in K} Pref_{ik}.x_{ijk} \qquad (1)$$

Subject to:

$$\forall k \in K, \sum_{j\in N} x_{djk} = 1 \qquad (2)$$

$$\forall k \in K, \sum_{i\in N} x_{ifk} = 1 \qquad (3)$$

$$\forall h \in N, \forall k \in K, \quad \sum_{i\in V\setminus\{f\}} x_{ihk} = \sum_{j\in V\setminus\{d\}} x_{hjk} \qquad (4)$$

$$\forall i \in N, \forall s \in S, \quad \sum_{j\in V\setminus\{d\}} \sum_{k\in K_s} x_{ijk} = m_{is} \qquad (5)$$

$$\forall i, j \in V, \forall s \in S : s \in S_i \ \&\ s \in S_j, \forall k \in K_s, \quad tdeb_{ik} + (T_{ij} + D_{is}).x_{ijk} \leq tdeb_{jk} + b_i.(1 - x_{ijk}) \qquad (6)$$

$$\forall i \in N, \forall s \in S_i, \forall k \in K_s, \quad a_i. \sum_{j \in N} x_{ijk} \leq tdeb_{ik} \leq b_i. \sum_{j \in N} x_{ijk} \quad (7)$$

$$\forall k \in K, \quad \alpha_k \leq tdeb_{dk} \leq \beta_k \quad (8)$$

$$\forall k \in K, \quad \alpha_k \leq tdeb_{fk} \leq \beta_k \quad (9)$$

$$\forall i \in N, \forall s \in S_i, \forall r \in S_i : r \neq s, \quad \sum_{k \in K_s} tdeb_{ik} - \sum_{k \in K_r} tdeb_{ik} \leq gap_{isr} \quad (10)$$

$$\forall i, j \in V, \forall k \in K, \quad x_{ijk} \in \{0, 1\} \quad (11)$$

$$\forall i \in V, \forall k \in K, \quad tdeb_{ik} \geq 0 \quad (12)$$

The objective (1) consists in minimizing the sum of non preferences and the total travelling time. Constraints (2) (resp. (3)) require each vehicle to leave, (resp. return) at the depot. Constraints (4) ensure the continuity of the routes and constraints (5) ensure that all customer demands are satisfied. The constraints (6) are the scheduling constraints allowing the coherence between the periods of visits. Constraints (7)-(9) ensure that all time windows are respected. Constraints (10) are the synchronization constraints between starting time of services at customers asking for more than one service. The remaining constraints of the model fix the nature of the decision variables.

In this formulation, waiting times are allowed before servicing a customer and at the depot. In a second model proposed below, the aim is also to minimizing the waiting times. These latter can have different causes, that could be the arrival of vehicle k at customer i before its earliest start time of service a_i, or the most common case when one customer requires several services to start simultaneously, but even when it is necessary to respect a certain time lag before starting the next service.

This model is then extended to treat further satisfaction of customer wishes. This new formulation of the problem is presented and named, NoWait-VRPTW-PS.

4 MATHEMATICAL MODEL FOR THE NoWait-VRPTW-PS

This model also involves two types of variables: binary routing variables noted x_{ijk}, y_{ik} and scheduling variable noted $tdeb_{ik}$, $tarriv_{ik}$ and $twait_{ik}$. where $tarriv_{ik}$ resp. $twait_{ik}$ indicate the arrival time of vehicle k at customer i and the waiting time at customer i if these latter are visited by vehicle k. This new problem can be modeled by the following MILP:

$$\min \sum_{i \in V \setminus \{f\}} \sum_{j \in V \setminus \{d\}} \sum_{k \in K} C_{ij}.x_{ijk} + \sum_{i \in N} \sum_{j \in V \setminus \{d\}} \sum_{k \in K} Pref_{ik}.x_{ijk} + \sum_{i \in N} \sum_{k \in K} twait_{ik} \quad (13)$$

Subject to:

$$\forall k \in K, \sum_{j \in N} x_{djk} = 1 \quad (14)$$

$$\forall k \in K, \sum_{i \in N} x_{ifk} = 1 \quad (15)$$

$$\forall h \in N, \forall k \in K, \quad \sum_{i \in V \setminus \{f\}} x_{ihk} = \sum_{j \in V \setminus \{d\}} x_{hjk} \quad (16)$$

$$\forall i \in N, \forall s \in S, \quad \sum_{j \in V \setminus \{d\}} \sum_{k \in K_s} x_{ijk} = m_{is} \quad (17)$$

$$\forall i \in N, \forall s \in S_i, \forall k \in K_s, \quad tdeb_{ik} \geq tarriv_{ik} - b_i.(1 - \sum_{j \in V \setminus \{d\}} x_{ijk}) \tag{18}$$

$$\forall i, j \in V, \forall s \in S : s \in S_i \cup S_j, \forall k \in K_s, \quad tarriv_{jk} \geq tdeb_{ik} + (T_{ij} + D_{is}).x_{ijk} - b_i.(1 - x_{ijk}) \tag{19}$$

$$\forall i, j \in V, \forall s \in S : s \in S_i \cup S_j, \forall k \in K_s, \quad tarriv_{jk} \leq tdeb_{ik} + (T_{ij} + D_{is}).x_{ijk} + \beta_k.(1 - x_{ijk}) \tag{20}$$

$$\forall i \in N, \forall s \in S_i, \forall k \in K_s, \quad twait_{ik} = tdeb_{ik} - tarriv_{ik} \tag{21}$$

$$\forall i \in N, \forall s \in S_i, \forall k \in K_s, \quad a_i.\sum_{j \in N} x_{ijk} \leq tdeb_{ik} \leq b_i.\sum_{j \in N} x_{ijk} \tag{22}$$

$$\forall k \in K, \quad \alpha_k \leq tdeb_{dk} \leq \beta_k \tag{23}$$

$$\forall k \in K, \quad \alpha_k \leq tdeb_{fk} \leq \beta_k \tag{24}$$

$$\forall i \in N, \forall s \in S_i, \forall r \in S_i : s \neq r, \quad \sum_{k \in K_s} tdeb_{ik} - \sum_{k \in K_r} tdeb_{ik} \leq gap_{isr} \tag{25}$$

$$\forall i, j \in V, \forall k \in K, \quad x_{ijk} \in \{0, 1\} \tag{26}$$

$$\forall i \in V, \forall k \in K, \quad tarriv_{ik} \geq 0 \tag{27}$$

$$\forall i \in V, \forall k \in K, \quad tdeb_{ik} \geq 0 \tag{28}$$

$$\forall i \in N, \forall k \in K, \quad twait_{ik} \geq 0 \tag{29}$$

Under this formulation, the objective function (13) includes the costs previously mentioned in the first model and the waiting times. Changed constraints are those called scheduling (18)-(21) compared to the previous one, which allow the coherence between the instants of visits, while taking into account the waiting time.

Model I		
Instance	Obj	CPU(s)
18 _4 _s	57.65	0.83
18 _4 _m	55.55	1.74
18 _4 _l	43.12	2.59
45 _10 _s	-16.38	37.61
45 _10 _m	-48.63	50.63
45 _10 _l	-109.16	67.34

Table 1: Computational results when the entire function is minimized - Model I

Model II		
Instance	Obj	CPU(s)
18 _4 _s	97.53	1.95
18 _4 _m	63.78	2.96
18 _4 _l	43.12	11.67
45 _10 _s	-2.67	224.11
45 _10 _m	-47.62	669.01
45 _10 _l	-109.16	33.4

Table 2: Computational results when the entire function is minimized - Model II

5 NUMERICAL RESULTS

To validate the developed models, tests were conducted using OPL Studio, Version 12.5 on a 3.20 GHz Intel(R) Core(TM) i5-3470 CPU computer with 8GB of RAM. A set of instances proposed by (Bredström and al., 2008) are specifically extended. The benchmark used is grouped into two different categories according to the number of customers and vehicles, which are equal to 18 clients with using 4 vehicles in the first group and 45 clients with using 10 vehicles in the second. In each group, the number of synchronization ranges from two to four, and the time windows can be small, medium or large. In these new instances, the customer demands (in terms of services needed) are randomly generated but all the rest of information remain the same as in (Bredström and al., 2008). Without loss of generality, we only treat the case when at most two services are requested. The results are shown in Table 1-6, where columns 1 describe the used instances. The first value represent the number of clients while the second represents the number of vehicles and the indexes s, m or l refer to the width of the customers time windows (small, medium or large). For example, instance 18_4_s expresses an instance with 18 clients, 4 vehicles and where the width of the time windows is small. Columns 2 represent the objective value for the optimal solution or the best found solution within the time limit of 3600 seconds and columns 3 illustrate the time (reported in seconds) used by solver Cplex. Tables (1) and (2) give the results corresponding to the entire objective function with model 1 and 2 respectively. These results show that only small instances can be solved to optimality within few seconds. On larger instances, the computational time growth considerably even when at most four synchronizations are considered. We also deduce that the computing time increases considerably depending on customers time windows width. Although, small time windows strongly constrain the problem and make it more difficult to solve. In (Bredström and al., 2008), the authors also attempt to optimize each criteria separately. Thus, Tables (3) resp. (4) show the results found by running the first model when minimizing only preferences resp. traveling times and finally Tables (5) resp. (6) give the results found by running the second model when minimizing both preferences and waiting times resp. traveling and waiting times. These results show that when the objective contains the minimization of the traveling time, it is much more harder to solve the problem to optimality.

Model I		
Instance	Pref	CPU(s)
18 _4 _s	-64.18	0.50
18 _4 _m	-66.52	0.55
18 _4 _l	-67.68	0.69
45 _10 _s	-295.63	11.75
45 _10 _m	-313.79	18.91
45 _10 _l	-317.96	163.40

Table 3: Computational results when preferences are minimized - Model I

Model I		
Instance	Trav	CPU(s)
18 _4 _s	98.1	0.72
18 _4 _m	91.8	1.50
18 _4 _l	86.7	2.68
45 _10 _s	204..3	44.55
45 _10 _m	190.5	3600
45 _10 _l	147.6	1290.35

Table 4: Computational results when traveling time is minimized - Model I

Model II		
Instance	Pref-Wait	CPU(s)
18 _4 _s	-40.31	2.76
18 _4 _m	-64.18	3.05
18 _4 _l	-67.68	3.40
45 _10 _s	-290.62	121
45 _10 _m	-309.91	182.02
45 _10 _l	-317.96	229.47

Table 5: Computational results when preferences and waiting time are minimized - Model II

6 CONCLUSION AND FUTURE WORKS

The work presented in this paper deal with a new vehicle routing problem taking into account important constraints highly needed in home care process, namely synchronization between multiple caregivers to care the same patient.

In this paper, a quick overview of routing problem with time-windows and synchroniza-

Model II		
Instance	Trav-Wait	CPU(s)
18 _4 _s	145.3	1.97
18 _4 _s	99.9	3.60
18 _4 _l	86.7	4.10
45 _10 _s	209.4	3600
45 _10 _m	193.8	3600
45 _10 _l	151.2	1395

Table 6: Computational results when traveling times and waiting times are minimized - Model II

tion constraints is given. Two mixed integer programming models are proposed for the case in which two types of synchronizations are considered. The objective function in the first one allows to minimize only the sum of the caregivers' traveling costs and the non preferences of customers, while the second one, take also into account the waiting times.

Computational experiments are carried out on 6 instances which are extended from the benchmark initially proposed by (Bredström and al., 2008). The computational time grows with the size of instances since the problem is NP-Hard. In fact, according to the results mathematical model can solve only small instances. However, structures generally handle more than 45 patients (such as around 100) the interest of this method becomes minor. For that, as future works, it would be necessary to develop exact, approximate and hybrid methods with acceptable running times.

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