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Textbooks design and digital resources

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Abstract:

In this paper we report on the comparison of the design and conceptualization of two very different French lower secondary mathematics textbooks: one which was developed, as it is ‘traditionally’ done, by ‘experts’ (teacher educators and researchers); and one which was developed, innovatively, by teachers using a digital platform. These different designs and conceptualizations had repercussions on the content, structure, potential and intended use of the books, which we investigated on the basis of specially designed questionnaires to the two groups of textbook authors. Our results point to re-conceptualisations of the notions of ‘quality’ and ‘coherence’ of resources such as textbooks, taking into consideration teachers’ documentation work, in particular their – often collective – work with digital resources.

Keywords: Digital teaching resources; Mathematics teaching resources; Quality process; Textbooks authors intentions; Teacher communities; Textbooks design; Teachers’ documentation work; Textbooks use.

Introduction: digital means and evolutions of textbooks design

Our contribution relates to theme C: “Design and use of text-based resources”. As stated in the ICMI study 22 discussion document, “most teachers use textbooks and/or online packages of materials as their total or main source of tasks.” Moreover, the textbook content and structure influence teachers’ choices (Pepin, 2009; Haggarty & Pepin 2002), for example in terms of their choice of tasks or sequencing of the topic area; and both content and structure depend on the textbooks design. We investigate the textbook design and its developments brought about by digital means. Textbooks are now often complemented by digital materials: files to be projected during the lesson by the teacher; animated figures and exercises using different software and providing feedbacks to students, available on a CD or on a website. Linking to theme C we focus on the following question:

“How can or should new digital formats influence textbook design: e.g. use of podcasts, twitter, and other social media; implications for design and coherence of materials (either original digital design or transfer from print) if teachers are able to select tasks in varied orders?” (ICMI study 22, discussion document, p.19).

We consider that this question is a very complex one. It encompasses indeed several aspects of the evolutions resulting from digital means, in particular the following: digital means provide new opportunities for the structuring of the textbooks for their use by teachers. They also open up new possibilities for design and
further evolution. In previous works we evidenced that digital means foster collective teacher work (Gueudet & Trouche, 2012): teachers discuss by e-mail; or online teacher associations create resources to be shared by all, not only members. Like many other kinds of teaching resources, now the textbook can also be designed by groups of teachers: “bottom-up” designs (in contrast to a traditional “top-down” design) are now developing, in particular in countries where national policies allow it. This constitutes a major evolution, which we retain as a central focus of our study.

Considering these developments, what does the concept of “coherence of a textbook” mean? The discussion document mentions (p.17), “the conceptual coherence”. This can be understood as the correctness of the mathematical content; an alignment or ‘consistency’ with the official curriculum; a sequencing of the notions and properties introduced that permits to avoid gaps in the mathematical progression; a correct articulation between the text of the course and the associated exercises and problems, or indeed to emphasise ‘abstraction’ as an ‘umbrella concept’ for the coherence of the textbook. In the case of a collective design, coherence can also mean that the individual mathematical intentions of the individual authors, their epistemological stances, are well coordinated; and that the whole textbook corresponds to the same “mode of address” (defined as the positioning of the user induced by the material, by Remillard, 2012). Moreover, in terms of implications for learning the most important issue linked to coherence is certainly the ‘coherence’ of what the teacher produces drawing on the textbook. Alike Shield and Dole (to appear), we consider that analysing textbooks can only inform about their potential to assist in teaching and learning, since teachers interact with textbooks in various ways.

Investigating the potential of a resource leads to an exploration of the concept of quality. We consider that the intrinsic quality of a resource has to be distinguished from its adequacy with respect to institutional and users expectations (Trouche, Drijvers, Gueudet & Sacristan to appear). The intrinsic quality encompasses mathematical, didactical, and ergonomic (ease of use) aspects. It also depends on the mathematical topic considered. For example, Trgalová, Soury-Lavergne & Jahn (2011) assessed the quality of dynamic geometry resources, and they differentiated between nine dimensions of quality: mathematical content; technical aspects; instrumental aspects; added-value of dynamic geometry; didactical implementations; pedagogical implementations; the potential of the resource integration into a teaching process/sequence; ergonomic aspects (e.g. presentation and adaptability); and metadata (e.g. accuracy facilitating searchability).

Traditional textbook analysis (e.g. in TIMSS: Valverde et al. 2002) proposes three aspects: content (e.g. number, measurement, geometry); performance expectation (knowing, using routine procedures, problem solving, mathematical reasoning, and communicating); and perspective (attitudes, careers, participation, interest, and habits of mind). Most textbook analyses focus on tasks (e.g. exercises and working tasks), and many studies have analysed problem-types (Zhu & Fan 2006), problem solving procedures (Fan & Zhu 2007), procedural complexity (Vincent & Stacey 2008), cognitive demand (Jones & Tarr 2007) or concept treatment (Cai, Lo & Watanabe 2002).

However, how textbooks deal with depth of understanding, for example in terms of mathematical abstraction, is largely left untouched. Without going into deep philosophical discussions, textbooks are without doubt didactical materials, and as such can be seen as providing ‘tools and products of abstraction’ (e.g. in their tasks, representations, contexts, etc.). Textbooks use different registers of representation (Duval 2006), e.g. usual language symbols; figures; representations of technological
tools. This variety, and the need for conversion between different representations, can be associated with ‘depth of understanding’. Another way of analysing depth of understanding might be by conceptualising ‘understanding mathematics’ in terms of ‘making connections’ (Pepin 2008): e.g. connections to what the pupils already knows; to authentic situations; across mathematical topics; across other subjects. This is in line with the literature on ‘learning mathematics with understanding’ (e.g. Hiebert et al. 1997). They contend that students build mathematical understanding by ‘reflecting and communicating’, and tasks should allow and encourage these processes. This means that such tasks should have the following features:

“First, the tasks must allow the students to treat the situations as problematic, as something they need to think about rather than as a prescription they need to follow. Second, what is problematic about the task should be the mathematics rather than other aspects of the situation. Finally, in order for students to work seriously on the task, it must offer students the chance to use skills and knowledge they already possess. Tasks that fit these criteria are tasks that can leave behind something of mathematical value for students.” (p.18)

In summary, and considering the above, the notion of ‘quality’ is complex: it involves the notion of coherence and depth of understanding in textbook analysis, and this in the light of the evolutions brought on by digital means. We discuss this further in what follows, drawing on the comparison of two differently designed textbooks. The main research questions are: What are the differences between a textbook designed by a team of experts (researchers, teacher trainers etc.); and a textbook designed by a group of teachers using a digital platform? Which are the ‘consequences’ of these different designs, in particular in terms of coherence and quality of the textbooks produced?

In the next section, we present the study and subsequently the findings and results. The findings are presented under three headings: (1) the two contrasting teams of authors and their different conceptualisations of their respective textbooks; (2) authors’ choices concerning content and structure; and (3) authors’ intentions concerning the use of their textbooks.

The study

Our study took place in France, where no institutional control of textbooks exists. France has a National Curriculum, which is presented as a text detailing mathematical objectives and accompanied by detailed comments for teachers, in addition to a booklet giving a structured list of pupil competences. For our study we selected two very different textbooks on the basis of contrasting cases. Both textbooks were grade 6 books (first year of lower secondary school, collège, in France). Since no official statistical figures were available on the most commonly bought mathematics textbooks for this grade, we cannot claim that these textbooks were the most used by teachers. However, they were the most commonly used by the teachers we worked with, and most mathematics teachers in the region knew of them. The differences between the two books did not only concern their content or the material they offered: the differences were also linked to the authors’ teams and the design processes. Therefore, we designed a questionnaire for each of the authors’ teams. The questionnaires were designed by drawing on our knowledge of previous studies on textbook analysis (e.g. Pepin & Haggarty 2001) and on our knowledge of the books, including what teachers told us about their use. The questionnaires included questions on the textbook’s design mode; the authors’ perspectives on mathematics and their teaching; the design choices, on a general level and on specific
aspects of the textbook concerned. The results we present here draw on a cursory analysis of the textbooks, but more importantly on the analysis of the responses to the questionnaires. The analysis involved the identification of similarities and differences, category generation and saturation based on constant comparison as advocated by Glaser and Strauss (1967) and a procedure similar to that described by Woods (1996). In particular, we have chosen to focus on the differences in this paper.

1. Different authors, different conceptualizations

In terms of textbook design Helice 6° (Chesné, Le Yaouanc, Coulange, & Grapin, 2009) has been developed by a team of four “experts”: three teacher educators, two of them with a master degree in mathematics education; and a researcher in mathematics education. A grade 6 teacher (not considered an author) “tested” some of the tasks in his class. Asked about the way they evaluated the relevance of the content, the authors declared that they trusted “research results, or [their] training experience” – we call it ‘expert evaluation’. These experts were clearly aware of their expert position.

Sesamath 6° (Sesamath, 2009) has been developed by a large group of authors: approximately 57 lower secondary school mathematics teachers (being involved in producing both the paper version and the digital complements), their work being coordinated by members of the Sesamath association1: teachers who are involved in the design of online resources. The Sesamath association (Gueudet & Trouche, 2012, Sabar & Trouche, to appear) designed many other resources: online exercises; a digital geometry software; and a complete virtual environment, LaboMep2. LaboMep allows the co-ordination of various kinds of resources, from Sesamath or from other sources on the web, and their preparation for student use. The “Sesamath 6°” textbook was published under a free license; it can be downloaded from a website3, or used online (figure 1). A paper textbook exists, corresponding to the text files (which are available in .pdf and .odt). Other complementary files (e.g. dynamic geometry files, spreadsheets, slides, online exercises) can only be accessed using the website.

Figure 1. Sesamath online textbook for grade 6. The text on the screen background corresponds to the content of the .pdf, or paper version. The “complements” window opens when the mouse is placed on a selected activity.

In terms of evaluation of the content relevance, the teacher authors and other

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1 The name of the association itself, Sesamath, is interesting and linked to « Open sesame », the famous phrase from the Arabian Nights. The motto of the association is “mathematics for everybody”.
2 LaboMep- Laboratory for Mathematics in the Pocket
3 http://manuel.sesamath.net/
members of the association used it in their classrooms and observed and evaluated it, in particular in terms of their students’ involvement with particular features of the book. Referring to a distinction introduced in the field of computer supported collaborative learning (Dillenbourg et al. 1996), we consider that the collective work of the authors in the case of Helice was collaborative (the authors sharing each part and step of the work). In the case of Sesamath, it consider the authors’ collective work as co-operative: different tasks had been assigned to different authors.

Another important difference (we develop it in the discussion part), was that the content of the digital textbook continuously evolved, according to the experiments and contribution of the teacher users. In fact, Sesamath proposed a website, ‘Sesaprof’, which was open for teachers and comprised of a ‘discussion forum’ section (discussing the textbook). These online discussion has led to modifications of the textbook’s content: for example, when a large number of users asked for the solutions of the textbook’s exercises, these were added in the digital textbook.

Beyond these differences, both teams claimed that they were constrained by commercial publishing and user expectations. For example, the Helice team explained that they had wanted to write more guiding comments for the teacher, but had been asked by the publisher to limit these comments. For the Sesamath team, the grade 6 book was the last in a series: they had started with and accomplished textbooks for grades 7, 8 and 9 (in this order) due to a curriculum changes. Thus, the structure of these earlier textbooks gradually became a model familiar to users, and the authors were obliged to keep a similar structure for the grade 6 textbook.

2. Authors’ choices concerning content and structure

The analysis of the textbooks and the authors’ answers to the questionnaire showed many differences concerning the textbooks content. We illustrate these statements with examples from the mathematical topic of ‘area’.

Organisation of the content

Concerning the textbook’s global structure and the organisation of the mathematical content, Sesamath provided a “classical” organisation: chapters were organised according to the headings of the official National curriculum. As an example, the topic of ‘area’ was in the section entitled “Area and Perimeter”, which was the second section of the chapter entitled “Quantities and Measures”.

Helice had a very special, spiral organisation, hence the title “Helice” (meaning “Helix”). The book was not structured in chapters, but in “units”, working with different ‘thematical’ lessons, and a ‘unit’ finished with problems and exercises linking the different notions learnt. The authors of Helice presented this (spiral organisation) as their central and original structure. Indeed, in France Helice was the only textbook retaining this composition. The authors specified that the intention had been that the learner revisited and deepened the same notion at different stages - a spiral curriculum. At every stage the notions were associated with different representations; links between different chapters were frequently made; and differentiation in terms of pupil learning (e.g. pace) was taken into account. The topic ‘area’ appeared twice in the table of contents: in the unit entitled “Distance and areas” (Unit 4- in a lesson called “Area: comparison and sharing”) and in the unit entitled “Division and computation of area” (Unity 5- in a lesson called “division and computation of area”). Moreover, the area of a circle was in fact presented in Unit 7, as an activity demonstrating that the area of a circle and the square of its radius are
We claim that this difference in terms of structure was influenced by the different design modes and author teams. Following a complex and coherent structure, such as the one retained by Helice, was only possible for a ‘steady’ author team, and likely to be very difficult for a large and ‘variable’ author group, such as the Sesamath group. A large ‘collective’ process, like Sesamath, required splitting the content into different parts, which then were designed by different authors who did not necessarily communicate. This splitting of tasks, we argue, has also influenced the coherence of the textbook. Bringing together the work of more than fifty authors and achieving a coherent didactical structure would require an enormous coordination effort, more than the one organised by the Sesamath team.

**Different methods, or a single expert method**

Helice authors stated that it was important for them to propose different representations and a rich vocabulary in their book. They also favoured exercises with different possible solutions.

In Sesamath, our analysis showed that some additional research activities suggested and fostered the search for several solutions. However, we contend that most of the exercises led to a single and final solution. In particular, the worked examples were all one-solution exercises- the authors called them ‘expert solution’ in the questionnaire (figure 2).

![Figure 2. Extract of Sesamath 6. Method 2: evaluating an area.](image)

*Figure 2. Extract of Sesamath 6. Method 2: evaluating an area.*

*Example 1. Determine the area of this figure by choosing the yellow triangle as the unit, then calculating the area of this rhombus. To find the area of the figure, you have to count the number of area units of the figure. The purple figure is made of 9 triangles. Its area is thus the area of 9 yellow triangles. The rhombus is made of two yellow triangles. The area of the purple figure, in number of rhombusses, is thus twice as smaller. Hence the area of the figure equals 4.5 rhombuses.*

**Primary-secondary link**

Another important aspect, given that the textbooks were for grade 6, was whether the primary-secondary link was explicitly addressed. Some of the Helice authors were teacher educators, both for secondary and primary school level. Thus they had considerable knowledge about the content taught at primary and secondary school. They identified several crucial cognitive and didactical changes in terms of transition from primary to secondary school mathematics. Concerning ‘area’, one important choice, according to them, was to start with activities on the comparison of areas of two figures, without using any kind of measures. The activity in figure 3 illustrates this choice.
Activity 2: Cutting and arranging. The objective of the activity is to demonstrate that all these figures have the same area as the square, by cutting and adjusting to obtain a superposition with the square.

The official curriculum mentioned “geometrical comparison of areas” as a teaching objective. This aspect was particularly developed in Helice; and this was a deliberate choice of the authors, in order to provide a better link to primary school notions, where comparing areas without using measure was a common task. Sesamath authors declared that they had only very limited knowledge of primary school mathematics curricular aspects, when they started writing the grade 6 book. Concerning ‘area’, the textbook focused on area computations, whereas the official curriculum also stipulated “geometric comparison of areas”. This geometric comparison, without measures, was not evident in the Sesamath textbook. With a better knowledge of primary school, where such tasks were frequent, the authors may have inserted it. However, the Sesamath authors paid attention to one ‘classical’ difficulty: the potential problem to distinguish between area and perimeter of a figure. This difficulty was known to be important for primary school, and also for grade 6 students, and as teachers of grade 6 students the textbook’s authors knew about it.

Writing the book raised the authors’ awareness of transition questions (e.g. the importance of mental arithmetic for grade 6.). The paper version of Sesamath did not reflect this, but in the associated online resources, in particular online exercises, the authors attended to this aspect of primary-secondary transition. Whilst the paper book remained the same since 2009, the authors claimed that they had attended to particular ‘shortcomings’ and that the online complements had considerably developed since (e.g. in LaboMep many online exercises of mental arithmetic for grade 6 had been added).

3. Authors’ intentions concerning the use of their textbooks

In the questionnaire, the Helice designers adopted a general stance about teachers’ adaptations of the book to their specific contexts. They said that “it [was] impossible to anticipate all possible adaptations” (in terms of contexts) and that they did not regard it as their “responsibility” to attend to these. This view was reflected in several statements throughout the questionnaire, e.g. the following: “the gap between what is planned and what happens in class is large- the gap between the authors’ intentions and the teachers’ use is even larger”. They also declared that they would anticipate that teachers would combine Helice with the use of other textbooks. However, the spiral progression made such a practice difficult and the very complex spiral structure was clearly an obstacle for the adaptation, or combination of several textbooks.

For Sesamath, the possibility to adapt the content was an important issue. The authors conceptualised the book, from its inception, as a digital textbook, each chapter file in .odt format. This offered opportunities for teachers to modify the texts of the
exercises, and of the lessons. The book also offered a large amount of exercises, with the intention of leaving their choice to teachers. According to the Sesamath authors, the digital textbook was a “collection of bricks”. Moreover, they stated that it should be thought of as belonging to a more general set of different kinds of resources available in the virtual environment LaboMep. Sesamath authors considered that it was the teachers’ responsibility to ‘build coherent lessons’ and a ‘coherent progression’. In their view this was made possible by providing a *large range of resources* to choose from, the textbook being only one of these. In addition, they wanted to provide ‘efficient tools’, such as LaboMep, for teachers to build their own teaching from these ‘initial bricks’.

**Discussion**

Going back to our initial questions (“What are the differences between a textbook designed by a team of experts (researchers, teacher trainers etc.) and a textbook designed by a group of teachers using a digital platform? Which are the ‘consequences’ of these different designs, in particular in terms of coherence and quality of the textbooks produced?"), we answer these by comparing the two contrasting textbook cases (Helice and Sesamath).

The investigation of the two textbooks showed that there are **differences** concerning essentially six levels (at least):

- the modes of design: more *collaborative* in the first case, more *co-operative* in the second;
- the nature of the structure: the first book is a *single whole*, with an organised structure (organised by the team of experts); the second is an *atomistic system* that can be arranged differently by different users;
- the organisation of the content: more *didactically original*, linked to the didactical choices of a ‘homogeneous’ team in the first case; more *aligned with* the institutional instructions in the second;
- the content: more *open* to a variety of ways for solving a given problem in the first case; more *driven by an expert solution* in the second;
- the integration into the whole grades 1-9 mathematics curriculum: links with primary school more taken into account in the first case than in the second; and
- the links to the users: the textbook provided as a *final product* given to the teachers in the first case; and as a *proposal* to be enriched by teachers’ contributions in the second.

For Helice, the **coherence** is insured by the authors’ didactical expertise, i.e. the mastering of the concepts at stake and of the potential difficulties and misconceptions for learning. It could be said that it is a *transcending* coherence. For Sesamath, the coherence is insured by the link with the curriculum and the institutional prescriptions; by teacher evaluation in class; and by the discussions among authors faced with the different contributions. Sabra and Trouche (to appear) describe, for example, the discussion in another Sesamath author team (for a grade 10 book) in terms of reaching a coherence and consistency between the introduction of equations (in one chapter), and the introduction of function (in another chapter). We argue that this is a **collective and institutional** coherence.

Helice was, we claim, of high didactical **quality**. It offered many rich tasks, organized according to a carefully considered and complex structure. It took into account central aspects of the primary-secondary transition. The Sesamath textbook
appeared to be, in its initial version, of a lower intrinsic quality: it offered less problems and less rich tasks. In terms of structure it simply followed the structure of the official National Curriculum.

However, the ‘digital additions’ and possibilities of Sesamath prompted us to re-consider the notions of ‘quality’ and ‘coherence’. Helice remained the same, in both its paper and pdf versions- we call this static quality. In contrast, the online version of Sesamath had already been modified several times, to take account of ‘user comments’, i.e. users’ experiences and needs. The digital means offered possibilities for modifications, and these were integrated by Sesamath in the process of re-design. The association perceived this as a necessity for meeting users'/teachers’ needs in order to insure the quality of the textbook- we call this dynamic quality. Both Helice and Sesamath authors recognized that teachers would select and adapt elements of the textbook for their teaching. However, only Sesamath supported these adaptations and drew on user contributions.

We argue that this was a major development linked to digital means: the involvement of a large group of teachers in the design of resources, which in turn continuously evolved. This also deepens our knowledge of already established phenomena: even teachers who were not involved in the design of textbooks could be considered as ‘designers’ of their own teaching materials, as teachers selected resources, combined them, set them up in class- a process that Gueudet & Trouche (2009) called teacher documentation work. This leads us to consider that the question raised in the discussion document: “[Which] implications for design and coherence of materials […], if teachers are able to select tasks in varied orders?” does not sufficiently recognize the complex link between ‘design’ and ‘coherence’, and the evolving role of teachers. Exploiting the potential of digital means for the design of teaching resources, including textbooks, requires the acknowledgement of this new role.

Drawing on the results presented here, we argue that textbooks can be considered as lived resource (Gueudet, Pepin & Trouche, 2012), or even ‘living resources’, as they get continuously enriched and renewed by teachers' experience. This new conceptualization of the textbook is likely to be associated with new forms of design, for example, in terms of reflection on meta-design (Fischer and Ostwald 2005): resources which support the design by teachers. It can also lead to favour teams of textbooks designers who combine and involve different ‘experts’ and expertise (e.g. teachers and other ‘experts’ such as researchers). From the literature (e.g. Kieran, Tanguay & Solares 2012), it is clear that teacher documentation work and professional knowledge are intrinsically intertwined, one leading to the evolution of the other (e.g. Gueudet & Trouche 2012; Pepin 2012). This has implications, also in terms of policy, for teacher professional development (of ‘users’ and of ‘designers’); mathematical task design and digital means/possibilities; and the re-conceptualization of the quality of resources in mathematics education.

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