On Spectrum Assignment in Elastic Optical Tree-Networks

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Pour répondre à la demande croissante du trafic d'Internet, une nouvelle génération de réseaux optiques est en cours de développement ; les réseaux optiques élastiques (EONs). La technologie EON permet d'utiliser le spectre optique de manière efficace et flexible. Cette flexibilité promet de résoudre les difficultés liées à la croissance et l'hétérogénéité du trafic. Toutefois, elle rend le problème d'allocation de ressources plus complexe. Dans ce papier, nous traitons le problème d'allocation de spectre dans les réseaux optiques élastiques en arbre. Dans ce type de réseau, bien que le routage soit fixé, l'allocation de spectre est NP-difficile. Nous présentons des résultats de difficulté et d'approximation pour des cas spéciaux où le réseau est une étoile ou un arbre binaire.

Keywords: Approximation algorithms, optical networks, routing and spectrum assignment, interval coloring

1 Introduction

Elastic Optical Networks (EONs) [GJLY12] have been proposed recently as a potential candidate to replace the traditional Wavelength Division Multiplexing (WDM) networks. In EONs, new technologies such as optical OFDM, adaptive modulation techniques, bandwidth variable transponders, and flexible spectrum selective switches are used to ensure an efficient utilization of the optical resources and to enable a fine-granularity grid as opposed to the WDM fixed-grid. In fact, the optical spectrum in EONs, is subdivided into small channels, called slots, which are finer than the 50GHz wavelengths used under WDM. With these slots, small bitrates are not over-provisioned and big bitrates can be satisfied as single entities, under the constraint of contiguity. This constraint dictates that the slots used by a request should be consecutive. This results in an efficient use of the spectrum but it also makes the problems of resource allocation in EONs more difficult than their counterparts in WDM.

The key resource allocation problem in Elastic Optical Networks is referred to as Routing and Spectrum Assignment (RSA). For static RSA, the input is a set of traffic requests and the objective is to allocate to each request, a path in the optical network and an interval of spectrum slots along that path, minimizing the utilized spectrum. The spectrum allocated to a demand has to be contiguous (contiguity constraint), it has to be the same over all links of the routing path (continuity constraint) and demands sharing a link should be assigned disjoint spectrum intervals (non-overlapping constraint). If the routing is fixed, i.e., a path is predefined for each request, RSA reduces to the problem of Spectrum Assignment (SA).

Related work. Spectrum Assignment is a generalization of the well studied problem of Wavelength Assignment (WA). Since WA has been proved NP-complete in [CGK92], SA is also NP-complete. In fact, SA remains NP-hard even in networks where WA is tractable, particularly in path networks. When the network is a path, SA is equivalent to the Dynamic Storage Allocation (DSA). Hence, as for DSA [BEJ+07], SA is strongly NP-complete even if the demand of each request is at most 2 slots. Recent papers have taken advantage of the relation between SA and other problems to draw some hardness and approximation results for restricted cases. In [TBL+14], SA is studied from a scheduling perspective. It is proved that

† Due to lack of space, proofs have been sketched or omitted. Full proofs are available in [Moa15]

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SA is NP-hard in path networks with as much as 4 links and unidirectional rings with 3 links. Afterwards, approximation algorithms of scheduling are used to find approximation for SA in path networks with few links. In [SZDS13], SA is studied from an interval coloring point of view. Algorithms for interval coloring are used to provide an $2+\varepsilon$-approximation algorithm for SA in path networks, $4+\varepsilon$-approximation in ring networks and $log(k)$-approximation for binary trees where k is the number of requests. To the best of our knowledge, no other paper presents results on SA in tree networks.

Contribution. In this paper, we study the spectrum assignment problem in trees. We focus on special cases where the tree is a star or where the demands of the requests are bounded by a constant and the tree is binary. By studying these special cases, we hope to gain more insight into the general problem and design a constant-factor approximation algorithm or prove that such algorithm does not exist. In our study, we follow the tendency and use relation to other problems to draw new results. We prove that SA is NP-hard in undirected stars of 3 links and in directed stars of 4 links, and show that it can be approximated in stars within a factor of 4 (Section 3). Afterwards, we use techniques used for the DSA problem to find constant-factor approximation algorithms for SA on binary trees when the demands are bounded (Section 4).

2 Definitions and notations

Given a graph $G=(V,E)$ modeling an optical network, and a set of requests $\mathcal{R}$ where each $r\in\mathcal{R}$ has a path p_r and a spectrum demand d_r (number of slots), a spectrum assignment of $(G,\mathcal{R})$ is a mapping f from $\mathcal{R}$ to $\mathbb{N}^*$ such that for every two requests $r,r'\in\mathcal{R}$, if $p_r\cap p_{r'}\neq\emptyset$ then $[f(r),f(r)+d_r-1]\cap[f(r'),f(r')+d_{r'}-1]=\emptyset$. The ***span of a spectrum assignment*** f, denoted $s(f)$, is the smallest integer b such that for each request $r\in\mathcal{R}$, $f(r)+d_r-1\leq b$. The ***span of an instance*** $(G,\mathcal{R})$, denoted by $s(G,\mathcal{R})$ is the minimum of spans over all possible spectrum assignments. We formulate the spectrum assignment problem as follows :

Problem 1 (Spectrum Assignment (SA)) *Given an instance $(G,\mathcal{R})$, compute $s(G,\mathcal{R})$.*

For an instance of SA, the ***load of an edge*** e is the sum of the demands of the requests using e and the ***load of an instance*** is the maximum load over all its edges. The ***greedy algorithm*** for SA is an algorithm which assigns spectrum to requests in a given order $r_1,\ldots,r_n$; a request r_i is assigned the smallest positive integer $g(r)$ such that $[g(r_i),g(r_i)+d_i-1]\cap[g(r_j),g(r_j)+d_j-1]=\emptyset$ for each r_j in $\{r_1,\ldots,r_{i-1}\}$ if $p_{r_j}\cap p_{r_i}\neq\emptyset$.

3 Spectrum Assignment in stars

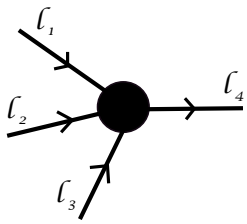
A star is a tree-network with at most one node of degree at least 2. The problem of wavelength assignment (WA) is NP-complete in undirected stars but polynomial in directed stars [Bea00]. The polynomiality of WA in directed stars was useful because optical networks are symmetrically directed and because it helped in the design of constant-factor approximation algorithms for WA in directed trees [Bea00]. Such algorithms cannot be extended to SA since we prove in this section that SA is not only NP-complete in undirected stars but also in directed stars with 4 links. On the positive side, we prove the existence of a 4-approximation algorithm and show that there are better approximation algorithms for stars with few links.

Theorem 1 *The problem of Spectrum Assignment is strongly NP-complete in undirected stars with 3 links.*

Sketch of proof. It is shown in [TBL+14] that the SA problem is NP-complete in a 3-link unidirectional ring. Let us consider an instance of SA in a 3-link ring $C=(l_1,l_2,l_3)$ with a request set $\mathcal{R}$. We build a star S with three edges e_1, e_2 and e_3, and a set of requests $\mathcal{R}'$ defined as follows. For each request $r\in\mathcal{R}$ using at most 2 links, we create a request r' in $\mathcal{R}'$ such that if the path of r is $p_r=l_i$, $i\in\{1,2,3\}$, then the path of r' is $p_{r'}=e_i$, and if $p_r=l_il_j$, then $p_{r'}=e_ie_j$. Solving SA in $(C,\mathcal{R})$ is equivalent to solving SA in $(S,\mathcal{R}')$. □

Theorem 2 *The problem of Spectrum Assignment is weakly NP-complete in directed stars with 4 links.*

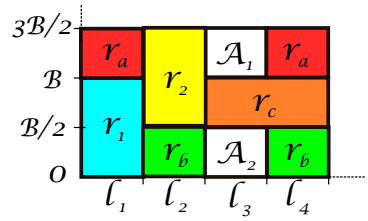
Sketch of proof. The proof is by reduction from the 2-PARTITION problem. Given an instance of the 2-PARTITION problem with a set of k integers $A=\{a_1,a_2,\ldots,a_k\}$ such that $B=\sum_{j=1}^{k}a_j$, we create an instance of spectrum assignment in a 4-links directed star network S (Figure 1a) and a set of requests $\mathcal{R}$. The set of requests $\mathcal{R}$ consists of the requests R in Figure 1b plus a request of size a_i for every integer a_i in the set A, all using link l_3. We prove that finding a spectrum assignment for $(S,\mathcal{R})$ with span $\frac{3}{2}B$ is



(a) Star S

Request	Demand	Path
r_a	$B/2$	$l_1 l_4$
r_b	$B/2$	$l_2 l_4$
r_c	$B/2$	$l_3 l_4$
r_1	B	l_1
r_2	B	l_2

(b) Requests R



(c) Spectrum assignment

FIGURE 1

equivalent to finding a partition of A into two sets A_1 and A_2 such that $\sum_{a_j \in A_1} a_j = \sum_{a_j \in A_2} a_j = \frac{B}{2}$. In fact, if there is a partition of A into A_1 and A_2 such that $\sum_{a_j \in A_1} a_j = \sum_{a_j \in A_2} a_j = \frac{B}{2}$, then we can assign spectrum as shown in Figure 1c. Now let us suppose there is a spectrum assignment for $(S, \mathcal{R})$ with span $\frac{3}{2}B$. There are two possible symmetric assignments to the requests on links l_1 and l_2. We suppose we assign to r_1, r_a, r_2 and r_b spectrum intervals $[0,B]$, $[B, \frac{3}{2}B]$, $[\frac{B}{2}, \frac{3}{2}B]$, and $[0, \frac{B}{2}]$, respectively (the analysis is similar for the other assignment). This assignment forces request r_c to use the interval $[\frac{B}{2}, B]$ and the other requests on link l_3 will have to be partitioned into two sets of the same weight $\frac{B}{2}$. □

Theorem 3 *The problem of Spectrum Assignment in directed stars with at most 3 links or exactly 2 ingoing links and 2 outgoing links can be solved in polynomial time.*

Sketch of proof. In any of these cases, the span is equal to the load and the greedy algorithm with specific orders can achieve the optimal span. □

Theorem 4 *There is a 4-approximation algorithm for the problem of Spectrum Assignment in stars (directed and undirected). Furthermore, there are approximation algorithms with ratios $\frac{7}{6}$ and 1.5 when the star has 3 and 4 links, respectively.*

The greedy algorithm in a specific order gives a 4-approximation as we prove in [Moa15]. Complete proofs of all the theorems in the paper can be found in [Moa15].

4 Spectrum Assignment with bounded demands in binary trees

In this section, we present constant-factor approximation results for SA in trees when the demands are bounded by a constant. It is important to recall here that routing on trees is unique and that even if the network is a path and the demands are bounded by 2, SA is still NP-complete. Let us also note, that the SA problem in binary tree is equivalent to the problem of Interval Coloring in chordal graphs [SZDS13]. This equivalence together with an approximation algorithm proposed for Interval Coloring in chordal graphs in [PPR05] allow to prove the following theorem.

Theorem 5 *There exists an approximation algorithm for the problem of spectrum assignment in binary trees with ratio $2\log_2(D)$ where D is the maximum demand.*

We aim at finding better approximations. For this purpose, we use techniques introduced in [LLQ04] to approximate DSA. Results in [LLQ04] can extend directly to SA in path networks giving approximation algorithms with factors $\frac{4}{3}$ and 1.7 when the spectrum demands are bounded with 2 and 3, respectively. In what follows we use the same techniques to design constant-factor approximations for SA in binary trees when the spectrum demand is bounded by 6.

Theorem 6 *There are approximation algorithms for the problem of Spectrum Assignment in binary tree networks of factors $\frac{3}{2}$, $\frac{19}{10}$, $\frac{67}{30}$, $\frac{659}{240}$ and $\frac{603}{200}$ when the maximum request demand is bounded by 2,3,4,5 and 6, respectively.*

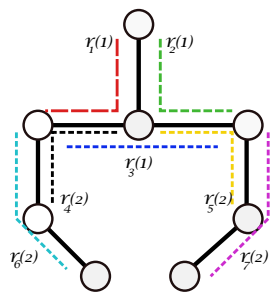


FIGURE 2: $L(4,2) \neq 4$. If we only have 4 slots, then due to conflict with r_7, r_5 can only use slots $[1,2]$ or $[3,4]$. If r_5 uses $[1,2]$, then r_3 and r_2 will use slots 3 and 4 and r_1 will be then forced to use slot 1 or 2. Due to conflict with r_6, r_4 can only use slots $[1,2]$ or $[3,4]$. This is impossible since one of the slots $[1,2]$ is used by r_1 and one of the slots $[3,4]$ is used by r_3 and r_1 and r_3 are conflicting with r_4.

Sketch of proof. The load on an edge e, with respect to a subset U of requests is the sum of the demands of the requests of U using e and the load of the subset U is the maximum load over all the edges. Let $L(d,h)$ denote the smallest W such that for each instance S of SA with load d and maximum demand h, there is a spectrum assignment $f(S)$ with $s(f) \leq W$ (if such W exists).

Key idea. The idea of the algorithms is to first compute $L(d,h)$ for small values of d and then use the results to solve the general cases as follows. In an instance of load D and maximum demand h, we partition the requests into multi-level blocks (subsets) with small densities. Namely, n_i level-i blocks of load d_i and minimum demand i, $i \in \{1,\ldots,h\}$. Afterwards, we use the algorithm used to compute $L(d_i,h)$ to allocate spectrum to each level-i block. The number of spectrum slots used at the end will be equal to $\sum_{i=1,\ldots,h} n_i L(d_i,h)$. Properties of the edge intersection graph of paths in a binary tree are used to compute $L(d,h)$ for small values of d and to assign requests to blocks in an optimal way [Moa15].

Example. For $h=2$, we prove that $L(2,2)=2$ and that $L(4,2)=5$ (Figure 2 illustrates why $L(4,2) \neq 4$). Afterwards, taking an instance $(T,\mathcal{R})$ of SA with load D and maximum demand h, we partition the requests into $n_1 = \lceil \frac{D}{4} \rceil$ level-1 blocks of load at most $d_1 = 4$ and $n_2 = \lceil \frac{D}{8} \rceil$ level-2 blocks of load at most $d_2 = 2$. By assigning spectrum to each block separately, we find a spectrum assignment f for $(T,\mathcal{R})$ with span $s(f) \leq n_1 L(4,2) + n_2 L(2,2) \leq \frac{3}{2}D + 7$ and since the load of an instance is always smaller than its span, the approximation follows.

□

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