Numerical end experimental investigation of hygro mechanical states of glass fiber reinforced polyester composites experienced by FBG sensors

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This study is focused on the finite element analysis of the hygro-elastic stresses experienced by unidirectional composite structures instrumented by Fiber Bragg Grating (FBG) sensors. One of the significant advantages of this method is that by taking into account the complex geometry of elaborated specimen and their different constituents with heterogeneous hygroscopic and mechanical properties. In the first time, the obtained results were compared with the results obtained owing to an analytical solution deduced from the simplified concentric cylinders approach. This comparison allows us to evaluate the reliability of the concentric cylinders approach which cannot account for every aspects of the actual geometry of the studied samples. Besides the numerical results were compared to the experimental characterization of the mechanical states deduced from the Fiber Bragg Grating measurements.

Keywords:
Finite element method
Hygroscopic aging
Fiber Bragg Grating sensors
Organic matrix composite
1. Introduction

Organic matrix composites are used in several engineering applications such as aerospace, marine, automobile, civil engineering and so on. Their application is in growth due to very good ratio of stiffness to weight. Composite structures are often submitted to humid environments during their life time [1,2]. It has been reported that organic matrices absorb significant amounts of moisture when exposed to humid environment [3–6]. Absorbed water may induce harmful effects on organic matrix composites [7–11]. In fact, the constituents of composites structures exhibit heterogeneous coefficients of moisture expansion and maximum moisture absorption capacity. As a consequence, multi-scale in-depth mechanical states profiles rise during the hygroscopic loading of organic matrix composites. According to the literature, the resulting mechanical states can eventually induce damage [12–14]. Thus, the durability of polymer composites may be impaired by hygroscopic loads. Monitoring the internal mechanical states experienced by reinforced polymer during the transient part of the diffusion process would actually be relevant in the aim of understanding and eventually being able to predict the durability of structural parts.

Recently, Fiber Bragg Grating sensors have been used to achieve the measurement of strain, temperature and other quantities of engineering interest [15–20]. FBG sensors have several advantages over conventional electrical and piezoelectric sensors, due to their capacity to investigate the internal mechanical states in the depth of composite specimen [21–23]. However, the presence of FBG sensors in the depth of a composite structure induces a stress field perturbation. Considerable efforts have been made by researchers in order to develop analytical models enabling to predict the mechanical states occurring during both the transient stage and the permanent regime of the moisture diffusion process of organic matrix composites submitted to hygro-mechanical loads [24–27]. In previous studies, the hygroscopic stress and strain experienced by unidirectional fiber-reinforced composites have been analyzed by considering the concept of concentric cylinders [28]. The authors have developed this approach to predict the transient hygroscopic stresses in composite structure. However the real geometry of complex specimen cannot be considered in the context of this simplified approach. Besides, reference [28] assumes that the moisture diffusion process obeys a one-dimensional Fick’s law, when a 3D kinetics would better represent the physics.

In this study, a finite element approach has been proposed in order to simulate the hygro-elastic stresses experienced, during an aging test, by composite specimens instrumented by FBG sensors. This method allows to properly account for the actual geometry of the elaborated specimen as well as their different constituents. The main objective of the present work consists in demonstrating the reliability of the analytical model based on the concentric cylinders approximation by comparing with the finite element method. The results of these two numerical methods were compared to experimental results also. This specific point was achieved by an experimental investigation of both the time-dependent moisture uptake and the strain field in composite samples owing to FBG sensors. In the previous work [28] the authors used an analytical solution for hygro-elastic stresses in multi-layered cylinders, In the present work the authors used the finite element method to analyze a hygro-elastic stresses where precise sample size was used. It should be noted that also in the previous work a one-dimensional Fick’s law (1D) was used to simulate the kinetics of moisture absorption, but in the recent study a three-dimensional Fick’s law (3D) was applied in order to moisture diffusion and hygroscopic stress modeling. Finally in the recent study, the FEM numerical results compared and consequently validate with experimental results.

2. Concentric cylinders approach

In order to simulate the presence of an optical fiber inserted in an unidirectional composite (Fig. 1) and estimate the mechanical stress field experienced by the structure during an hygro-elastic load, a concentric cylinders model has been proposed in [28]. In this model, the hygro-elastic behavior of the central and the external cylinders were considered as isotropic and isotropic transverse, respectively. The optical fiber is surrounded by a protective coating considered as a third cylinder. This protective coating will be represented by an intermediate cylinder (Fig. 2) having isotropic properties. The radius of the optical fiber is a, whereas the outer radii of the protective coating and unidirectional composite are \( r_0 \) and \( b \), respectively.

A list of symbols (Table 5) has been provided due to the large number of different symbols used in this paper.

2.1. Hygroscopic problem

The three cylinders are supposed initially dry. The moisture movement is supposed to be the solution of the following system of equations given by the transient diffusion problem and secondly by the hygro-elastic stress field. This system is given by the following set of equations:

\[
\frac{\partial C}{\partial t} = \nabla \cdot (D \nabla C) + \dot{M}
\]

where \( C \) is the moisture content, \( D \) is the moisture diffusion coefficient, and \( \dot{M} \) is the rate of moisture absorption.

The moisture content is related to the moisture strain by the following equation:

\[
\varepsilon = E \left( \frac{C}{C_0} - 1 \right)
\]

where \( E \) is the moisture modulus and \( C_0 \) is the moisture saturation.

Fig. 1. Optical fibers inserted parallel to the reinforcing fibers (white dashed line circles illustrate, respectively the optical fiber, protective coating and unidirectional composite).

Fig. 2. Schema of the concentric cylinders model.
between the cylinder 1 and cylinder 2. This moisture content jump \( \chi \) is considered between cylinders 2 and 3 since both their densities and maximum moisture absorption capacities are heterogeneous [29]. This quantity \( \chi \) corresponds to the ratio of the maximum moisture absorption capacity of the second cylinder by that of the third cylinder (see system 2 below).

It is also assumed that moisture flow is zero at the interface between the cylinder 1 and cylinder 2. This hypothesis is justified by the hydrophobic property of optical fiber represented by cylinder 1.

\[
\frac{\partial m_i}{\partial t} = D_i \left[ \frac{\partial^2 m_i}{\partial x^2} + \frac{1}{r} \frac{\partial m_i}{\partial r} \right] \quad a < r < b, \quad t > 0, \quad i = 2 \text{ to } 3
\]

(1)

\[
\begin{align*}
&\frac{\partial m_2 }{\partial t} = 0 \\
&m_2 (r, t) = m_3 (r, t) \\
& D_2 \frac{\partial m_2}{\partial r} (r, t) = D_3 \frac{\partial m_3}{\partial r} (r, t) \\
&m(b, t) = m_0 \\
&m(r, 0) = 0
\end{align*}
\]

(2)

According to [30], the time and space dependant moisture content, \( m_2 (r, t) \), solution of the problem (1) and (2) is:

\[
m(r, t) = m_0 \left[ 1 + \pi \sum_{m=1}^{\infty} \frac{\exp(-D_0 \chi r)}{m_0 f_{m}(bo_m) - f_{1}(bo_m)} Y_1 (bo_m) J_0 (ro_m) \right] \]

(3)

In order to solve the hygro-mechanical problem, the strain–displacement relations (5), as well as the compatibility and equilibrium Eq. (6) were considered [32]. By replacing the displacement components \( u, v, w \) in the cylindrical coordinate system \( (x, \theta, r) \), and employing the equilibrium equation we find the following relations

\[
\begin{align*}
&\varepsilon_{33} = \frac{\partial w}{\partial r}, \quad \varepsilon_{22} = \frac{w}{r}, \quad \varepsilon_{11} = \frac{\partial u}{\partial r} \\
&\frac{\partial \sigma_{33}}{\partial r} + \sigma_{33} - \frac{\sigma_{22}}{r} = 0
\end{align*}
\]

(5)

(6)

where \( \varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \sigma_{11}, \sigma_{22}, \sigma_{33} \) are respectively the axial, hoop and radial strain and stress components.

The displacement \( u(x) \) does not depend on the moisture content field. Additionally the radial component of the displacement \( w(x) \) could be obtained by applying the solution of the analytical transient moisture content. (Eq. (3)). The radial component of the displacement field in the material 2, denoted \( w^{23} \), satisfies the following equation:

\[
\begin{align*}
& \left( r^2 \frac{\partial^2 w^{23}}{\partial x^2} + r \frac{\partial w^{23}}{\partial r} - w^{23} \right) = \frac{K_1}{r} \left( \chi \right) \left( \frac{\partial^2 w^{23}}{\partial r^2} \right) \\
& \text{with } K_1 = L_{12} \beta_{11} + L_{23} \beta_{22} + L_{33} \beta_{33}
\end{align*}
\]

(7)

Since the optical fiber is hydrophobic, the following simplified purely elastic relations (8) are considered for the displacement field which takes place within the cylinder 1 [33,34].

\[
\begin{align*}
&u^{2}(x) = A^{2} x \\
&w^{2}(r) = B^{2} r
\end{align*}
\]

(8)

(9)

With \( \alpha_m \) the mth positive root of \( J_0 (bo_m) Y_1 (bo_m) - J_1 (bo_m) Y_0 (bo_m) = 0 \), where \( J_0, J_1, Y_0, \) and \( Y_1 \) are Bessel’s functions of first or second kind.

Transient moisture concentration was determined also via the Finite difference method with an explicit formalism [31].

2.2. Mechanical problem

Initially, the cylinders are assumed to be stress free, so the hygro-elastic behavior (4) is written as follows:

\[
\begin{align*}
\sigma_{11} &= \left[ L_{11} L_{12} \right] \left[ \varepsilon_{11} - \beta_1 \Delta m \right] \\
\sigma_{22} &= \left[ L_{12} L_{22} L_{23} \right] \left[ \varepsilon_{22} - \beta_2 \Delta m \right] \\
\sigma_{33} &= \left[ L_{12} L_{23} L_{33} \right] \left[ \varepsilon_{33} - \beta_3 \Delta m \right]
\end{align*}
\]

(4)

where \( L_1 \) and \( \beta_1 \) respectively stand for the elastic stiffness tensor and the coefficients of hygroscopic expansion.

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Transmit moisture concentration was determined also via the Finite difference method with an explicit formalism [31].

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\end{align*}
\]

(4)

where \( L_1 \) and \( \beta_1 \) respectively stand for the elastic stiffness tensor and the coefficients of hygroscopic expansion.
Fig. 3. Schematic view of the internal section of instrumented specimen.

Fig. 4. Positioning of the FBG sensor in the elaborated sample with associated mesh refinement in the case when the FBG sensor positioned in the direction perpendicular to the reinforcement (considering with symmetry plane condition).
10 mm, we can define two regions in the middle of the instrumented specimen. The first zone is constituted by the fiber embedded by the polyester resin and the composite. The length of this zone is 10 mm. In the second zone the optical fiber is embedded by the acrylate coating instead of polyester resin. The length of the second zone is 80 mm. In order to represent more realistically the instrumented samples, we used the finite element method to simulate the stress and strain evolution in the composite and the optical fiber. The results of these simulations will enable, to check the reliability of the approximations considered in the concentric cylinders approach.

3.1. Size and shape of the specimen

The tested unidirectional composite samples were made of E-glass fibers embedded in a polyester resin. The FBGs employed in this study were inscribed in standard optical fiber (SMF-28 e) of the diameter of which is 250 µm. The optical fiber containing Bragg gratings was inserted between two composite plates bonded with polyester resin. The FBG were either aligned with the reinforcing fibers or set perpendicularly to them. These types of specimen architectures enabled us to obtain instrumented samples, the final size of which is 90 × 90 × 3 mm³. The optical fiber is surrounded by the composite plates considered as homogeneous equivalent materials. The main objective of this work consists in determining the moisture content evolution through the composite plate. Consecutively the strain and stress in the composite and the optical fiber induced by absorbed moisture will be predicted. For the purpose of the numerical simulations, in order to reduce the calculation time, three symmetry planes following the principal axis (1; 2; 3) have been considered. This allows us to simplify the model by studying only one-eighth of the instrumented sample (Fig. 4). The sample size is reduced down to 45 × 45 × 1.5 mm³ by applying this simplification.

All FE simulations have been performed using the COMSOL Multiphysics® code. The mesh is constructed with tetrahedron elements and is refined at the fiber core near the Bragg grating, as well as at the fiber–matrix interface where the calculated fields may experience strong variations (Fig. 4).

Precisely our finite element model contains 126255 tetrahedron elements of degree 2 (9 nodes per element with quadratic shape function). A refined mesh was chosen in the specific zone where FBG sensor is inscribed. Additionally this area is particularly interesting for comparison with the experimental results obtained by optical Bragg grating sensor positioned in this point. In this zone the number of elements for the fiber (silica), acrylate coating, and composite plate are, respectively 18,108, 48,905, and 10,307. Besides, the moisture and mechanical fields in the rest of the specimens have been simulated with a less refined mesh in order to reduce the calculation time.

The following figure (Fig. 4) represent the positioning of the FBG sensor in the elaborated sample with associated mesh refinement in the case when the FBG sensor is positioned in the direction perpendicular to the reinforcement.

3.2. Moisture diffusion and hygroscopic stress modeling

The glass fiber reinforced polyester composite and the neat polyester resin samples exhibit a Fickian diffusive behavior as described in [35]. Accordingly, a three-dimensional Fick’s law (10) was used to simulate the kinetics of water absorption:

\[ \frac{\partial c}{\partial t} = D_1 \frac{\partial^2 c}{\partial x^2} + D_2 \frac{\partial^2 c}{\partial y^2} + D_3 \frac{\partial^2 c}{\partial z^2} \]

Subscripts i and s denote the moisture concentration values at the initial and permanent stages of the diffusion process, respectively. L, I, and e are, respectively the length, the width and the thickness of the considered sample.

In order to simulate the hygroscopic stress induced by moisture diffusion, the hygro-elastic Hooke’s law (12) has been used:

\[ \sigma = \lambda : (\varepsilon - \beta \Delta c) \]

The numerical model requires knowing the mechanical and hygroscopic properties of the various sample constituents, namely: the composite plate, the polyester resin layer, the acrylate protective coating and the fiber core (silica).

The hygroscopic properties of the unidirectional composite plate and the polyester resin layer, i.e. the maximum moisture content and the moisture diffusion coefficient values were, respectively measured and determined by using the gravimetric tests and identification methods described in previous study [36,37]. Besides, the values given in the literature [38,39] were used for the acrylate protective coating. The mechanical properties provided by the supplier were used for the fiber core. Finally, some physical properties of the composite plate such as the moisture expansion coefficient have been determined experimentally via optical sensor, whereas its density was calculated from the rule of mixtures.

The mechanical and hygroscopic properties of the sample constituents are given in Tables 1 and 2.

4. Results and discussion

4.1. Transient moisture diffusion simulation

The FBG sensors were positioned parallel and perpendicular to the reinforcing fibers in the elaborated samples. Thus, we simulate

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Hygroscopic properties of the sample constituents, properties.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D11 (mm²/s)</td>
</tr>
<tr>
<td>Composite(17%)</td>
<td>2.31 × 10⁻⁷</td>
</tr>
<tr>
<td>Composite(21%)</td>
<td>2.31 × 10⁻⁷</td>
</tr>
<tr>
<td>Composite(22%)</td>
<td>2.31 × 10⁻⁷</td>
</tr>
<tr>
<td>Resin</td>
<td>2.31 × 10⁻⁷</td>
</tr>
<tr>
<td>Acrylate coating</td>
<td>5.64 × 10⁻⁷</td>
</tr>
<tr>
<td>Fiber core</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Mechanical and physical properties of the sample constituents.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E1 (GPa)</td>
</tr>
<tr>
<td>Composite(17%)</td>
<td>13.18</td>
</tr>
<tr>
<td>Composite(21%)</td>
<td>15.16</td>
</tr>
<tr>
<td>Composite(22%)</td>
<td>15.66</td>
</tr>
<tr>
<td>Resin</td>
<td>3.00</td>
</tr>
<tr>
<td>Acrylate coating</td>
<td>3.20</td>
</tr>
<tr>
<td>Fiber core</td>
<td>73.10</td>
</tr>
</tbody>
</table>
hygro mechanical behavior of these samples in two principal directions. Fig. 5 depicts the transient moisture content of the specimen containing 22% glass fiber. The moisture content variation through the sample thickness, was plotted from point \((44.96, 44.985, 0)\) mm to point \((44.96, 44.985, 1.5)\) mm. Three constituents with different moisture diffusion properties yield the existence of three zones with specific moisture uptakes in this direction (axis3).

It is found that the strongest gradient of moisture content occurs nearly to the sample edge. Then, the moisture diffuses through the whole composite cylinder, reducing the moisture content gradients (until the equilibrium state is finally attained). We can also observe a moisture uptake jump at polyester resin – composite interface due to their heterogeneous hygroscopic properties. The ratio between the maximum moisture content of the resin and composite plate allows us to determine the precise value of the moisture uptake jump named \(\zeta\). As an example \(\zeta\) is equal to 1.92 for the elaborated sample with 22% \(\nu_f\). The saturation of this specimen is attained within less than one year (300 days).

4.2. Hygroscopic strain simulation

Figs. 6(a)–(c) illustrate the radial, hoop and axial strains in the optical fiber, polyester resin layer and composite plate, calculated during the transient stage, in the case when FBG sensor is positioned parallel to the reinforcing fibers. On these figures, the strain/stress values are interpolated via the shape function regarding to the nodes. The radial strain is discontinuous at the interface between the composite plate and the polyester resin layer, where the moisture content jump occurs. The radial strain reaches \(3500 \times 10^{-6}\), \(11,000 \times 10^{-6}\), and \(400 \times 10^{-6}\) in the composite plate, polyester resin layer and optical fiber, respectively at equilibrium state. The radial strains predicted for any of the three constituents, according to this method are reasonable, regarding to their moisture uptake.

Contrary to the case of the radial strain, a fairly uniform hoop strain is experienced by the composite plate during the moisture diffusion process. The hoop strain is more important in the composite than in the optical fiber.
The axial strain represents the hygroscopic deformation along the reinforcing fibers. In fact in this direction the hygroscopic strain is small, $(70 \times 10^{-6})$ due to the hydrophobic properties of glass fibers, as well as their high stiffness in that specific direction. This effect has been illustrated by Fig. 6(c). Actually, the axial strain is nearly constant in the composite plate and the fiber core (silica). On the contrary a tiny fluctuation was observed for the axial strain experienced by the polyester resin layer because of its hydrophilic properties. Globally the axial strain at saturation state (after 300 days) is negligible in comparison with radial and hoop strain.

Fig. 7. Axial, hoop and radial stress in transient state obtained by finite element method (dots) and the analytical approach (lines), in the case when FBG sensor is positioned parallel to the reinforcing fibers.
4.3. Numerical comparison of hygroscopic stress with FE results and concentric cylinder results

The importance of the performed simulation is that the actual geometry of the instrumented sample and the constituents distribution has properly been taken into account during FE analysis, whereas the concentric cylinder method is based upon approximations. In order to evaluate the influence of this factor on the hygro-mechanical states, the radial, hoop and axial stresses predicted by finite element method and calculated by concentric cylinder approach, have been compared.

Fig. 7 represents the axial, hoop and radial stress in transient state obtained by both the theoretical approaches. This figure corresponds to the case of a sample with 22% of reinforcing fibers. In addition the FBG sensor was considered parallel to the reinforcing fibers.

The two methods are consistent on many aspects: as an example, compressive stress gradients for both hoop and axial stresses are predicted near to the external surface in the transient stage. Strong discontinuities of the hoop and axial stresses can be observed at the boundaries between the interphase material, the optical fiber and the composite.

The computations show that the maximum tensile hoop and axial stresses are reached during the diffusion process in the composite subdomains. High radial stress gradients which increase as a function of the time take place in the resin region (Fig. 7(c)) according to both the calculation methods.

Globally, the axial and hoop stress results obtained by both methods in the composite, resin and optical fiber subdomains are consistent. On the opposite, the radial stress obtained by the two methods present significant discrepancies, as high as 13 MPa, particularly in the optical fiber subdomains. This could be explained by the main assumptions involved in the concentric cylinder model.

In fact the FE method takes into account the actual structure of instrumented sample, precisely in the vicinity of the optical fiber where Bragg grating is inscribed. Notably at this measurement point, the acrylate protective coating was removed. This zone has been filled by the polyester resin during the manufacturing process. Knowing that the moisture diffusion, and consequently, the moisture uptake, are dissimilar for these two materials, can constitute an explanation for the discrepancy with the calculated hygroscopic strains.

Since the maximum moisture capacity of the acrylate coating is two times higher than that of the polyester resin (3.1% against 1.5% [39]), we could expect the hygroscopic strain experienced by the acrylate to be higher than that of the polyester. The acrylate coating is placed between the composite plate and optical fiber core. This induces a different strain profile through the thickness of the instrumented sample, where we observe a compression state for the polyester resin layer (Fig. 3).

This modification of the hygro-mechanical state in the core of sample in the vicinity of FBG sensor, due to the presence of an acrylate coating with different hygroscopic property, is not taken into account by the concentric cylinder approach.

4.4. Comparison with experimental results

4.4.1. Hygroscopic strain measurement by FBG sensors

Hygroscopic strains have been measured by FBG sensors inserted in composite plates in order to validate the obtained numerical results. The FBG sensors were positioned perpendicular and parallel to the reinforcing fibers in order to provide information in both the principal directions. The samples were intended to provide the time-dependent evolution of the internal strains states throughout the moisture diffusion process. They were eventually immersed in de-ionized water at the beginning of the aging tests.

Mechanical strains measurements through FBG have been presented in various papers [40]. A FBG consists in a series of grating planes formed along the fiber axis. If the Bragg condition is fulfilled, a light signal propagating into the device can interfere constructively to the waves reflected by each of the grating planes. As a result, a back reflected signal with a center wavelength commonly known as the Bragg wavelength \( \lambda_b \) is formed. In the case when the FBG is submitted to a homogeneous axial strain \( \varepsilon_z \) and uniform temperature change \( \Delta T \), the Bragg wavelength experiences a deviation \( \Delta \lambda_b \) from the reference value \( \lambda_{b0} \) corresponding to the unloaded state \( (\varepsilon_z = 0; \Delta T = 0) \):

\[
\frac{\Delta \lambda_b}{\lambda_{b0}} = \frac{\lambda_b - \lambda_{b0}}{\lambda_{b0}} = a \varepsilon_z + b \Delta T
\]  

(13)

Coefficients \( a \) and \( b \) depend on the nature of the optical fiber and the FBG inscription parameters. According to relation (13), at isothermal conditions, the Bragg wavelength shift \( \Delta \lambda_b \) is proportional to the axial strain experienced by the optical fiber.

The measured Bragg’s wavelength shift enables to determine the strain experienced in the center of the instrumented samples, from relation (13), provided that the thermal contribution, namely, the product \( b \Delta T \) is negligible by comparison to the quantity of interest: \( a \varepsilon_z \).

4.4.2. Experimental set up and data exploitation

A broadband optical source provided by pumping an erbium-doped fiber, has been used in this experimental set up. The source spectral width is about 30 nm centered at 1550 nm (1550 nm ± 30 nm). The FBGs were inscribed in the standard SMF-28e single mode optical fiber of 250 μm diameter via phase mask method. The Bragg grating length is 10 mm. The reflected signal is received back by a JDS Uniphase coupler (1 × 2). The reflected signal is processed by a spectrum analyzer (Anritsu MS9710B) with 70 pm graphic resolution.

The coupler actually guides the reflected signal to the Spectrum Analyzer. The spectral signature of the Bragg grating is numerically analyzed in order to determine the measured strain during the hygroscopic aging test imposed to the samples.

Besides, the hygroscopic properties of the composite and the pure polyester resin samples have been determined owing to the gravimetric test. In particular, the maximum water uptake, was deduced from those measurements. This makes it possible to plot the curve of the hygroscopic strain as a function of the water uptake. From this curve the coefficients of moisture expansion of the studied samples can be determined. The values of these coefficients are presented in Table 3.

According to the obtained results, the strain value and consequently the moisture expansion coefficient of specimens is higher in the direction perpendicular to the reinforcing fibers. In fact the moisture expansion coefficient in the direction parallel to the reinforcement fiber weakly depends to the fiber volume fraction (\( \rho_f \)). Knowing that the fiber volume fraction difference for these composites is about 1%, the low difference in beta 11 (\( \beta_{11} \)) is conceivable. Accordingly the measured value in that direction is of the same order of magnitude than the uncertainty.

### Table 3

<table>
<thead>
<tr>
<th>( \psi_i )(%)</th>
<th>( m_i )(%)</th>
<th>( \varepsilon_{b1} )(10^-6)</th>
<th>( \varepsilon_{b2} )(10^-6)</th>
<th>( \beta_{11} )</th>
<th>( \beta_{22} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>0.85</td>
<td>180</td>
<td>4000</td>
<td>0.022</td>
<td>0.472</td>
</tr>
<tr>
<td>21</td>
<td>0.79</td>
<td>330</td>
<td>3140</td>
<td>0.042</td>
<td>0.397</td>
</tr>
<tr>
<td>22</td>
<td>0.78</td>
<td>200</td>
<td>3100</td>
<td>0.026</td>
<td>0.397</td>
</tr>
</tbody>
</table>
This should obviously be attributed to the strongly anisotropic hygroscopic behavior of fiber-reinforced composites [41].

4.4.3. The experimental and numerical results confrontation

The data collected through the experimental tests; in particular, the measured hygroscopic strains, were confronted with the numerical results in order to validate the applied numerical approach. It should be remembered that the FBG sensors were initially positioned either parallel or perpendicular to the reinforcing fibers. As a result, the experimental investigation provides both the axial and circumferential strains in the transient state in the immediate vicinity of the sensors. However, the mechanical strain was not investigated in the direction normal to the fibers of the composite plies due to the very small sample thickness (less than 3 mm). Therefore, the confrontation of the numerical approaches with the experimental method is not possible in the direction normal to the fibers of the composite plies. This is due to the lack of experimental reference values in this direction. Actually, the two predictive approaches used in the present work predict significantly different levels of stress in the direction normal to the fibers, as evidenced by the analysis described in Section 4.3.

However, a very good agreement between the theoretical predictions and the measured profiles was eventually found in the others principal directions. Knowing that the FE method takes into account the actual geometry of the instrumented specimen and the spatial distribution of its constituent, particularly in the vicinity of the FBG sensor, we utilized the hygroscopic strains simulated by this method for this comparison. It should be noted that the measured circumferential strains were applied for this comparison and the FBG sensor was considered perpendicular to the reinforcing fibers.

At the saturation state, the FBG sensor measured the value of $3484 \times 10^{-6}$ for hoop strain which is very close to the value of $3600 \times 10^{-6}$ predicted by the FE simulations. The relative deviation in this case reduces to 3.3%. Several factors, particularly the uncertainty of the measurement could explain this small discrepancy. This comparison justifies also the reliability of the materials parameters employed in the constitutive hydro-elastic equation, for the needs of the FE simulation.

The results obtained by these simulations have been grouped in Table 4.

5. Conclusions

This study is devoted to the hydro-elastic behavior of unidirectional composite structures. The investigation was achieved through measurements and numerical computations. Internal strains states in the longitudinal and transverse directions to the reinforcing fibers were determined owing to Fiber Bragg Grating sensors, during the transient stage of aging tests (immersion in de-ionized water).

The moisture content distribution through the instrumented sample thickness was computed as a function of time owing to Finite Element Method. The heterogeneous diffusion coefficients and maximum moisture absorption capacities of the various constituents of the studied specimen were accounted for. The local strain and stress fields in the composite structure have been calculated by the same approach. The influence of an interphase, here the polyester resins subdomain between two unidirectional composite plates, and the existence of an acrylate protective coating between the fiber core (silica) and the composite plates have been commented. The results obtained by the Finite Element Model were compared to the values predicted by a simplified analytical solution established owing to the concentric cylinder method.

A very good agreement between the two computational approaches shows the reliability of the concentric cylinder method. Finally, the hydro-mechanical properties of elaborated sample have been measured via FBG sensors in order to validate the obtained numerical results. Good agreement of experimental results and numerical results represents the adjustment of FE method to predict the hygro-mechanical state of elaborated specimens. In addition, it attests the reliability of the parameters used in the constitutive hygro-elastic equation, during the FE simulation.

The FE method was used in order to simulate the hygro-elastic behavior of unidirectional composite structures only in the present

Table 4
Comparison between the results predicted by finite element simulation and the Fiber Bragg Grating sensor measurement for elaborated composite with 22% fiber volume fraction.

<table>
<thead>
<tr>
<th>FE Simulation prediction</th>
<th>FBG sensor measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial ($\varepsilon_{11}$)</td>
<td>$100 \times 10^{-6}$</td>
</tr>
<tr>
<td>Hoop ($\varepsilon_{22}$)</td>
<td>$3600 \times 10^{-6}$</td>
</tr>
<tr>
<td>Radial ($\varepsilon_{33}$)</td>
<td>$500 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 5
A list of symbols used in this paper.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{0(i)}$</td>
<td>The transverse diffusion coefficient in the $i$th cylinder</td>
<td>$c$</td>
<td>The Moisture concentration</td>
</tr>
<tr>
<td>$D_{1(2)}$</td>
<td>The transverse diffusion coefficient in the 2th cylinder</td>
<td>$D_{1}$</td>
<td>Longitudinal diffusion coefficient</td>
</tr>
<tr>
<td>$D_{1(3)}$</td>
<td>The transverse diffusion coefficient in the 3th cylinder</td>
<td>$D_{2}$</td>
<td>Transverse diffusion coefficient</td>
</tr>
<tr>
<td>$m_1$</td>
<td>The moisture content in the 1th cylinder</td>
<td>$c_i$</td>
<td>The initial moisture concentration.</td>
</tr>
<tr>
<td>$m_2$</td>
<td>The moisture content in the 2 cylinder</td>
<td>$c_e$</td>
<td>The maximum equilibrium Moisture concentration.</td>
</tr>
<tr>
<td>$m_3$</td>
<td>The moisture content in the 3 cylinder</td>
<td>$L, l, e$</td>
<td>The length, the width, and the thickness of the considered sample.</td>
</tr>
<tr>
<td>$x$</td>
<td>The moisture content jump between cylinder 2 and 3 (in concentric cylinder approach)</td>
<td>$E_1$</td>
<td>Longitudinal elastic modulus</td>
</tr>
<tr>
<td>$m(t, t)$</td>
<td>The time and space dependent moisture content</td>
<td>$E_2$</td>
<td>Transverse elastic modulus</td>
</tr>
</tbody>
</table>
| $\omega_m$ | The $\omega_{m}^{max}$ positive root of $J_1(b)\omega_m b
 -$ $J_1(\omega_m b) = 0$ | $\nu$ | Poisson’s ratio |
| $J_0, J_1, Y_0, Y_1$ | Bessel’s functions of first or second kind | $\rho$ | Density |
| $\sigma_{11}$ | Axial stress | $\eta$ | Fiber volume fraction |
| $\sigma_{22}$ | Hoop stress | $\varepsilon$ | Axial strain in cylindrical coordinate |
| $\sigma_{33}$ | Radial stress | $\zeta$ | The moisture uptake jump at polyester resin – composite interface (in finite element method) |
| $\varepsilon_{11}$ | Axial strain | $m_e$ | The maximum equilibrium Moisture content. |
| $\varepsilon_{22}$ | Hoop strain | $U(X)$ | Axial component of displacement |
| $\varepsilon_{33}$ | Radial strain | $W'(X)$ | Radial component of displacement |
| $\beta_{11}$ | Axial coefficient of hygroscopic expansion | $u^R(X)$ | Axial component of displacement in the 1th cylinder |
| $\beta_{22}$ | Radial coefficient of hygroscopic expansion | $w^R(t)$ | Radial component of displacement in the 1th cylinder |
study. Such a method could be developed in order to simulate the hygro-elastic behavior of cross-ply composite laminate.

References


