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# Optimal selection of machining direction for three axis milling of sculptured parts

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**ABSTRACT** In the field of free form surface machining, CAM software allow to manage various modes of tool-path generation (zig-zag, spiral, z-level, parallel plan, iso-planar, etc.) leaning on the geometry of the surface to be machined. Various machining strategies can be used for the same shape. Nevertheless the choice of a machining strategy remains an expert field. Indeed there are no precise rules to facilitate the necessary parameter choice for tool-path computation from the analysis of the numerical model of a part and the quality requirements. The objective of this paper is to provide a method to assist in the choice of the machining direction for parallel plane milling of sculptured parts. The influence of tool-path on the final quality according to the intrinsic geometrical characteristics of the latter (curves, orientation) was studied. The directionnal beam are introduced and defined from the local surface parameter. Finally a methodology to optimize machining time while guaranteeing a high level of quality was developed and applied to examples.

**KEYWORDS** Machining strategy, Surface roughness, Finishing process, Sculptured parts, three axis milling

## 1 Introduction

In this paper, machining strategies for finishing processes of sculptured surfaces are studied. This research is especially focused on the choice of necessary parameter for tool-path computation in 3 axis milling. A machining strategy is a methodology used to compute an operation with the aim of carrying out a geometrical entity in its final form [1]. The choice of a machining strategy depends on various factors e.g. form deviation, surface roughness. The geometry of the tool, the cutting conditions (feed rate, cutting speed) and the adjustment parameters of the tool-path computation (transversal and longitudinal step, machining direction) are characterized by the machining strategy [2].

In a competitive climate, it is necessary to reduce the costs and respect the design requirements. So, the choice of a machining strategy is an optimization problem under constraints. This problem is based on the geometry of the manufactured surface. The machining time depends on the part geometry and the tool-path. The surface roughness and the form deviation are related to the machining strategy and the surface geometry.

The choice of a machining direction in order to optimize the machining time is studied under various headings:

- Reduction in effective machining time. The machining direction is chosen according to an increase in material removal rate (feed rate, tool engagement) [3,4]. Currently no methodology is presented to choose a machining direction for the whole surface.
- Reduction in non-cutting time  $T_{nc}$ . The machining direction is chosen to decrease non-cutting tool-path [5].

Few formalized studies [6,7,8,9] presents methods to select a machining direction according to quality requirements of the machined surface. The reduction of the effective machining time leads to an increase in material removal rate. That requires an increase in the real feed rate and the transversal step.

The objective of this research work is to develop a methodology of machining direction selection according to the local parameters of the surface (curvature radius, orientation).

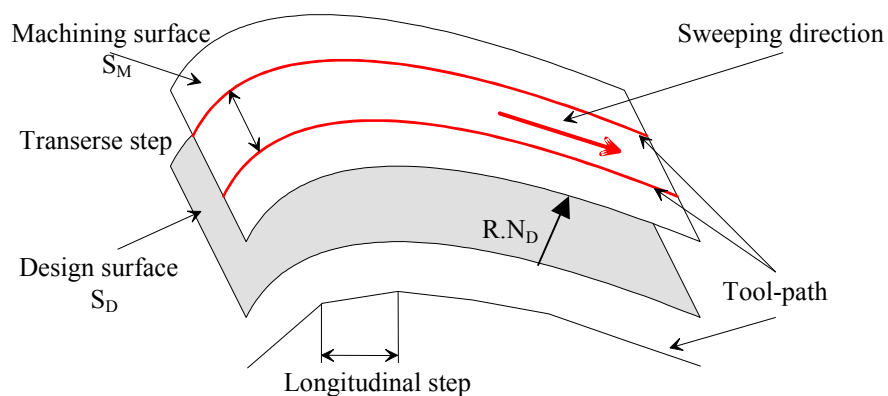
This choice allows an optimization of the machining time, while respecting the constraints on the manufactured surface. The selected machining direction has to minimize the machining time and respect requirements on the surface. Then we study the influence of the machining direction on the material removal rate  $A=f_c \cdot p$  and the relation between these same parameters and the surface roughness.

## 1.1 Free form tool path computation

Machining of sculptured surface permits to obtain the part in its final form respecting the design requirements (form deviation and surface roughness). The main parameters are [2]:

- the machining direction,
- the transversal step,
- the longitudinal step.

Numerous tool-path computation methods are available in CAM software, such as z-level, parallel plane, iso-parametric. Some research work have been carried out in this field to create new methods or improve existing ones [10,11]. Contrary to the roughing process, the manufacturing time during finishing does not take priority over the geometric specifications of the surface.



**Fig. 1.** Description of the tool-path construction

Considering the method proposed by Kim and Kim [12], the tool-path (Fig. 1) is computed starting from an offset surface  $S_M$ , theoretically defined by :

$$S_M(u,v) = S_D(u,v) + R \cdot N_D \quad (1)$$

Where  $N_D$  is the normal with  $S_D$  at point  $S_D(u,v)$  and  $R$  the tool radius. The normal of the surface can be computed as :

$$N_D(u,v) = \frac{\frac{\partial S_D(u,v)}{\partial u} \times \frac{\partial S_D(u,v)}{\partial v}}{\left\| \frac{\partial S_D(u,v)}{\partial u} \times \frac{\partial S_D(u,v)}{\partial v} \right\|} \quad (2)$$

The manufactured surface quality results from the linkage between the computed tool-path and the primary cutting motion. During the tool-path calculation, the theoretical path presented previously can not be directly communicated to the numerical control. The tool-path must be expressed according to an adapted interpolation format. In this way, the parameter of machining tolerance is defined to carry out the calculation. If the linear interpolation is used, the machining tolerance (Fig. 2a) allows to calculate the longitudinal step between two successive positions of the tool. The longitudinal step is computed according to the curvature radius of the tool path [13]. The higher the radius of curvature, the larger the longitudinal step.

In addition, the machining of the surface is obtained by sweeping, it remains a material scallop due to the form and the dimension of the selected tool. The scallop height can be parameterized by a transversal step. The transversal step  $p_t$  (Fig. 2b) is defined in a plane perpendicular to the tool axis  $\mathbf{k}$ , the local transversal step  $p$  is defined in the plane  $(\mathbf{n}', \mathbf{d}_T)$ .

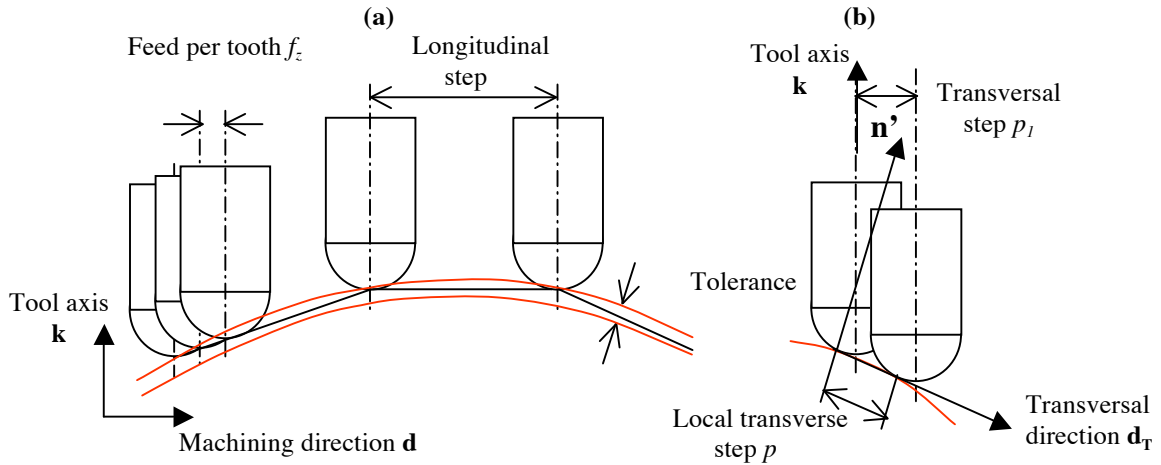
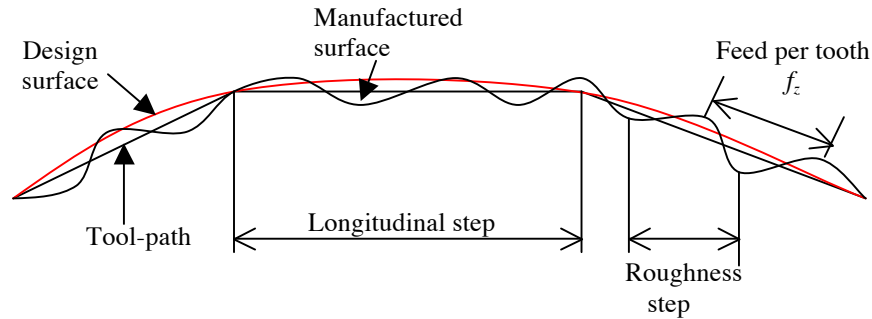


Fig. 2. Parameter description

Considering the cutting motion, the feed rate and the rotation of the tool must be taken into account. This periodic phenomenon is combined with the tool-path to generate the machined surface. If the longitudinal step is larger than the feed rate, the tool-path computation error [2] is of a higher order than surface roughness (Fig. 3).



**Fig. 3.** Surface roughness and tool-path generation error

The machining strategy parameters could be modified to decrease the machining time and respect the design requirements (form deviation and roughness). Next table (tab. 1) shows that all the different parameters influence machining time and surface quality.

Phenomenon	Geometric parameters	Design requirements	Parameter of the machining strategy	Influence on machining time
Feed rate variation	Longitudinal curvature		Machining direction	$\propto$
			Feed rate	
Cover rate	Transversal curvature		Machining direction	$\propto$
	Orientation surface/tool axis		Feed rate	
Cutting speed variation	Surface orientation compare to the tool	Surface roughness	Machining direction	
Mark of the cutting edge		Surface roughness	Feed rate	$\propto$
Scallop height	Transversal curvature	Surface roughness	Machining direction	
	Orientation surface/tool axis		Longitudinal step	
Load variation	Longitudinal and transversal curvature	Form deviation	Machining direction	$\propto$
	Surface orientation compare to the tool	Surface roughness	Feed rate	
Facet	Longitudinal curvature	Form deviation	Longitudinal step	
		Surface roughness		

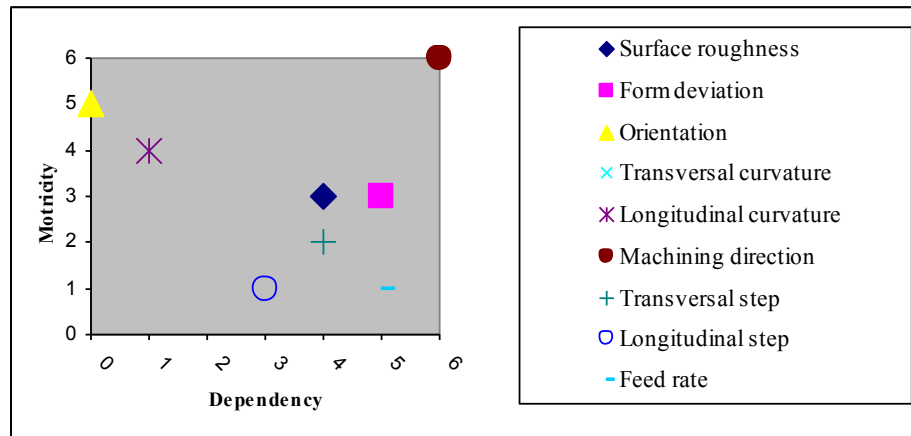
**Table 1.** Links between the parameters

## 1.2 Influence of the machining direction

To highlight the interrelationship between the parameters, the MICMAC method is applied [14] from the previous table (Tab. 1). Two characteristics are used to ensure the classification : motricity and dependency. The motricity of a variable corresponds to the number of parameters influenced by a variation this one. Conversely the dependency of a variable is the number of parameters which could modify the latter. It is then possible to classify the set of parameters according to their motricity and

their dependence. Indeed, figure 4 shows that the machining direction is the most significant parameter concerning machining time and surface roughness variation and highlights that :

- the machining direction is a key parameter in the choice of a machining strategy,
- the motor parameters (parameters with the highest motricity) are the geometrical parameters, surface orientation compared to the tool, the transversal and the longitudinal curvature of the tool-path.



**Fig. 4.** Motricity and dependency of the parameters

The machining direction is one of the most important parameters for the reduction of the machining time (Fig. 4). Usually [3,4,15] this direction is chosen according to an optimal feed rate or depth of cut. And there is no methodology linking the surface and the the machining strategy parameters with surface roughness requirements.

Only a few research works present methods for the optimal choice of the machining direction. Chiou et al. [4] propose to define locally the machining direction allowing to increase the radial step. The tool-paths are then computed by connecting the different points using a curve tangent to these directions. Feng et al. [15] define for each point the machining direction allowing to maximize the feed rate while respecting the form deviation. They analyze the variation of the cutting forces according to the surface (approximated locally by a plane) and the machining direction.

Chen et al. [3] define a machining direction allowing the greatest material removal. This direction is defined by the greatest depth of cut according to a defined deviation. For each point, they show that this direction corresponds to the projection of the surface gradient on an orthogonal plane to the tool axis.

The objective of this work is to reduce the machining time by increasing the material removal rate. However, locally, this material removal rate is proportional to  $A=p_1.f_z$ .  $p_1$  is the transversal step on the surface, and  $f_z$  is the programmed feed rate. But  $p_1$  is directly associated [15] to the machining direction at a given point according to the studied surface. So this work proposes to choose the optimal machining direction according to the feed rate, the transverse step and the geometry of the surface.

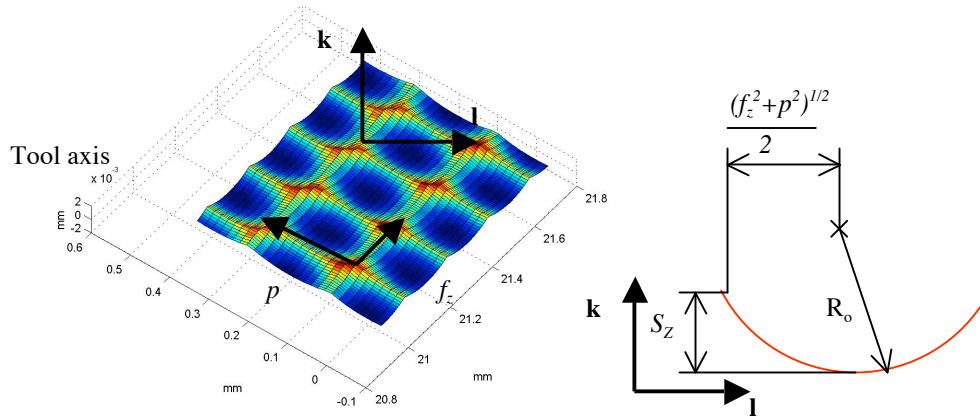
Initially a model of surface roughness is defined. This model permits to define the acceptable maximum transverse step at each point of the surface. From this maximum step a fitness function could be introduced. This function will define the influence of a machining direction over the machining time.

## 2 Surface roughness criterion

The objective is to define a criterion of surface roughness according to the surface geometry and the machining direction. Various studies on the characterization of the influence of the orientation of a ball end mill tool were undertaken. Some studies have an experimental point of view [6,7] and are more interested in observing the wear of the tool than the final surface quality. Other studies [16] analyze the mark left by a tool for various slopes and propose a model to predict the surface quality, but the criterion of surface roughness used is along one direction and not directly defined from the surface. The model defined by Kim [17] and developed in a previous work [18] shows that the pattern obtained is constructed from numerous spherical segments. The radius of this segment is the tool radius  $R_o$ . This pattern depends only on the feed rate ( $f_z$ ), the local transverse step ( $p$ ) and the radius of the tool ( $R_o$ ).

Currently there is no standardized criterion to describe this pattern. Moreover, traditional criteria of characterization [19] ( $R_a$ ,  $R_t$  defined in ISO 4287) do not highlight the link between the feed rate ( $f_z$ ) and the transversal step ( $p$ ) [20,21] because of their uni-directional definition. To define the surface roughness corresponding to this pattern, the criterion used is the surface criterion  $S_z$  [22] defined in the standard draft ISO 12085. This parameter corresponds to the height deviation between the lowest and highest points of the surface. For a spherical segment of  $R_o$  radius, and dimension  $f_z$  and  $p$ , the maximum height  $S_z$  is expressed by (Fig. 5):

$$S_z = R_o - \left( R_o^2 - \frac{f_z^2 + p^2}{4} \right)^{1/2} \approx \frac{f_z^2 + p^2}{8.R_o} \quad (3)$$



**Fig. 5.** Computation of the maximal deviation

## 3 Real feed rate and machining time

At first approximation, the real feed rate is considered equal to the programmed feed rate during machining. In this case, the real feed per tooth  $f_z$  is equal to the programmed one without any influence of the selected machining direction. In this case, the machining direction which minimizes the tool-path length, minimizes the machining time.

But the study of the high speed milling machine shows that the real feed rate is not always equal to the programmed one. The feed rate evolution depends on the power of the motion axes used and in

particular of the jerk value, maximum acceptable accelerations and speeds. According to the literature [23], the two principal factors of deceleration are:

- the tool-path continuity,
- the tool-path curvature radius.

In the case of sculptured surface, the tool-paths are assumed to be continuous and only the tool-path curvature radius is taken into account. According to maximum axis acceleration and the local curvature radius  $R_c$  of a tool-path, the maximum real feed per tooth is given by:

$$V_{f \max} = \sqrt{R_c \cdot a_{\max}} \Rightarrow f_{z \max} = \frac{\sqrt{R_c \cdot a_{\max}}}{Z \cdot N} \quad (4)$$

Thus, for a given point of surface, if the machine needs to slow down, the feed per tooth is reduced. Hence the surface roughness criterion  $S_z$  presented in 3 is reduced but the selected direction of machining can be less powerful.

## 4 Definition of a machining direction

The objective of this work is to propose a method for machining direction choice. The direction chosen allows at the same time:

- to ensure the respect of roughness requirement by modelling the local surface roughness,
- to reduce the machining time, by evaluating the real feed rate at any point and choosing the suitable machining direction.

The presented method permits to determine the most powerful machining directions (minimizing the machining time) according to the local surface parameters. The concept of directional beam is introduced. These beams are defined starting from a fitness function. This fitness function highlights the performance of a machining direction related to the machining time. The directional beams represents for a point the set of acceptable machining directions, i.e. the set of machining directions allowing a satisfactory level of performance.

Then the concept of the directional beams is used in order to establish criteria of surface feasibility. The methodology suggested determines the set of the machining directions maximizing the performance criterion.

### 4.1 Fitness function

To define the most powerful machining direction, the fitness function is introduced. This function links the machining direction at a point and the cutting parameters with time reduction. This function is described by:

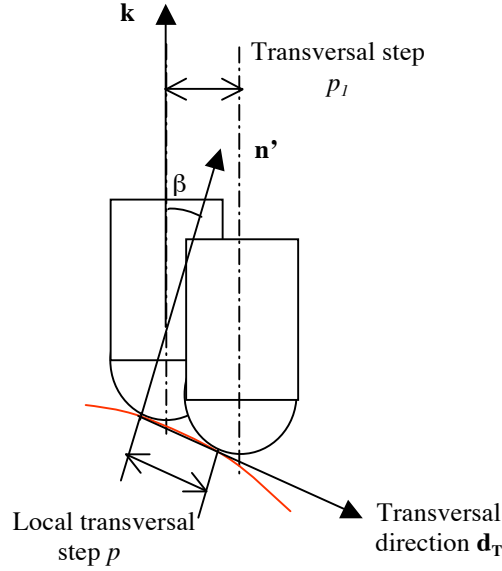
$$\left( \begin{array}{l} (D, S_M, P_c) \rightarrow \mathfrak{R} \\ (d, M, P) \mapsto G_p(d, M, P) \end{array} \right) \quad (5)$$

$D$  is the set of machining directions and  $d$  a direction in  $D$ .  $M$  is a point on surface  $S_M$ . Finally,  $P_c$  represents the set of cutting parameters ( $f_z$ , transversal step) and  $P$  is an element of this set. This function  $G_p$  expresses the performance at point  $M$  on the surface  $S_M$  of a machining direction  $d$  for parameters  $P$ .

A reduction in machining time could be achieved by increasing the material removal rate. Thus the fitness function could be defined by:



$$\forall (d, M, P) \in (D, M, P_c) \quad G_p(d, M, P) = p_1 \cdot f_z \quad (6)$$



**Fig. 6.** Definition of  $\beta$

Locally, the tool-path is assumed to be defined on the tangent plane of the surface. The transverse direction  $\mathbf{d}_T$  and the vector  $\mathbf{n}'$  (Fig. 6) are defined by:

$$\mathbf{d}_T = ((\mathbf{d} \times \mathbf{n}) / (\|\mathbf{d} \times \mathbf{n}\|)) \quad (7)$$

$$\mathbf{n}' = \mathbf{d}_T \times \mathbf{d}$$

Where  $\mathbf{d}$  is the machining direction. Generally, using  $\beta$  as  $\beta = \cos^{-1}(\mathbf{n}' \cdot \mathbf{k})$ , the local transversal step could be defined by  $p = p_1 / \cos(\beta)$  [15]. The effective height of pattern  $S_z$  is specified in the preceding paragraph. At the considered point  $M$ :

$$S_z = ((p^2 / \cos^2 \beta) + f_z^2) / (8 \cdot R_o) \quad (8)$$

$\beta$  directly depends on the machining direction selected and on the normal the surface at the considered point  $M$ . Replacing  $\beta$  in the equation 8, the calculation of the satisfactory step at the point considered  $M$  becomes:

$$p_1^2 = (8 \cdot S_z \cdot R_o - f_z^2) \left( \left( \frac{\mathbf{d} \times \mathbf{n}}{\|\mathbf{d} \times \mathbf{n}\|} \times \mathbf{d} \right) \cdot \mathbf{k} \right)^2 \quad (9)$$

The maximum feed rate of the machine tool depends on the maximum acceleration of the slowest axis of the machine and also on the curvature radius  $R_c$  of the tool-path [23]. This speed is expressed by:

$$V_{f \max} = \sqrt{R_c \cdot a_{\max}} \quad (10)$$

This enables us to evaluate the real feed rate of the machine at the determined point with a defined speed  $V_f^{\text{programmed}}$  by:

$$f_z = \min(V_f^{\text{programmed}}/Z.N, V_{f_{\max}}/Z.N) \quad (11)$$

A fitness function taking into account the constraint of surface roughness could be defined by:

$$\forall (d, M, P) \in (D, S_M, P_C) \quad G_p(d, M, P) = p_1 \cdot f_z \quad (12)$$

$$G_p(d, M, P) = \sqrt{(8.S_z.R_o - f_z^2)} \cdot \left( \left( \frac{d \times n}{\|d \times n\|} \times d \right) \cdot k \right) \cdot f_z$$

## 4.2 Definition of the directional beams

For a given point  $M$  on the surface and fixed cutting parameters  $P$ , it is possible to define direction  $d$  which maximize the fitness function  $G_p$ . The maximum of  $G_p$  is noted:

$$G_{p_{\max}}(M, P) = \max_{d \in D} (G_p(d, M, P)) |_{(M, P) \text{ fixed}} \quad (13)$$

From this maximal value, the machining direction  $d$  which enables us to obtain this maximum value, are gathered in a set  $D_{\max(M, P)}$ . It is defined by:

$$D_{\max(M, P)} = \{d \in D, G_p(d, M, P) = G_{p_{\max}}(M, P)\} \quad (14)$$

**Directional Beam:** A directional beam is a set of machining directions for a given point  $M$  and cutting parameters  $P$ . The fitness function  $G_p$  is close (with a coefficient  $\alpha$ ) to its maximal value  $G_{p_{\max}}$  for all directions.

$$Fd(M, P) = \{d \in D, G_{p_{\max}}(M, P) - G_p(d, M, P) \leq (1 - \alpha) \cdot G_{p_{\max}}(M, P)\} \quad (15)$$

These beams enable to introduce flexibility in the choice of machining direction. They describe the concept of neighborhood between two machining directions.

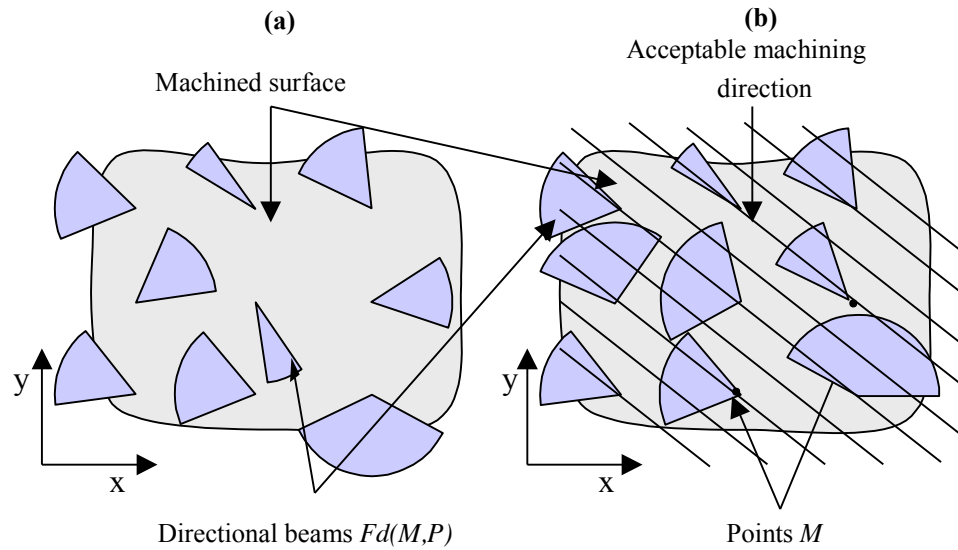
If two points  $M_1$  and  $M_2$  present two beams ( $Fd(M_1, P)$  and  $Fd(M_2, P)$ ) non-disconnected, they could be machined with a common machining direction. This direction guarantees a machining time close to the optimal value in  $M_1$  and  $M_2$  (Fig. 7).

## 4.3 Machining feasibility criterion

To determine a machining direction for all points  $S_M$ , it is sufficient to calculate the beam intersection. We note:

$$I = \bigcap_{M \in S_M} Fd(M, P) \quad (16)$$

- If  $I \neq \{\emptyset\}$  then a machining direction permits to machine the surface close to the optimal fitness at each point. All the direction in  $I$  are suitable.
- If  $I = \{\emptyset\}$  then no machining direction permits to machine the surface close to the optimal fitness at each point for a rate  $\alpha$ .

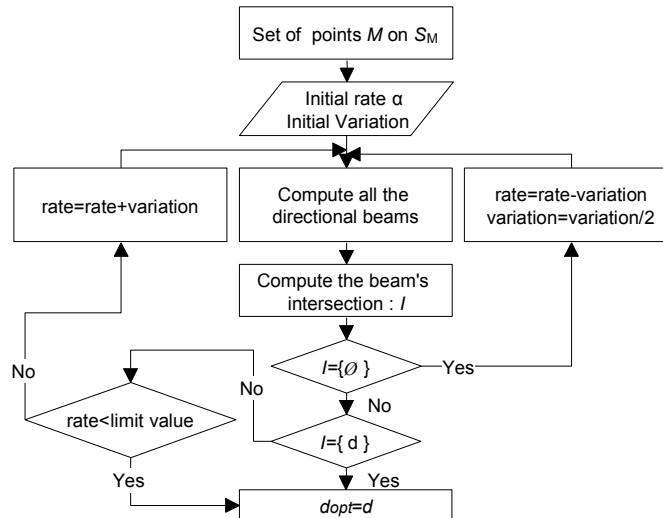


**Fig. 7.** Optimal machining direction (a)  $I \neq \{\emptyset\}$  (b)  $I = \{\emptyset\}$

If the beam intersection is not empty, the selected direction guarantees that for each point the fitness function is close to its maximum (Fig. 7). But there is no guarantee that the direction chosen is the best one. To find the best direction, we must increase  $\alpha$  in order to reduce the directional beams.

#### 4.4 Algorithm of choice

The optimal machining direction must be located at the beam's intersection and maximized the rate  $\alpha$ . To determine this direction, the algorithm described in figure 8 is proposed. This algorithm begins with a low value of  $\alpha$ . Then this value is increased in order to reduce the number of elements at the intersection. If the intersection turns empty then the step chosen to increase rate  $\alpha$  is reduced and the process starts again. This algorithm maximizes  $\alpha$  and determines a machining direction close to optimal ( $G_{pmax}$ ) at any point.



**Fig. 8.** Machining direction choice algorithm

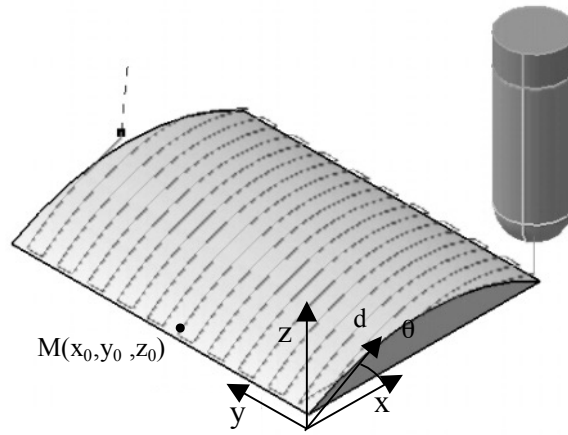
## 5 Application

### 5.1 Case 1

The previous methodology is now applied on a first example (Fig. 9). For a given point on the surface, the machining direction is defined by  $d$  with  $d=(\cos\theta,\sin\theta,0)^T$ .  $\theta$  defines the orientation of  $d$  from the  $x$  axis in the  $(x,y)$  plane.

The machined surface is defined as a portion of a cylinder of axis  $y$  and radius  $R_{cyl}$ . In a coordinate system  $(x,y,z)$  with  $z$  as the tool axis, this surface is written as:

$$z = \sqrt{R_{cyl}^2 - x^2} \quad (17)$$



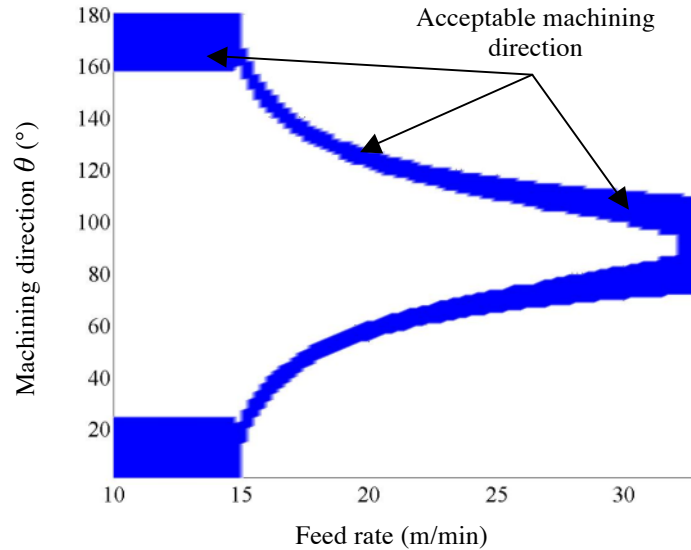
**Fig. 9.** Description of the cylinder milling

For a machined surface  $S_D$  defined previously, the radius of curvature  $R_c$  of the tool-path at a given point  $(x_0, y_0, z_0)$ , is expressed according to the selected machining direction  $d$  and the surface parameters.

$$R_c = \left| \frac{-1}{\cos^2 \theta} \cdot \sqrt{(R_{cyl} + R_{tool})^2 - x_0^2 \cdot \sin^2 \theta} \right| \quad (18)$$

The direction along the cylinder axis can increase the feed rate. Indeed, in these directions the tool-path curvature is null and the programmed speed is reached at each point. On the other hand, the transversal direction penalizing for speed allows a greater transversal step. A large radius of cylinder makes it possible to obtain the programmed feed rate apart from the selected direction. In this case the optimal machining direction  $d$  should be the transverse one.

The previous algorithm is applied to this surface for a ball end tool of diameter  $10 \text{ mm}$  ( $R_{tool}=5 \text{ mm}$ ), a roughness criterion  $S_z=0.01 \text{ mm}$ , a radius of cylinder  $R_{cyl}=10 \text{ mm}$  and a maximum acceleration of  $3 \text{ m/s}^2$ . The results obtained are represented in figure 10. This figure describes the acceptable machining direction according to the programmed feed rate.

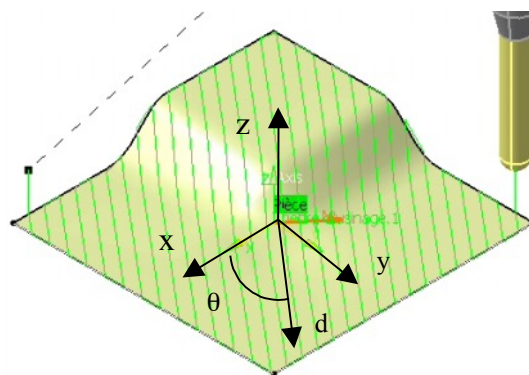


**Fig. 10.** Optimal machining direction according to the feed rate

These results highlight that:

- In the case of low programmed speeds, the transverse direction ( $0^\circ$  and  $180^\circ$ ) is not very penalizing for the feed rate of the machine. The optimal direction is this one because it allows the greatest transversal step.
- In the case of high programmed speeds, the maximum speed of the machine is more significant on the choice of machining direction. The optimal direction is maximizing the real feed rate. So the optimal machining direction is parallel to the cylinder axis.

## 5.2 Case 2

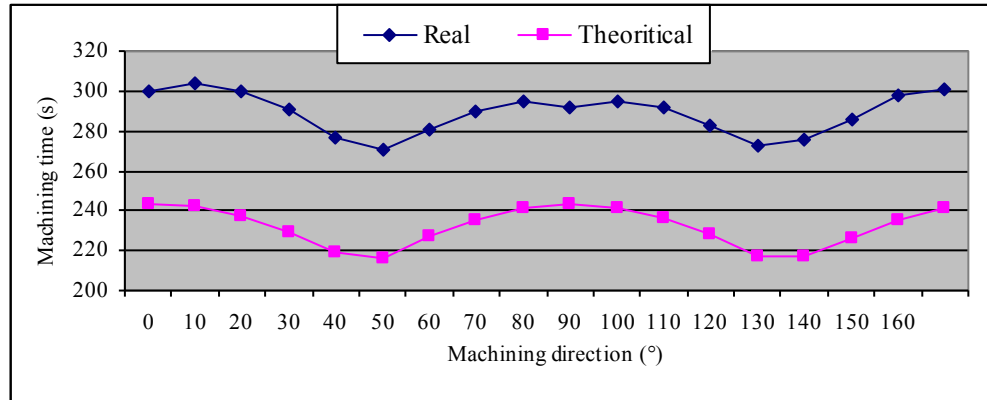


**Fig. 11.** Description of the machined surface (case 2)

This method has been tested on the surface defined in figure 11. This surface is contained in a parallelepiped of  $100 \times 100 \times 15 \text{ mm}^3$ . To machine this surface, the tool used is a 10 mm diameter ( $R_{\text{tool}} = 5 \text{ mm}$ ) ball-end mill with a feed per tooth  $f_z = 0.2 \text{ mm/tooth}$ . The algorithm ends up in a direction of  $\theta = 45^\circ$ .

In order to check this result, tool-paths have been programmed on CAM software (Catia V5). The feed rate used was 6000 mm/min ( $N = 15000 \text{ rpm}$ ). For these cutting conditions, timekeeping on

machine tool (HURON KX15 with a Siemens 840D numerical control unit) were carried out. Figure 12 shows the real (time keeping on machine tool) and theoretical (time given by CAM software) machining time evolution according to machining direction. The lowest machining time is obtained for an orientation of  $\theta=50^\circ$ . This orientation is closed to the optimal machining direction of the algorithm. The results confirm a notable reduction in real machining time (12%) with an orientation of  $\theta=50^\circ$  compared to  $\theta=90^\circ$ . The difference between the theoretical and real machining time is constant. This gap comes from the the tool-path linking. The curvature radius of the linking shape imposes a feed rate reduction.



**Fig. 12.** Machining time evolution

## 6 Conclusion

In the framework of 3-axis milling of a sculptured part, the machining direction is a key parameter of the machining strategy. The choice of this direction should make it possible to reduce the total machining time, while respecting the surface requirements.

The objective of the presented work is to propose a methodology to the machining direction choice. This choice is carried out by respecting the surface roughness requirements and by assessing the real feed rate at each point.

The concept of directional beam has been introduced in this article. These beams represent in each point the set of machining directions guaranteeing a maximum rate of performance. The intersection of this beams defines a set of optimal machining directions. From this intersection a machining direction is selected on the surface. This methodology is applied to simple case. The results correspond to expected ones and highlight the best machining directions according to the programmed feed rate.

The presented method of direction choice is currently being validated on a simple case and is going to be applied on more complex parts like forging dies and thus to be able to provide an assistance tool to C.A.M programming.

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