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TEMPO CURVING AS A FRAMEWORK FOR INTERACTIVE COMPUTER-AIDED COMPOSITION

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ABSTRACT

We present computer-aided composition experiments related to the notions of polyrhythmic structures and variable tempo curves. We propose a formal context and some tools that enable the generation of complex polyrhythms with continuously varying tempos integrated in compositional processes and performance, implemented as algorithms and user interfaces.

1. INTRODUCTION

Tempo variations in musical performances significantly influence musical and rhythmic perception. Expressive musical timing and tempo curves (or *time maps*) are the object of previous studies in the field of computer music [1,2]. In general the timing of beats and musical events is computed by the integration of tempo curves [3], and the compositional challenges are concentrated on the joint specification of these curves (or other expressive timing controls) and of a certain level of synchrony between simultaneous voices.

As a slightly different approach, we concentrate here on the notion of rhythmic equivalence in the context of time-varying tempos. Rhythms are prescriptive structures denoted by sequences of durations (also called *temporal patterns* [4]) when associated with a given (and possibly varying) tempo. Intuitively, it is possible to imagine that two different rhythms produce an equivalent temporal pattern if played following adequate tempo curves. Considering a rhythm as the convolution of another rhythm and a tempo curve, or as a superimposition of other rhythms and tempo curves, can be attractive musically as different representations of the same musical material can be suggestive of different interpretations in performance. In this paper, we explore and actualize this idea through computer-aided composition tools and techniques.

2. PRELIMINARY DEFINITIONS

In this section we introduce simple conventions that will be used throughout this paper. Our intention is not to discuss or overlap with the rich literature on rhythm theory and formalisation, but to provide keys for the reading and understanding of the subsequent parts.

2.1 Rhythmic Figures / Durations / Tempo

It is important to first distinguish the notated/compositional form of a rhythmic structure (e.g. $\downarrow \downarrow \downarrow$) from the corresponding “phenomenological” rhythm, that is, the resulting sequence of durations or temporal pattern (e.g. 2s, 1.5s, 0.5s). These two forms are related functionally by a tempo value (in the previous examples, $\downarrow = 60$, i.e. a quarter note corresponds to 1s).

We will use the upper-case character R to identify notated rhythms and lower-case r for the rendered temporal patterns. We note $\otimes$ the rendering operation associating a tempo τ to a rhythm R , yielding to $r : R \otimes \tau = r$.

2.2 Equivalence

We call *equivalent* two rhythms yielding equal temporal patterns and note this equivalence $R_1 \equiv R_2$. In other words:

$$\forall \tau, R_1 \equiv R_2 \Leftrightarrow R_1 \otimes \tau = R_2 \otimes \tau. \quad (1)$$

Recent works have delved into formalisms that allow one to search for equivalent notations for a given rhythm R , that is, $R_i \neq R$ such that $R_i \equiv R$ [5]. Most of the time, the tempo τ is used to convert rhythmic figures into actual durations, or conversely, to guide the search for the rhythmic notation that will best match a given temporal pattern (rhythmic quantification [6]). In both cases, it is the same on the two sides of the equality (as in Eq. 1).

In order to integrate the tempo as a variable parameter, we will group the rhythms and tempos in pairs (R, τ) and now call *equivalent* two pairs (R_i, τ_i) and (R_j, τ_j) which verify $R_i \otimes \tau_i = R_j \otimes \tau_j$. We also note this equivalence $(R_i, \tau_i) \equiv (R_j, \tau_j)$. Given a pair (R_i, τ_i) , a limited number of rhythms $R_j \neq R_i$ will verify $(R_i, \tau_i) \equiv (R_j, \tau_j)$ if $\tau_j \neq \tau_i$.¹

2.3 Polyphony

In order to work with polyphonic rhythmic structures, we also introduce the operator $\oplus$ which perceptually merges several rhythms or temporal patterns into a single one. We will use it for instance to compose complex rhythmic lines from simpler ones, or conversely to find sets of rhythms $\{R_1 \dots R_n\}$ which verify: $\oplus_{i=1}^n R_i \equiv R_T$ (where R_T is called a “target” rhythm).²

¹ Rhythms verifying this property are equivalent rhythms ($R_j \equiv R_i$) modulo a “speed factor” τ_i/τ_j (e.g. $\downarrow \downarrow \downarrow$ and $\downarrow \downarrow \downarrow$).

² We simplify the notation here using R_i for expressing rhythms in general, that is either (R_i, τ_i) pairs or τ_i .

Note that the $\oplus$ operator is hard to define and implement in the notation domain (see Figure 1a), but it is trivial in the “real” time domain, where there is a direct mapping between the clock time and the notes’ onsets and durations (see Figure 1b).

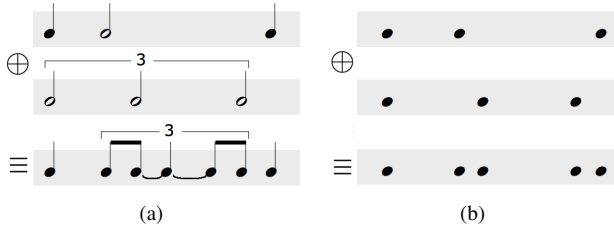


Figure 1: Merging rhythms a) in the notation domain and b) in the time/durations domain.

2.4 Varying Tempo

We now note $\tau_i(t)$ the function giving the value of the tempo τ at time t .

Considering the tempo as a variable function of time, we can assume that for any pair (R, τ) there exist an infinity of $(R_i, \tau_i(t))$ which verify $(R_i, \tau_i(t)) \equiv (R, \tau)$, and as a corollary, that for any given rhythms R_1 and R_2 and for any tempo τ_1 there exist a tempo function $\tau_2(t)$ such that $(R_1, \tau_1) \equiv (R_2, \tau_2(t))$. Conversely, given a *target* rhythm $r_T = (R_T, \tau_T)$ and a tempo curve $\tau(t)$ there must exist a rhythm R which verifies $(R, \tau(t)) \equiv (R_T, \tau_T)$.

Finding $\tau(t)$ or R here, or a combination of rhythms and tempos (R_i, τ_i) such that $\oplus_{i=1}^n (R_i, \tau_i) \equiv (R, \tau)$, is an appealing challenge from a musical point of view: it will allow us to render predetermined target rhythms (R_T, τ_T) using poly-temporal structures computed from time-varying tempo curves (see next section).

This problem is hard to solve with purely formal or algorithmic methods, and the search gets even more complex when combinations of rhythms are involved. As we will see below, supervised/interactive tools and heuristics provide interesting opportunities for compositional exploration.

3. COMPOSITIONAL CONTEXT

Musical polytemporality has been explored by many composers throughout the 20th and 21st centuries, however, the challenges involved in the composition and representation of polytemporal music have prevented many from progressing beyond experimentation to the development of a praxis. Gérard Grisey (*Tempus ex machina*), Iannis Xenakis (*Persephassa*), György Ligeti (*Kammerkonzert, 3rd mvmt.*), and Conlon Nancarrow (*Studies for Player Piano*) all produced works that explored the textures that become available to a composer when the clock that unifies performers is removed. However, the limited number of polytemporal works produced by these composers is representative of the challenges of constructing compositional systems and intuition in this domain. Dobrian [7] recently published a survey of compositional and technical issues related to

polytemporal composition. In this section, we present recent works by John MacCallum that serve as a case study highlighting the need for a set of compositional tools that facilitate experimentation, exploration, and situated action.

3.1 Motivation

The conceptual motivation behind the works described below is predicated on the idea that synchrony between multiple musicians is a fictional construct of music notation. The concept of a musical “now” is not an infinitesimal, but rather, a small window, the width of which varies continuously between the imperceptibly small and the unacceptably large. As performers use the visual and auditory cues around them to negotiate and approximate a common tempo, they construct a system that is not *synchronous*, but rather, *plesiochronous* in nature, i.e., *nearly* together, or close enough for the intended purpose. One’s attention is rarely drawn to this fundamental feature of performance, except in the most extreme moments when the system begins to diverge from plesiochrony and approach true synchrony or diverge off to asynchrony. The works below foreground the human aspect of performance, albeit in a representative way, and push Platonic ideals inherent in music into the background.

3.2 Virtual and Emergent Tempos

In MacCallum’s recent works, performers listen to a click-tracks that vary smoothly in tempo over time, independent of one another. The compositional challenge in these works is to construct musical material that unifies the different parts that are no longer bound by a common clock. *aberration* for percussion trio,³ is an investigation into the use of composite rhythm and the emergence of a “virtual tempo” as a means of producing coherent ensemble material. In this work, tempo curves $\tau_i(t)$ were chosen using a random process and, in many sections of the piece, the rhythms R_i are chosen using a simulated annealing algorithm with the goal of producing $\oplus_{i=1}^3 (R_i, \tau_i(t)) \equiv (R_T, \tau_T(t))$ where R_T represents a sequence of $1/16^{th}$ notes in a tempo $\tau_T(t)$ that can be imagined as the ideal tempo continuously “running in the background” that the musicians are trying to approximate.

The form of *aberration* was constructed largely independently of $\tau_i(t)$, and the material itself was algorithmically derived and then altered to draw the listener’s attention to certain features of the relationship between the tempos at a given moment. The result is a complex rhythmic texture with a number of emergent properties unforeseen by the composer at the time of its creation. It is largely these underdeveloped textures that became the focus of a more intuitive and less systematic/process-oriented exploration in *Delicate Texture of Time* for eight players.

3.3 Methods

The composition of *aberration* relied heavily on a carefully constructed plan that was designed to project a small number of textures of interest to the composer who had, at

³ <http://john-maccallum.com/index.php?page=/compositions>

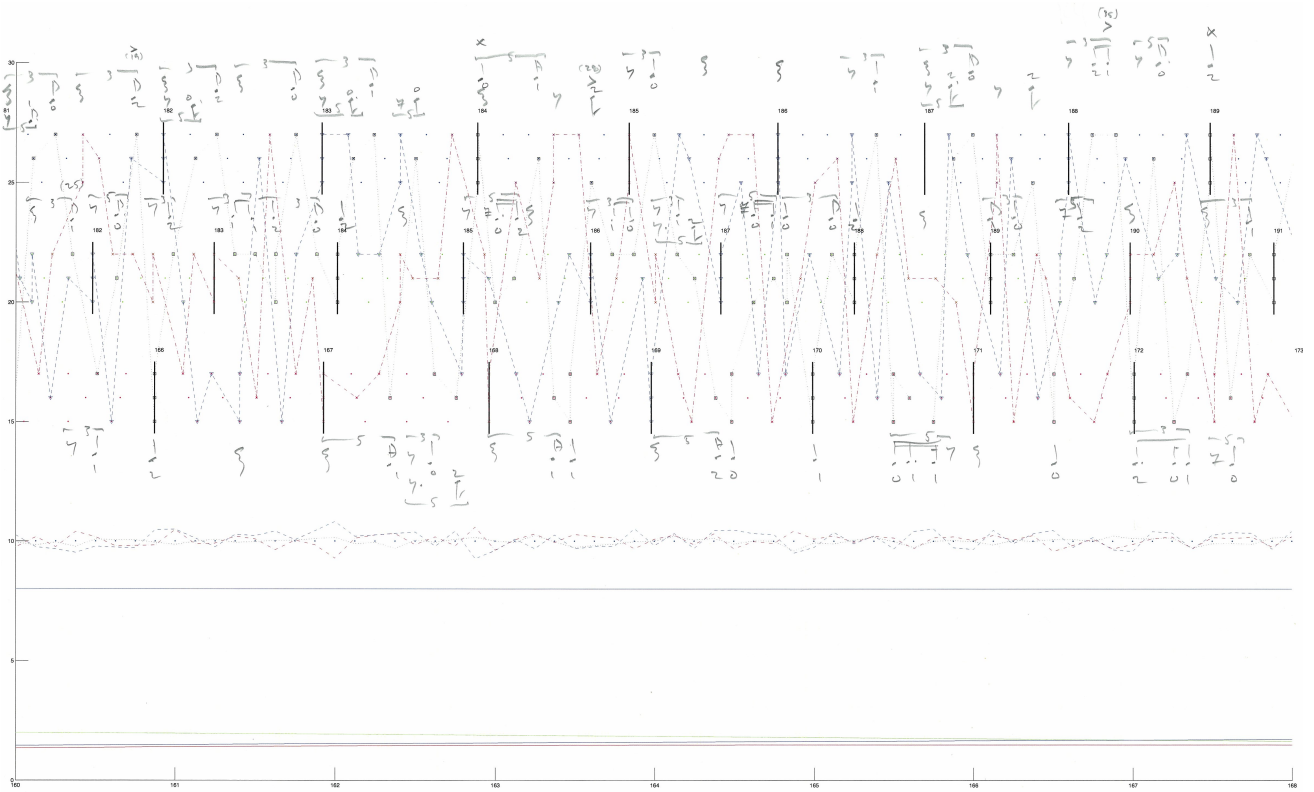


Figure 2: Compositional sketch of MacCallum's *aberration*.

that time, no intuitive sense of how they would be enacted in performance. The piece was constructed as follows:

1. $\tau_i(t)$ were created using a random process designed to produce curves that generally oscillate between a maximum and minimum tempo and change direction with some average frequency.
2. The time of every beat and subdivision (triplets, sixteenth notes, and quintuplets in this case) was computed for every voice from $\tau_i(t)$ and written to a file.
3. A simulated annealing algorithm was run to find R_i such that $\oplus_{i=1}^3 (R_i, \tau_i(t)) \approx (R_T, \tau_T(t))$.
4. Matlab was used to create a template showing the position of every bar, beat, and subdivision for all voices, along with lines overlayed to show different outputs of step 3.
5. Steps 2–4 were repeated until the simulated annealing algorithm produced output with a high degree of voice exchange without too many instances of polyrhythms containing more than two subdivisions in half a beat.
6. Simple compositional sketches would be made to investigate the features of $(R_i, \tau_i(t))$.
7. Steps 1–6 were repeated until a suitably interesting output was produced.
8. The composition was then done directly on top of the template in pencil (Figure 2), and the results transcribed using Sibelius for the parts and OmniGraffle for the score (Figure 3 – see also Section 4.4).

There are a number of difficulties inherent in the steps listed above:

- The distance of the approximation $\oplus_{i=1}^3 (R_i, \tau_i(t)) \approx (R_T, \tau_T(t))$ is directly dependent on $\tau_i(t)$ which were chosen *a priori* using a random process rather than being treated as free variables. This is not necessarily a problem, indeed, in the case of *aberration*, this was a feature of the work and a point of compositional investigation.
- Steps 1–7 offer little in the way of compositional intervention. When the simulated annealing algorithm produced output that was deemed unusable for one reason or another, it was difficult to apply constraints to have it avoid similar conditions during future execution.
- Step 8 is extremely cumbersome, error-prone, and forces the composer to commit to a given output once the composition of material has begun. If changes to any of the $\tau_i(t)$ need to be made at a later time, any material created must be discarded.
- Step 8 must be completed with little or no audition of material during the compositional process, preventing the composer from experimenting with material.

Delicate Texture of Time was produced in a manner similar to the method listed above, with the exception that the tools had become easier to use and more robust, and Adobe Illustrator was used in place of OmniGraffle, however the problems listed above remained present in the process.



Figure 3: Score of MacCallum's *aberration*.

4. COMPUTER-AIDED COMPOSITIONAL PROCESS

In this section we present an implementation of the procedure described previously aided by the *timewarp~* external for Max/MSP [8] and computer-aided composition processes implemented in the OpenMusic environment [9].

The compositional objective driving our discussion is the determination of n rhythmic lines (corresponding to n instrumental performers), each following a given tempo $\tau_i(t)$ ($i = 1 \dots n$) and merging to produce a target rhythm (R_T, τ_T) .⁴ The target rhythm can come from previous compositional processes or material, or it can be arbitrarily decided and specified by the composer. It can be expressed with rhythmic notation (R_T, τ_T) or (equivalently) directly as a sequence of durations (r_T) .

Our problem is therefore to find R_i ($i \in \{1 \dots n\}$) given r_T and $\tau_i(t)$, such that:

$$\oplus_{i=1}^n (R_i, \tau_i(t)) \equiv r_T$$

The combination of n lines exponentially increases the search space of this problem, which makes it slightly more complex than the equivalence issues mentioned in Section 2.4. As a first step, we will consider that $n = 1$ (and eventually drop the subscripts i to simplify the notation). As we will see, the presented methods easily scale to greater numbers of rhythmic lines.

4.1 Resolution with One Voice ($n = 1$)

As in Step 2 (Section 3.3), a first simplification we make to the problem is to preliminarily choose a number of possible

⁴ We suppose — especially in the case of complex tempo variations — that each performer will be assisted, typically by a click-track.

pulse subdivisions for each voice R_i . Each subdivision (S) yields a regular rhythmic pattern (notated R_i^S) which will be used to compose a “quantification grid”. Given these patterns R_i^S and the tempo curve $\tau(t)$ a sequence of duration $r^{S_j} = R^{S_j} \otimes \tau(t)$ can be computed for each subdivision (see Figure 4). Currently this part of the process is performed in Max/MSP using the *timewarp~* external as described in [8]. The results (r^{S_j}) are communicated to OpenMusic through a simple file export/import protocol.

Note that at this point, if the subdivision is known and the tempo curve $\tau_i(t)$ does not change, the conversion of r^S back into R is relatively straightforward.

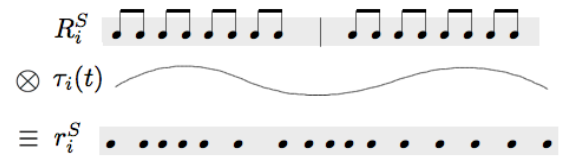


Figure 4: Generating a sequence of durations r_i^S starting from a beat subdivision S and a tempo curve $\tau_i(t)$ ($S = 2$).

The same procedure is applied for different values of S ($S_1, S_2, \dots$) yielding a superimposition of lines (r^{S_j}) following the same tempo curve $\tau(t)$ (Figure 5).

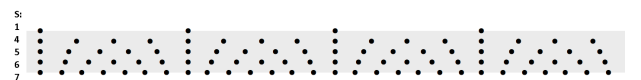


Figure 5: Superimposition of r^{S_j} ($S_j = \{1, 4, 5, 6, 7\}$).

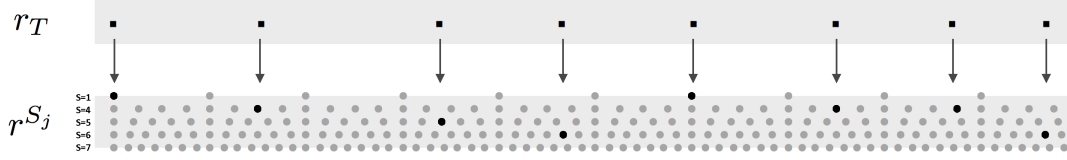


Figure 6: Finding elements of r_T in r^{S_j} .

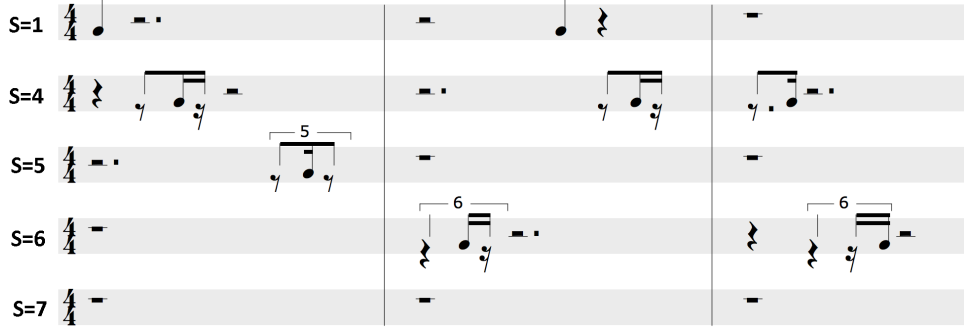


Figure 7: Reconstitution of R^{S_j} for each subdivision S according to the selection in Figure 6. (Note that $R^{S_7} = \emptyset$.)

The search procedure then consists in finding and marking an element in one of the different r^{S_j} which best matches each of the elements in the target r_T (see Figure 6). Constraints that govern acceptable combinations of subdivisions can also be applied to this search procedure to offer a degree of control over the general polyrhythmic complexity of the output.

From these marks it is easy to reconstitute a simple rhythm R^{S_j} for each value of S , containing one single rhythmic figure or subdivision (S_j) and considering every marked element in r^{S_j} as a played note, and every non-marked element as a silence (see Figure 7). An “abstract” rhythm R is then created regardless of the tempo curve $\tau(t)$, which will be equivalent to r if played back following $\tau(t)$.

A delicate part in the process is the merging of the different lines R^{S_j} back into a single voice R . As the tempo curve has been abstracted (it is the same for every R^{S_j}), some OpenMusic tools such as the *merger* function can be used for this operation [10]. Another solution is to perform a “local” quantification of the sequence r obtained from the combination of $r^{S_j} = R^{S_j} \otimes \tau_\alpha$, where τ_α is an arbitrary value of the tempo [6]. This quantification process is generally straightforward and reliable using existing tools (e.g. *omquantify*⁵), given the known tempo τ_α and the limited set of allowed subdivisions corresponding to the different S_j . Figure 8 shows the rhythm $R_0 = R^{S_1} \oplus R^{S_4} \oplus R^{S_5} \oplus R^{S_6} \oplus R^{S_7}$ merging the lines R^{S_j} from Figure 7. This rhythm corresponds to the target sequence r_T from Figure 6, if played following the initial tempo curve $\tau(t)$: $R_0 \otimes \tau(t) \equiv r_T$.

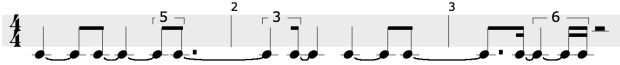


Figure 8: Rhythm merging the R^{S_j} from Figure 7.

4.2 Resolution with n Voices

The previous procedure is easily adapted to more than one voice. Considering our initial problem of finding R_i ($i \in \{1 \dots n\}$) such that $\oplus_{i=1}^n (R_i, \tau_i(t)) \equiv r_T$, we just need to reproduce n times the process of generating the lines $r_i^{S_j}$ as in Figure 5.

The search is then extended so as to look up in the n different voices for an element of $r_i^{S_j}$ matching each element of r_T . According to the selection, separate sets of rhythms $R_i^{S_j}$ will be generated for each voice, merged into R_i and gathered in a polyphony as a result of the overall process.

Here as well, the graphical representation and alignment of polyphonic rhythmic structures with different, time-varying tempos is a tricky aspect of the polyphonic extension, but stands out of the scope of our present discussion. The OpenMusic *poly* editor allows for the assignment of tempo changes approximating $\tau_i(t)$ at every pulse of the n different voices, and to visualize/play these voices as a single score (see Figure 9).

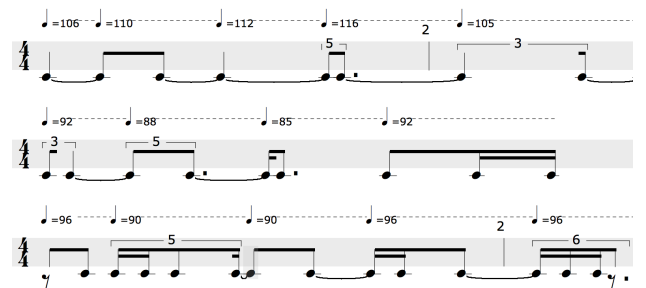


Figure 9: Aligned representation of 3 voices (R_1, R_2, R_3) in the OpenMusic *poly* editor, with tempo changes approximating $\tau_1(t)$, $\tau_2(t)$ and $\tau_3(t)$.

⁵ <http://support.ircam.fr/docs/om/om6-manual/co/Quantification.html>

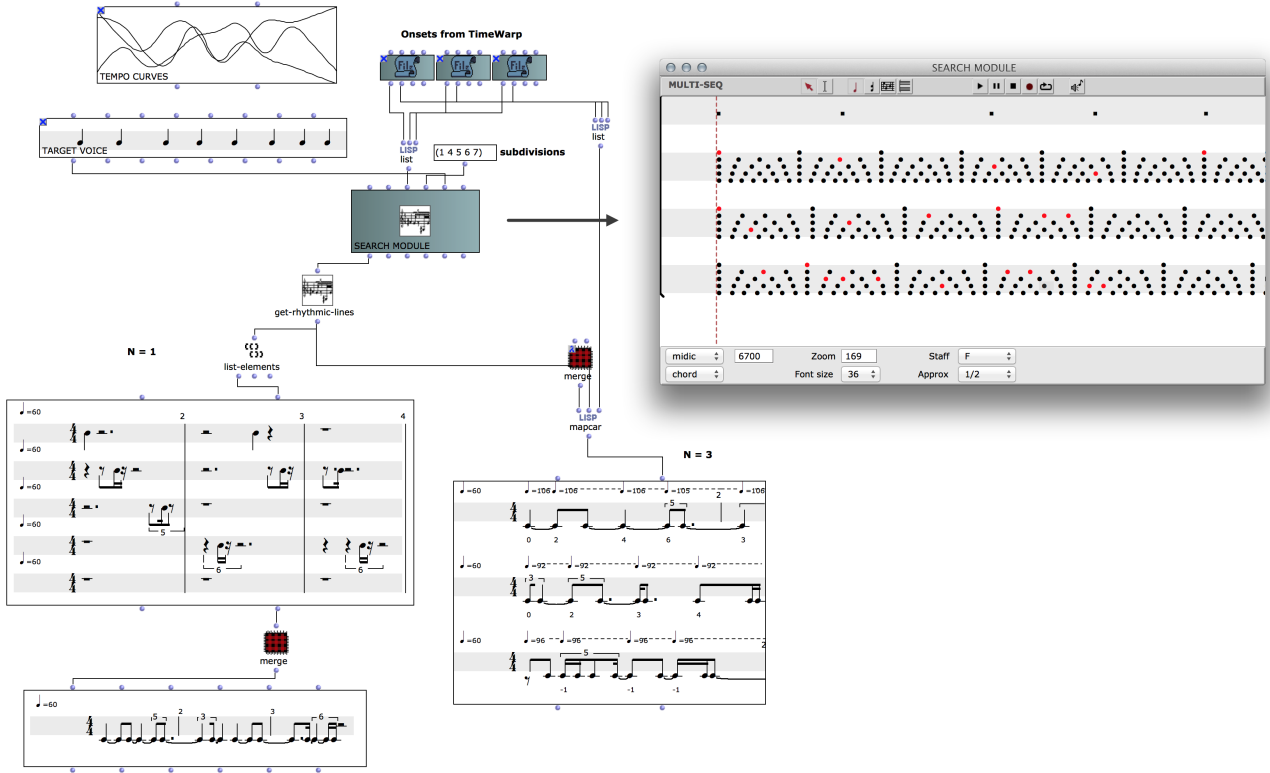


Figure 10: OpenMusic visual program implementing the rhythm matching process. The rhythm matching interface at the right allows to visualize/edit the matching between the target rhythm r_T and the various lines' tempo-varying grids $r_i^{S_j}$.

4.3 User Interface and Supervision of the Process

The process presented in the previous sections in principle could be completely automated and packaged in a black box. The main choices (inputs) for the user are the tempo curves $\tau_i(t)$, and the allowed rhythmic subdivisions S_j in the different voices. Still, visualizing the different steps is crucial to understand the process and eventually tweak these parameters in order to obtain relevant results, hence the choice and importance of a visual programming environment like OpenMusic where each of these steps is materialized by a module, and where all intermediate results can be inspected and edited (see Figure 10).

More importantly, composers' choices can be taken into account in the search part of the process where elements of the different $r_i^{S_j}$ are selected to compose the rhythms R_i . The algorithm may make unfortunate choices in the case of equivalent matches, and sometimes the “best” choice in terms of distance may not be the best in terms of readability, playability of the result, or because of any other compositional reason (e.g. controlling voice exchange or density, see for instance Step 5 in Section 3.3).

The main module of the visual program in Figure 10 therefore comes with a specific user interface (visible at the right on the figure) which extends the traditional *multi-seq* object of OpenMusic and allows the composer to visualize and make the choices of the elements in $r_i^{S_j}$ according to visual judgements or other arbitrary motivations. Computation can therefore temporarily stop here to leave space for manual edition and operations, before proceeding to downstream parts of the data processing.

4.4 A Note on Score Notation

In the works presented above, the parts that the musicians read from are typeset according to standard notational conventions in which spacing between notes is set in order to minimize page turns without sacrificing readability. The score, however, is prepared in such a way that the horizontal distance on the page between two notes N_i and N_j is proportional to the duration of N_i (see for instance on Figure 3). This redundant representation of time (rhythmic and proportional) allows one to see clearly the intended temporal relationships between the individual parts and to easily correlate moments in the score with the notation as seen by the performers.

Proportional notation that allows for complete and accurate control over the space between notes is impossible in most environments necessitating the use of graphic design software such as OmniGraffle for *aberration* or Adobe Illustrator for MacCallum's more recent works. The use of two separate environments for the score and parts can lead to differences between the two causing confusion in rehearsal.

Currently, OpenMusic scores represent time proportionally (*chord-seq*) or notationally (*voice*), but not both simultaneously. Recent work has been done to extend the notation objects and provide a hybrid (redundant) representation of time.

5. FROM COMPOSITION TO PERFORMANCE

One of the goals of the compositions described in Section 3 is the representation of the inherently plesiochronous nature of human musical performance. It is the musical material itself, however, that carries this representation; those pieces do nothing to elicit a performance that would foreground this feature the way, for example, the distribution of the musicians across a large distance in space would. We present in this section two recent projects designed with this performative aspect in mind, and which also problematize the relationship between music notation and its realization.

5.1 Windows of Musical Time

If the “musical now” is a small window of continuously varying width, what would we find if we could construct a performance context that would increase the scale of that window to the point that its contents become musical material and even form? MacCallum’s recent work *Hyphos* for alto flute, bass clarinet, viola, and electronics is a compositional study meant to explore this idea. As in *aberration* and *Delicate Texture of Time*, the performers listen to click-tracks to aid them as they perform long nonlinear *accelerandi* and *decelerandi*, however, in *Hyphos*, they are meant to only use the click-tracks in rehearsal and dispense with them in performance. The use of different slow, gradual, continuous changes in tempo for the three performers is designed to defamiliarize the performance context by removing the fictional shared tempo. As the performers attempt to follow their individual temporal trajectories, their vertical relationships vary over time with respect to those prescribed by the score, and the “window of the now”, bounded by the two musicians leading and trailing the others, is brought to the foreground.

The compositional challenge here is to construct material that satisfies musical and aesthetic goals despite potentially extreme variation in performance. To this end, a set of tools that reconfigure the score to represent the temporal relationships of the individual parts during a given performance is essential for composers looking to gain a deeper understanding of the nature of the performative variability and develop compositional strategies and intuition that rely on it.

This work highlights the latent dualistic role of the score as providing a set of prescriptive instructions for performance, as well as being a representation of that which was performed. In the case of a score for two musicians, one may be able to follow the prescribed score and mentally reconcile the visual and auditory representations of the composition, however, as the number of parts increases, the complexity becomes unmanageable and the score, as a notational representation of the performance, is no longer of any value. Without a visual aid describing what actually happened, constructive communication with and between performers is hindered.

5.2 External Sources of Temporal Control

Hyphos, described in Section 5.1, was a study in preparation for a collaborative project between MacCallum and choreographer Teoma Naccarato that remains ongoing at the time of this writing. In *Choreography and Composition of Internal Time*,⁶ pulses extracted from wireless electrocardiogram (ECG) units worn by dancers serve as click-tracks for musicians in real-time. The musicians render a score, but as in *Hyphos*, the temporal relationships between the different parts is in constant flux as the dancers perform choreography designed to affect change in their cardiac function that approximates the general contour of the precomposed $\tau_i(t)$. The use of biosensors here is intended to foreground the limits and variation of human bodies in performance, as well as to intervene in the compositional and choreographic processes.

6. CONCLUSION

We presented formal and compositional approaches for dealing with poly-temporal rhythmic structures in computer-aided composition. These formalisms and general workflow emphasize both computational and interactive considerations at manipulating musical time and rhythmic notations. Computer-aided composition provides interactive musical representations at the different steps of the process and allows for the combination of systematic/automated procedures with compositional interventions.

The presented framework is suitably general to be used for the generation and manipulation of rhythmic structures. It can, for example, be seen as a supervised rhythmic quantification tool, enabling the production of notated rhythmic approximations of a given sequence of linear onsets, using variable tempo tracks and/or polyrhythmic scores. We have also emphasized, in the discussion of recent compositional projects, how it is likely to be used in more interactive situations such as when the tempo information, for example, becomes a reactive input causing the different steps and views of the corresponding musical representations to update on the fly.

Acknowledgments

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⁶ <http://ccinternaltime.wordpress.com>

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