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Weighted Path as a Condensed Pattern in a Single Attributed DAG *

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Abstract

Directed acyclic graphs can be used across many application domains. In this paper, we study a new pattern domain for supporting their analysis. Therefore, we propose the pattern language of weighted paths, primitive constraints that enable to specify their relevancy (e.g., frequency and compactness constraints), and algorithms that can compute the specified collections. It leads to a condensed representation setting whose efficiency and scalability are empirically studied.

1 Introduction

Graphs are ubiquitous in many data analysis settings. Recently, richer graph models have been considered where, for instance, vertices or edges are labelled by sets of attributes or properties (i.e., itemsets) instead of a single one. For example, a social network can be represented as a large graph where each vertex denotes a person and its associated domains of interests. These graphs are called attributed graphs or itemset-associated graphs [Fukuzaki et al., 2010]. Quite often, for instance when time is concerned, edges are oriented and graphs turn to be acyclic.

Among others, we are concerned with the analysis of spatio-temporal data that can be modeled by means of attributed directed acyclic graphs (a-DAG). In an a-DAG, vertices may denote spatial objects characterized by a set of attributes and/or events while edges may represent the spatio-temporal proximity between objects (i.e., neighbouring objects in consecutive timestamps). Our goal is to tackle such data analysis problems where studied objects are not stationary. Indeed, a specificity of geographical data is that "everything is related to everything else, but near things are more related than distant things" (Tobler’s first law of geography). This statement reflects the concept of spatial dependence between geographical objects. In such a spatial analysis setting, a DAG is a natural representation to model spatial dependences between objects at different times (i.e., modeling the influence of one object on an other). For example, it can be used to model the spatial dependences between living areas when studying the diffusion of a virus, or to model dependences between erosion objects and their environments when looking for a better understanding of soil erosion dynamics. Indeed, in an a-DAG graph representing the spread of a vector-borne disease in a city (e.g., the Dengue fever), vertices could be the city districts at a given timestamp and they may be characterized by a set of attributes or events. Here, edges may express the disease propagation from one area to another at successive timestamps. If we consider now soil erosion analysis, vertices may be geological objects like, e.g., gullies, that are observed at a given date. Their characteristics could be expressed by itemsets, and edges would be used to symbolize geological events like the merge or the division of the related objects (see Fig. 1).

Usually, when tackling such application domains, well known diffusion models are assumed and exploited. We are looking for a generic data mining perspective that does not rely on any a-priori domain knowledge. We study frequent weighted path mining in a single a-DAG where each weight denotes the frequency of a transition. Frequent paths are useful to support the analysis of the causal relationship between sequences of events and/or attributes. Since the number of path patterns can be huge, we design a condensed representation of such collections.

Figure 1: Example of an a-DAG built from successive timestamps containing several areas characterized by a set of attributes. An edge represents the evolution of an object to another through two consecutive timestamps.

Our work is related to sequential data mining and graph mining. Many algorithms have been proposed to support fre-
quent sequence mining. Closedness, which has been exten-
sively studied for itemset mining [Pasquier et al., 1999] has
been applied to sequences as well [Yan et al., 2003]. A pat-
tern is closed if there is no super-patterns (w.r.t. a specializa-
tion/generalization relation) with the same support (i.e., oc-
curring in the same transactions or sequences). Also, sev-
eral algorithms have been proposed to mine patterns in graph
transactions [Inokuchi et al., 2000; Borgelt and Berthold,
2002; Washio and Motoda, 2003; Yan and Han, 2002]. Con-
densed representations of frequent graph patterns have been
studied [Yan and Han, 2003; Termier et al., 2007]. For exam-
ple, [Yan and Han, 2003] proposes to mine closed frequent
graphs in graph transactions, and [Termier et al., 2007] adapts
this approach for transactions of DAGs.

These studies focus on frequent pattern mining in trans-
actions of graphs while we want to consider a single (large)
DAG. The graph-transaction setting and the single-graph set-
ting share common properties but the algorithms developed
for the former cannot be used for the latter whereas the op-
opposite is true [Kuramochi and Karypis, 2005]. One of the
first problems in the single-graph setting is how to define
pattern frequency. Indeed, it cannot be defined as the num-
ber of transactions in which it occurs. Several authors have
studied this issue [Kuramochi and Karypis, 2005; Fiedler and
Borgelt, 2007; Bringmann and Nijssen, 2008]. Most of them
define a frequency based on pattern occurrences but this is not
simple: several occurrences may overlap, leading to a non-
monotonic frequency measure. However, exploiting such fre-
quency measures, several algorithms have been proposed to
mine frequent patterns in a single graph [Cook and Holder,
1994; Matsuda et al., 2000; Kuramochi and Karypis, 2005;
Gudes et al., 2006]. When tractable, the number of frequent
patterns can be huge and it makes sense to look for condensed
representations of them, for instance closed ones. To the best
of our knowledge, closedness has not yet been studied within
the single-graph setting.

Moreover, most of graph mining algorithms process la-
beled graphs, i.e., graphs with only one label associated to
each vertex and/or edge. Mining our attributed graphs leads
to a combinatorial explosion (i.e., the exploration of search
spaces for both graphs and label itemsets). Few works have
considered attributed graphs. [Miyoshi et al., 2009] mines a
labeled attributed graph, i.e., a graph with labels and quanti-
tative itemsets in vertices. By keeping labels, frequent pattern
mining is simplified and decomposed in two steps: mining the
labeled graph and mining the itemsets. [Moser et al., 2009;
Fukuzaki et al., 2010] focus on mining cohesive patterns and
itemset-sharing patterns, i.e., patterns representing subgraphs
with shared itemsets.

Computing frequent paths has been already studied as
well. For example, [Chen et al., 1998] mines frequent path
traversal patterns in a labeled directed acyclic graph re-
presenting user accesses in web pages. Traversal patterns
have also been considered in [Borges and Levene, 2000;
Nanopoulos and Manolopoulos, 2001; Geng et al., 2007].
However, in these works, the authors only consider classi-
cal labeled graphs and not attributed graphs. Moreover, their
definition of a path is different from the one we use: they can
push more stringent constraints like avoiding repeated ver-
tices and/or edges.

The same data mining problem has been already tackled
by some of us in [Mabit et al., 2011]. In this paper, the au-
thors propose to flatten the graph and then to mine frequent
sequences using available algorithms from the shelf. Unfor-
unately, their experiments show that this is not scalable due
to the intrinsic complexity of this mining task. Here, we avoid
a sequence mining approach for two reasons:

- A graph contains structural information that cannot be
  exploited if it is decomposed into sequences. This can
  motivate for a direct single graph mining approach.
- If we still want to decompose the graph into several
  sequences, we must choose to either duplicate nodes or
delete edges. The last solution seems unacceptable since
we would loose information. Furthermore, duplicating
nodes will over-express information and thus add a bias.
Although this duplication can be managed, it remains
ineffective [Mabit et al., 2011; Selmaoui-Folcher and
Flouvat, 2011].

Our contribution is twofolds. First, we propose an original
definition of what could be an (exact) condensed represen-
tation in a single-graph setting (see Section 3). Second, we
propose an efficient algorithm to mine frequent patterns, i.e.,
weighted paths, in a single a-DAG (see Section 4). Our expe-
riments highlight its good performances and the high compres-
sion rate provided by the designed condensed representation
(see Section 5).

2 Preliminaries

Let us introduce some needed definitions or concepts about
attributed directed acyclic graphs and weighted paths.

2.1 Attributed directed acyclic graphs

Attributed DAG. An attributed DAG (or a-DAG) \( G =
(V_G, E_G, \lambda_G) \) on a set of items \( I \) consists of a set of vertices
\( V_G \), a set of directed edges \( E_G \subseteq V_G \times V_G \) and a labelling
function \( \lambda_G : V_G \to P(I) \) that maps each vertex of the DAG
\( G \) to a subset of \( I \) (\( P(I) \) denotes the power set of \( I \)). An
example of such a graph is given in Fig. 2.

Figure 2: An a-DAG example.
Path pattern and path occurrence. Let \( P \) be a succession of itemsets \( I_i \in \mathcal{P}(\mathcal{I}) \) denoted \( P = I_1 \rightarrow I_2 \rightarrow \cdots \rightarrow I_n \). \( P \) is called a path pattern iff there exist a succession of vertices \( v_1, v_2, \ldots, v_{|P|} \in V_G \) satisfying \( \forall I_i \in P, I_i \subseteq \lambda_G(v_i) \) and such that each \( v_i \) is a parent of \( v_{i+1} \) in \( G \). The succession of vertices \( O = v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_{|P|} \) is called a path occurrence of \( P \). For example, given the a-DAG in Fig. 2, the occurrences of the size-3 path \( ah \rightarrow cd \rightarrow i \) are \( 2 \rightarrow 3 \rightarrow 6 \), \( 2 \rightarrow 3 \rightarrow 8 \), \( 2 \rightarrow 4 \rightarrow 7 \), \( 2 \rightarrow 5 \rightarrow 7 \), and \( 5 \rightarrow 7 \rightarrow 8 \). The occurrence \( 2 \rightarrow 3 \rightarrow 6 \) supports the paths \( ah \rightarrow cde \rightarrow bi \), \( a \rightarrow cde \rightarrow bi \), \( h \rightarrow cde \rightarrow bi \), \( h \rightarrow c \rightarrow bi \), and so on.

Let \( P_i \) denote the \( i \)-th itemset of \( P \), \( O_i \) be the \( i \)-th vertex of \( O \), and \( occur_G(P) \) be the set of \( P \) occurrences in \( G \).

2.2 Weighted paths

Simple path patterns describe properly a sequence of events in an a-DAG. It would be however useful to know the contribution of every single edge to pattern occurrences. Therefore, we propose the pattern language of weighted paths.

Weighted path. Weighted paths are paths with a weight on each edge that represents the number of its different occurrences over all the \( P \) occurrences. For example, in the data from Fig. 2, the path \( P = ah \rightarrow cd \rightarrow i \) whose occurrences have been listed earlier provides the pattern:

\[
ah 
\begin{array}{c}
\rightarrow
\end{array}
\begin{array}{c}
5
\end{array}
\begin{array}{c}
\rightarrow
\end{array}
\begin{array}{c}
i
\end{array}
\]

Indeed, the number of different occurrences of \( ah \rightarrow cd \) in \( occur_G(P) \) is 4 and the number of different occurrences of \( cd \rightarrow i \) in \( occur_G(P) \) is 5. Such a presentation supports pattern interpretation: it tells that itemset \( ah \) occurs 4 times before path \( cd \rightarrow i \) occurs, or that itemset \( i \) occurs 5 times after path \( ah \rightarrow cd \) occurs. From now on, \( \omega_G(P_i \rightarrow P_{i+1}) \) designates the weight of the edge between itemsets \( P_i \) and \( P_{i+1} \).

Inclusion relation. We define the operator \( \subseteq \) on a couple of weighted paths as follows: \( P \subseteq P' \) iff \( |P| \leq |P'| \) and \( \exists k \in [0, |P'| - |P|] \) such that:

\[
\begin{cases}
\forall i \in [1, |P|], & P_i \subseteq P'_{k+i} \\
\forall j \in [1, |P'|], & \omega_G(P_{j} \rightarrow P_{j+1}) = \omega_G(P'_{k+j} \rightarrow P'_{k+j+1})
\end{cases}
\]

Inclusion of itemsets and weights equality are checked both. A weighted path \( P' \) absorbs/captures another weighted path \( P \) if we can find \( P \) in a sub-sequence of \( P' \). We say that a weighted path \( P' \) is a super-weighted path of \( P \), or that \( P' \) contains \( P \).

2.3 Problem statement

A popular data mining problem concerns frequent pattern discovery, i.e., looking for patterns with a support/frequency that is greater than a given threshold. Based on [Bringmann and Nijssen, 2008], we define an anti-monotonic support value of a pattern \( P \) in an a-DAG \( G \) denoted \( \sigma_G(P) \):

\[
\sigma_G(P) = \min_{1 \leq i < |P|} \{(O_i \rightarrow O_{i+1} / O \in \text{occur}_G(P)) \}
\]

\[
= \min_{1 \leq i < |P|} \omega_G(P_i \rightarrow P_{i+1})
\]

While being easily computable and anti-monotonic, this support fits well with the notion of frequency in a single DAG: it distinguishes paths occurring at different locations in a DAG from those finishing on or beginning from few edges. In other words, we want totally distinctive paths to be better valued than paths that share more edges.

Weighted paths actually help to better describe the evolution from an itemset to another (see Fig. 3). Simple paths would have blurred such distinction between some patterns having the same support but occurring in varying ways.

Figure 3: Two a-DAGs where path \( a \rightarrow b \rightarrow c \rightarrow d \) occurs in different ways but has the same support (1).

The collection of frequent paths in \( G \) is the set of patterns \( P \) such that \( \sigma_G(P) \geq \text{minsup}, \text{minsup} \) being a user-defined threshold. However, the number of frequent paths in \( G \) can be huge. In such a situation, it makes sense to look at the concept of condensed representation of frequent patterns [Calders et al., 2004].

Given an a-DAG \( G \), we look for the condensed representation denoted \( \text{cond}(G) \) of all weighted paths: each frequent weighted path and its support must be derivable from \( \text{cond}(G) \).

3 Exact condensed representation of frequent paths

Many authors have been working on closed pattern mining (see, e.g., Pasquier et al., 1999; Yan et al., 2003; Yan and Han, 2003). In these settings, (frequent) closed patterns form an exact condensed representation of the frequent patterns: the computed collection of closed patterns is much smaller while it is possible to deduce the set of all frequent patterns from them.

Closed and condensed. Let us first highlight the differences between condensed representations from either a single graph or graph transactions. In the latest, the most popular form of condensed representation of frequent patterns is the
collection of closed patterns. It exploits the Galois connexion that holds between transactions and patterns. An important property of the closure operator in this context is the support preservation property: patterns and their closures have the same support. This property relies on the support definition: a pattern is counted once per transaction. In the single-graph setting, the support definition is quite different: it is the minimum weight over the path edges. Moreover, the Galois connexion defined for closed patterns cannot be applied since we do not have transactions here. Another problem is that a pattern has only one associated closed pattern. However, a weighted path can be deduced from several different super-weighted-paths, as shown in Fig. 4. It turns out that we can hardly use a closure approach in our setting.

<table>
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<tr>
<th>1:ab</th>
<th>2:b</th>
<th>3:c</th>
<th>4:d</th>
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Figure 4: A weighted path can be included in several super-weighted paths.

A condensed representation of paths. As the support of a weighted pattern is directly encoded in it (i.e., the support is the minimum weight over all its edges), we can define the set of condensed weighted paths in an a-DAG $G$ as follows:

$$
cond(G) = \{ P / \exists \bar{P}' \text{ s.t. } P \subseteq \bar{P}' \}
$$

Note that there is no need to check for support equality since the support information is already attached to the pattern itself.

**Theorem 3.1.** Each path of $G$ as well as its support can be deduced from $\text{cond}(G)$.

**Proof.** a) From the definition of weighted paths and $\sigma_G$, the latter can be deduced from the former (minimum of weights). Reading a weighted path provides its support. b) Inclusion relation uses both information about attributes and edges weights; Then, a weighted path can be deduced from any of its super-weighted paths.

## 4 Mining condensed weighted paths

We propose a two-steps algorithm to mine condensed weighted paths directly from the graph structure. Unlike [Mabit et al., 2011] who first tries to flatten the graph, we avoid such an expensive and unnecessary candidate generation. It can handle the inherent high memory complexity of the problem (as shown later in Section 5). In addition, the structural information is kept that enables to push structural constraints if needed.

First, we mine every single size-2 weighted path (with one edge) that is condensed regarding the set of size-2 (only) weighted patterns. Then, we use a depth-first search to retrieve the condensed set by extending previously computed size-2 weighted paths.

### 4.1 Mining size-2 weighted paths

Size-2 weighted paths can naturally be represented by triplets like $\{\omega_G(P_{\text{orig}} \leftarrow P_{\text{dest}}), I_{\text{orig}}, I_{\text{dest}}\}$. Those triplets match frequent triadic concepts such as defined by [Cerf et al., 2008]. We use their algorithm to mine size-2 weighted paths.

**Proposition 4.1.** Given the set of size-2 weighted paths called $L2W$, and the following ternary relation:

1. The first dimension is $P(E_G)$, the second and the third are $P(I)$,
2. Given an edge $v_1 \rightarrow v_2 \in E_G$, the equivalent tuple $T \in (E_G, P(I), P(I))$ is $T = (v_1 \rightarrow v_2, \phi_G(v_1), \phi_G(v_2))$.

the closed 3-sets (i.e., size-2 paths) noted $LC2W$ are equivalent to condensed size-2 weighted paths w.r.t $L2W$.

**Proof.** Let us take up the definition of closed 3-set from [Cerf et al., 2008]: a 3-set $S$ is closed iff there is no other 3-set $S'$ such that $\forall i \in \{1,2,3\}, S' \subseteq S'$. As Dimension 2 and Dimension 3 represent respectively origin and destination itemsets, the definition of closed patterns on $(E_G, I, I)$ is the definition of condensed size-2 weighted paths (but only condensed w.r.t. size-2 weighted paths). Their weight is the number of different edges, which is actually the size of $S'$.

For example, we use the closed 3-set $\{1 \rightarrow 3, 1 \rightarrow 4, \{b, c\}, \{e, d, e\}\}$ to provide the weighted path:

$$
P = bc \rightarrow cde.
$$

Having such sets of size-2 weighted paths enables to proceed to a standard depth-first search extension from any previously found size-2 weighted path.

### 4.2 Extending weighted paths

The goal of the second step is to extend weighted paths until they get condensed. To do so, we use the previously computed set of size-2 weighted paths which are ensured to be maximal w.r.t. itemsets on both origin and destination vertices. Their extension must be executed both downward (by adding children) and upward (by adding parents). A single a-DAG vertex can have several children and parents. To deal with the many possible combinations of extensions, we use two prefix trees (one for each extension direction), as shown in Fig. 5. Due to space limitations, we only present downward extension. Upward extension is done following the same principle and properties (by simply considering parent vertices instead of child vertices).

Given a pattern $P$, we define $V_{\text{dest}}(P) = \{ v_i \} \in \text{occur}_G(P)$ and $L_i(P) = \{ i \in I, \text{ such that } \forall v \in V_{\text{dest}}(P), v \text{ has at least one child } u \text{ s.t. } i \in \phi_G(u). \}$. $L_i(P)$ is the list of items belonging to at least one child of each destination vertex of $P$. The extension method is based on the following proposition that is derived from Proposition 4.1.

**Proposition 4.2.** Let $P$ be a weighted path to be extended (downward). Let the projected binary relation of $P$ called $DB|P$ be the following:

1. The first dimension is $P(E_G)$, the second is $P(L_i(P))$, 


The last recursively generated super-weighted paths are then closed and obtained by successive projections of the base. Can be extended with itemset extension instead of children. It is only used to define an extension towards parents (upward extension) instead of children.

Algorithm 1: Process each size-2 weighted path.

Input: \( LC2W, \) a-DAG \( G \)

1. while \( LC2W \neq \emptyset \) do
2. Pick and remove \( e_2w \) from \( LC2W \)
3. Create two prefix trees \( pt_{down} \) and \( pt_{up} \)
   // Extend downwards and upwards
4. ExtendPath \((G, LC2W, e_2w, pt_{down})\)
5. ExtendPath \((G^{-1}, LC2W^{-1}, e_2w^{-1}, pt_{up})\)
6. Generate solutions from \( pt_{down} \) and \( pt_{up} \)

Algorithm 2: Extend path.

Input: \( LC2W, G \), weighted path \( P \) applying for extension and \( pt \) the prefix tree

Output: \( LC2W, pt \)

1. \( V_{dest}(P) := \{ \text{destination vertices of } \text{occur}_G(P) \} \)
2. if \( \exists v \in V_{dest}(P) \) such that \( v \) has no child then stop
3. \( Li(P) := \{ i \in I, \text{such that } \forall v \in V_{dest}(P), v \text{ has at least one child } u \text{ such that } i \in \lambda_2(u) \} \)
   // \( Li(P) \) is the list of items that are at least in one child of each destination vertex of \( P \)
4. \( DB|_P := \{ \text{transactions } T = \{ v \rightarrow u, i_1i_2 \ldots i_N \} \text{ s.t. } i_k \in Li(P) \cap \lambda_G(u) \} \)
5. \( LCI := \{ \text{closed itemsets } CI \text{ of } DB|_P \text{ such that } \sigma(CI) \geq minsup \} \)
   // Support of each \( CI \) is the number of edges
6. foreach \( CI \in LCI \) do
7. \( e_2w := P|_P^{-1} \rightarrow CI \)
8. \( P' := \text{Extend } P \text{ with } e_2w \)
9. if \( V_{dest}(P') \supseteq \{ \text{origin vertices of } \text{occur}_G(e_2w) \} \) then
   // \( P' \supseteq e_2w \), no need to extend \( e_2w \)
   Remove \( e_2w \) from \( LC2W \)
   // We append the extension to the prefix tree
11. Append child \( pt_{child} := \{ P', \sigma(P' \cup e_2w) \} \) to \( pt \)
12. ExtendPath \((G, LC2W, P', pt_{child})\)

Example. Consider the first a-DAG in Fig. 2 on which we apply the algorithm with \( minsup = 3 \).

We take the size-2 weighted path \( cd \rightarrow bi \), whose occurrences are \( 3 \rightarrow 6, 4 \rightarrow 7, \) and \( 5 \rightarrow 7 \), and we try to extend it downwards (its destination vertices are then 6 and 7), as shown in the right part of Fig. 5.

Candidate items for extension (those that belong to at least one child of each destination vertex) are \( f, g, h, \) and \( z \) but not \( e \) since Vertex 6 has no child containing \( e \).

Algo. 2 Lines 4,5 provides the closed itemset \( h \) supported by the edges \( 6 \rightarrow 8, 7 \rightarrow 8, \) and \( 7 \rightarrow 9 \) (but not itemset \( fghi \) because its support is 2, and thus lower than \( minsup \)).

We can now add itemset \( h \) to the path as shown in Fig. 5.
on which occurrences (not stored in the prefix trees) are represented below each size-2 weighted path. Crossed branches indicate that the extension violates the support constraint.

As the extension \( bi \rightarrow^3 h \) actually covers all occurrences of \( bi \rightarrow h \) in the a-DAG, this weighted path is not condensed. It is thus removed from \( LC2W \) (Algo. 2 Lines 9,10).

The two prefix trees given in Fig. 5 represent all the upward and downward extensions of

\[ cd \rightarrow^3 bi. \]

At the end, it generates the two condensed paths:

\[ ah \rightarrow^3 cd \rightarrow^3 bi \rightarrow^3 h, \]

\[ a \rightarrow^5 cd \rightarrow^3 bi \rightarrow^3 h. \]

### 5 Experiments

We implemented the algorithm in C++. Experiments were performed on a computer running Ubuntu 12.04 LTS and based on a Intel Core i5 @ 3.20GHz with 8GB main memory. Experiments have been run on a real ‘Dengue disease’ dataset and four synthetic datasets. The real dataset\(^3\) has 223 vertices, 984 edges, and each vertex has 10 or 11 attributes. The two synthetic datasets generated from V20K-E60K have 20000 vertices and 60000 edges (with 1-5 items and 5-10 items among 15) while the two datasets generated from V40K-E120K have 40000 vertices and 120000 edges (with 1-5 items and 5-10 items among 15). The number of attributes follows a normal distribution whose mean is the desired size (respectively 3 and 7,5).

Fig. 6 shows a performance comparison between mining the whole set of solutions and mining only its corresponding condensed set for the Dengue dataset. The baseline method consists in applying the same search strategy as in Algo. 2 without trying to choose closed itemsets in projected databases (every candidate itemset has a chance to extend the current weighted path). One can appreciate the compression rate, which is \( \sim 10 \) to up \( \sim 10^4 \) for the largest solution sets.

![Figure 6: Run times and solution sizes (condensed vs. non condensed) for the ‘Dengue’ dataset](image)

Fig. 7 shows the scalability of our approach on relatively large synthetic datasets. Top graphs (where each vertex has between 1 and 5 items among 15) illustrate the impact of the a-DAG size on the execution time. The bottom graphs (where each vertex has between 5 and 10 items among 15) show that itemsets size has a deeper impact on performance (\( \text{minsup} \) could not be as much lowered due to a lack of memory).

![Figure 7: Time and size of condensed sets for synthetic datasets.](image)

### 6 Conclusion

In this paper, we studied frequent pattern mining in a single attributed DAG. We introduced a new type of pattern and we proposed for the first time an exact condensed representation in the single-graph setting. Our experiments on real and synthetic datasets highlight the high compression rate of our condensed representation and the efficiency of our algorithm. A future work could be to study the influence of size-2 weighted paths on algorithm performances. Another perspective would be to extend this work to propose a condensed representation for frequent subgraph mining in a single-graph setting.

### References


